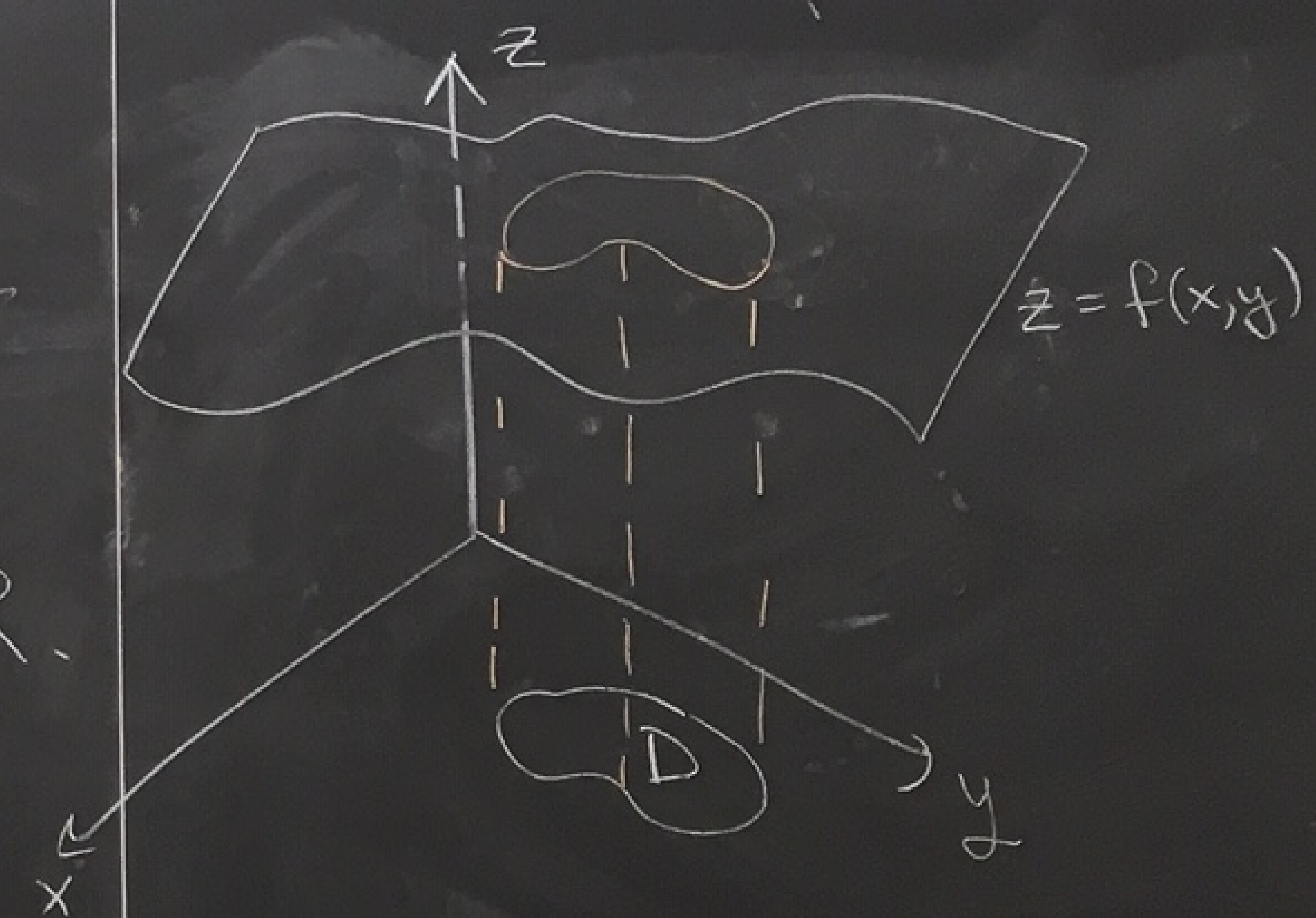
Tues 10/15

13.2 — Double integrals over General regions

We know how to integrate a function $f(x,y)$ over a rectangle R.

What if D is some bounded region in the xy-plane?

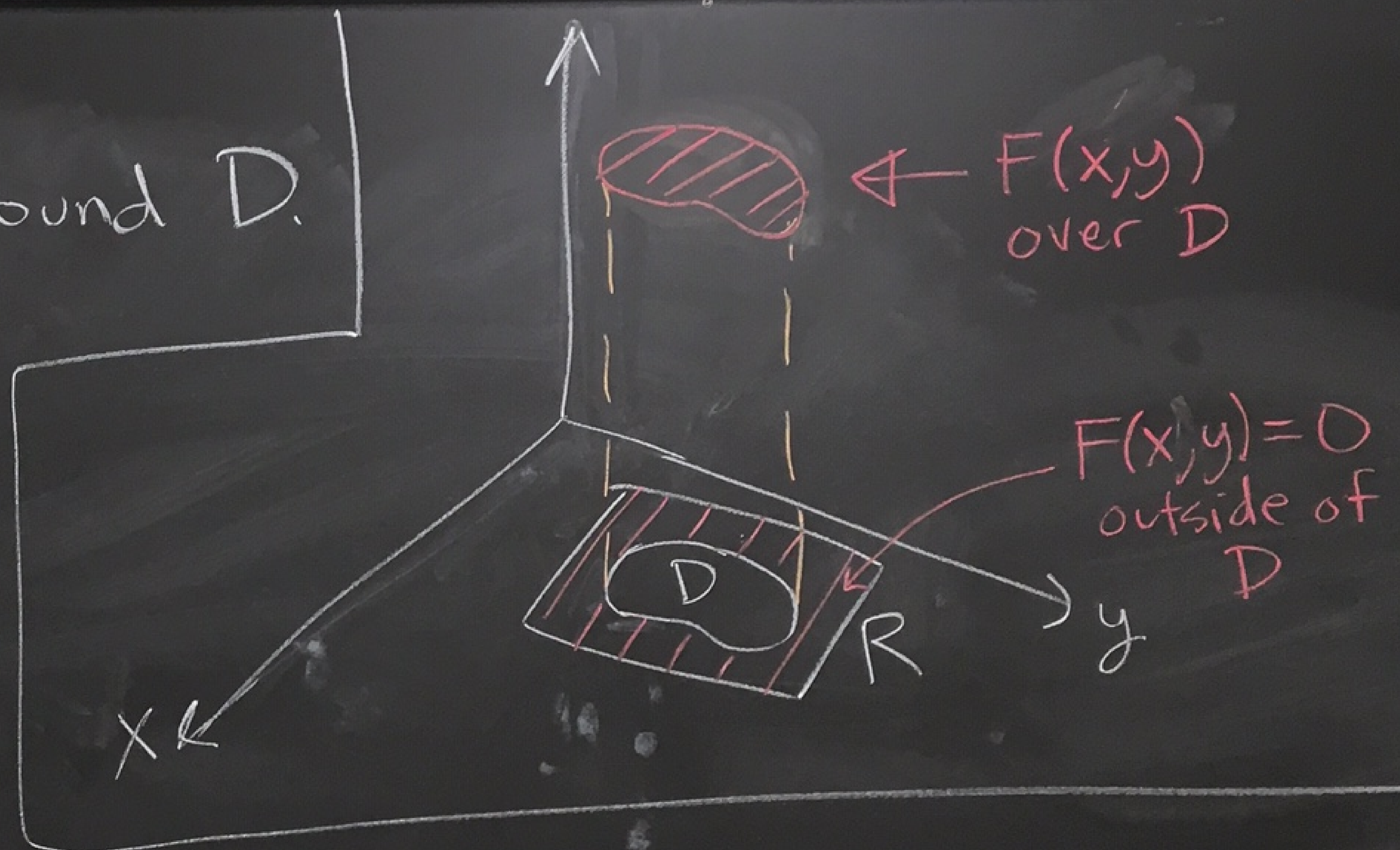


To integrate f over D.

① Put a rectangle R around D.

② Define

$$F(x,y) = \begin{cases} f(x,y), & \text{when } (x,y) \text{ is in } D \\ 0, & \text{when } (x,y) \text{ is not in } D \text{ but is in } R. \end{cases}$$

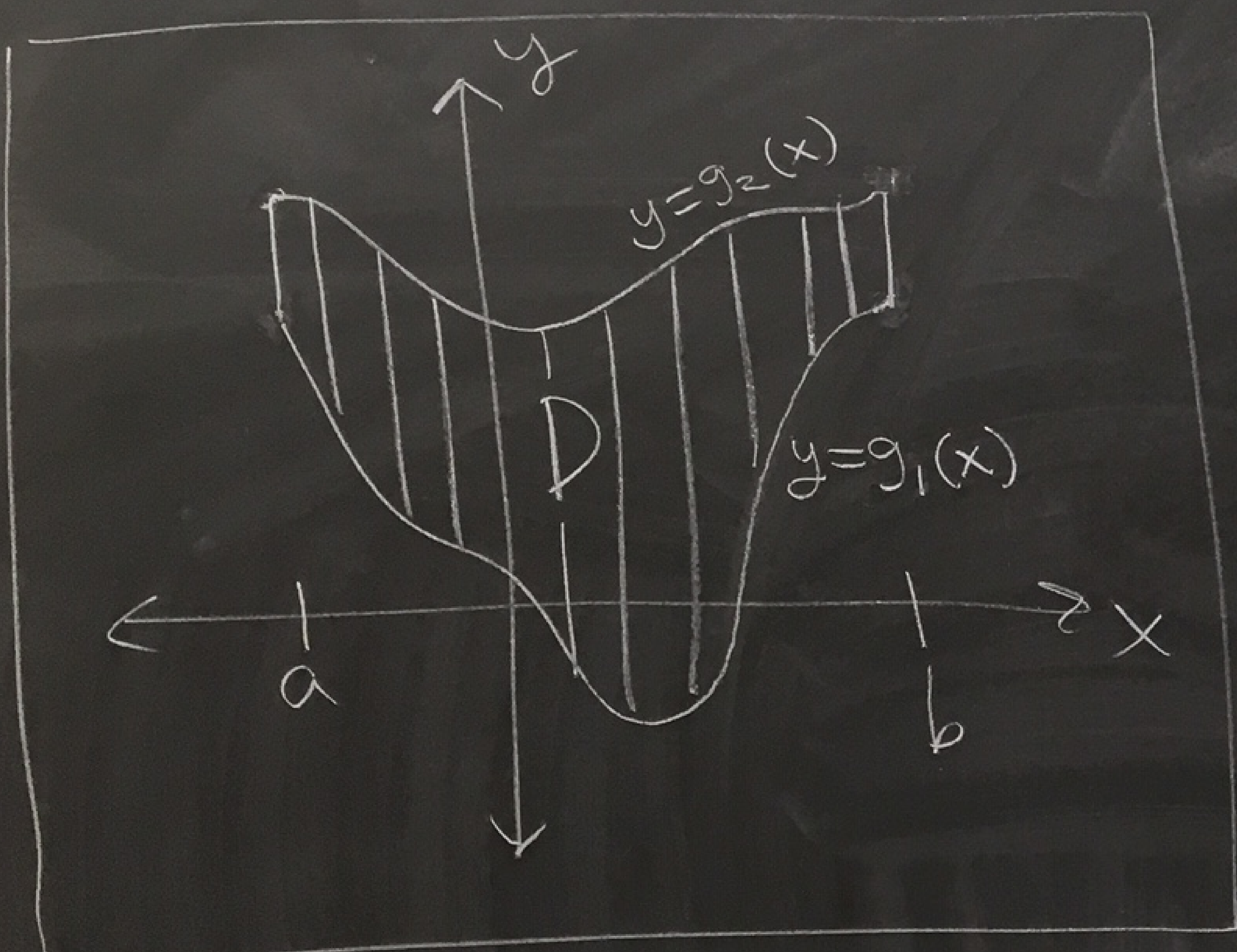


Define

$$\iint_D f(x,y)\,dA = \underbrace{\iint_R F(x,y)\,dA}_{\text{we defined this in 13.1}} \quad \text{- if it exists.}$$

$$D = \{(x,y) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$$

where $g_1(x)$ and $g_2(x)$ are continuous for $a \le x \le b$.

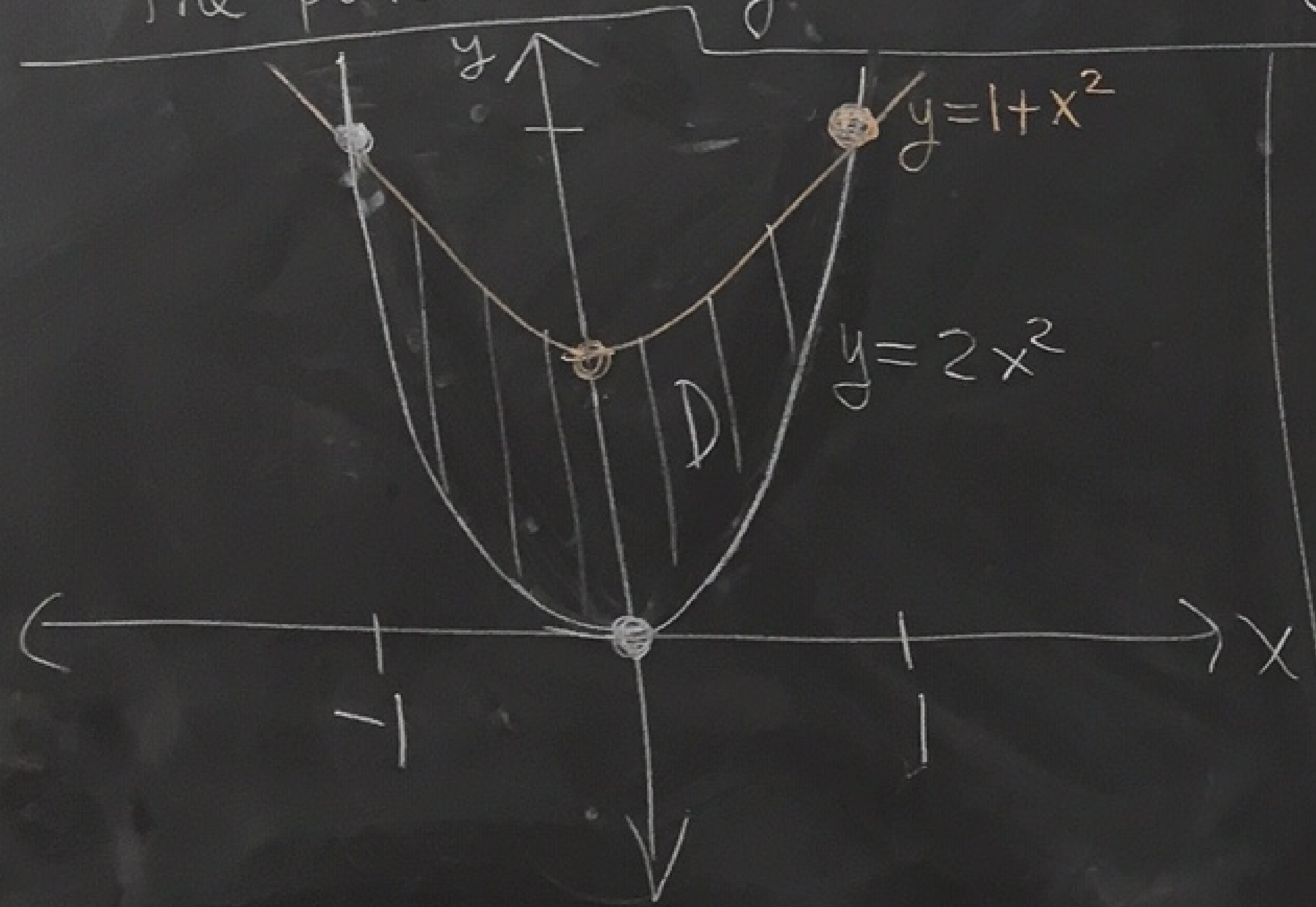


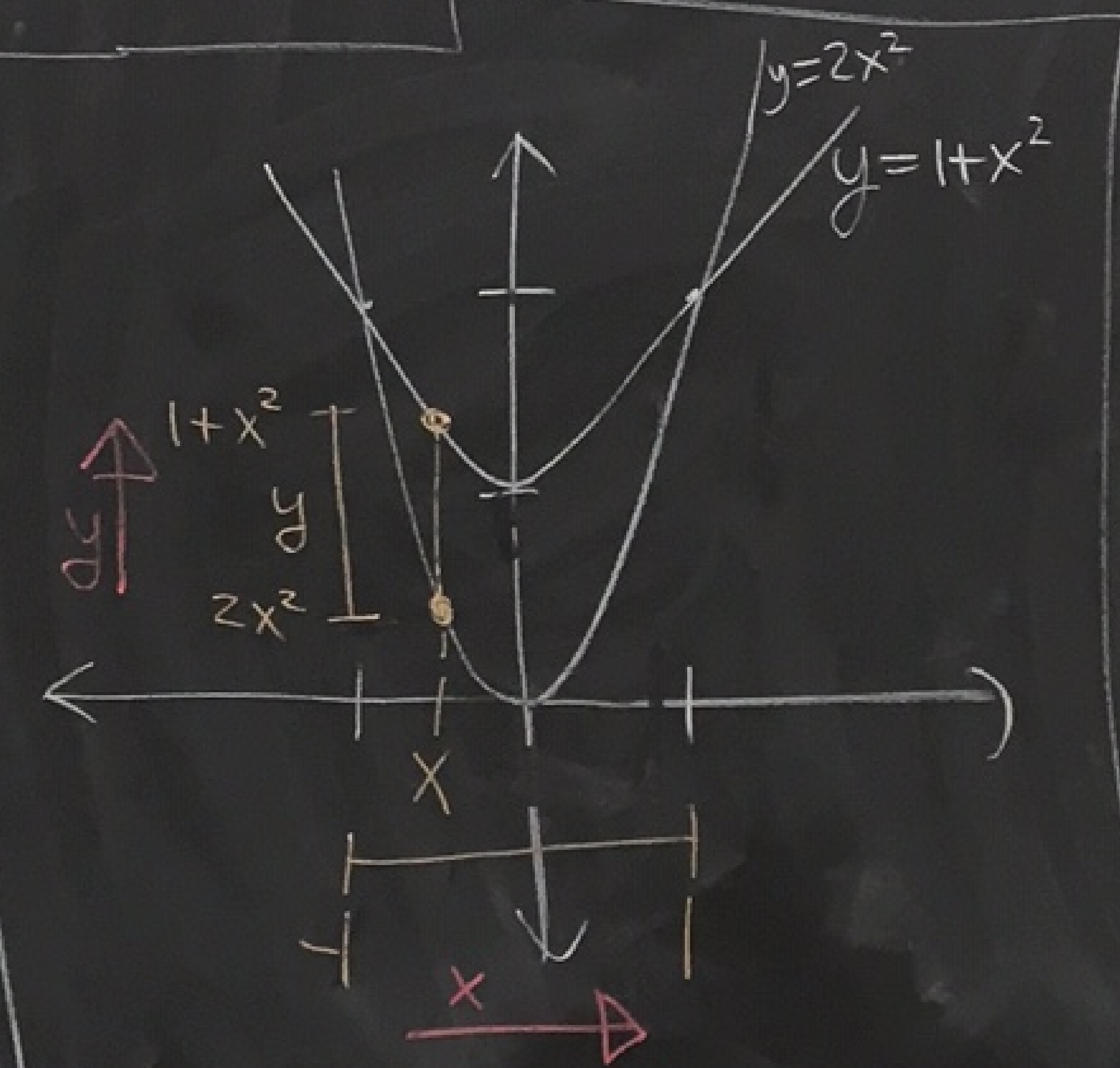
$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

dx

Ex: Evaluate $\iint_D (x+2y)\,dA$

D is the region bounded by the parabolas $y=2x^2$ and $y=1+x^2$





$$\iint_D (x+2y)\,dA = \int_{-1}^{1}\left[\int_{2x^2}^{1+x^2}(x+2y)\,dy\right]dx$$

$$= \int_{-1}^{1}\left[xy+2\frac{y^2}{2}\right]_{y=2x^2}^{1+x^2}dx$$

$$= \int_{-1}^{1}\left[\underbrace{\left(x(1+x^2)+(1+x^2)^2\right)}_{y=1+x^2} - \underbrace{\left(x(2x^2)+(2x^2)^2\right)}_{y=2x^2}\right]dx$$

$$= \int_{-1}^{1}\left(x+x^3+1+2x^2+x^4\right)-\left(2x^3+4x^4\right)dx$$

$$= \int_{-1}^{1} \left(-3x^4 - x^3 + 2x^2 + x + 1\right) dx$$

$$= -3\frac{x^5}{5} - \frac{x^4}{4} + 2\frac{x^3}{3} + \frac{x^2}{2} + x \Bigg|_{-1}^{1}$$

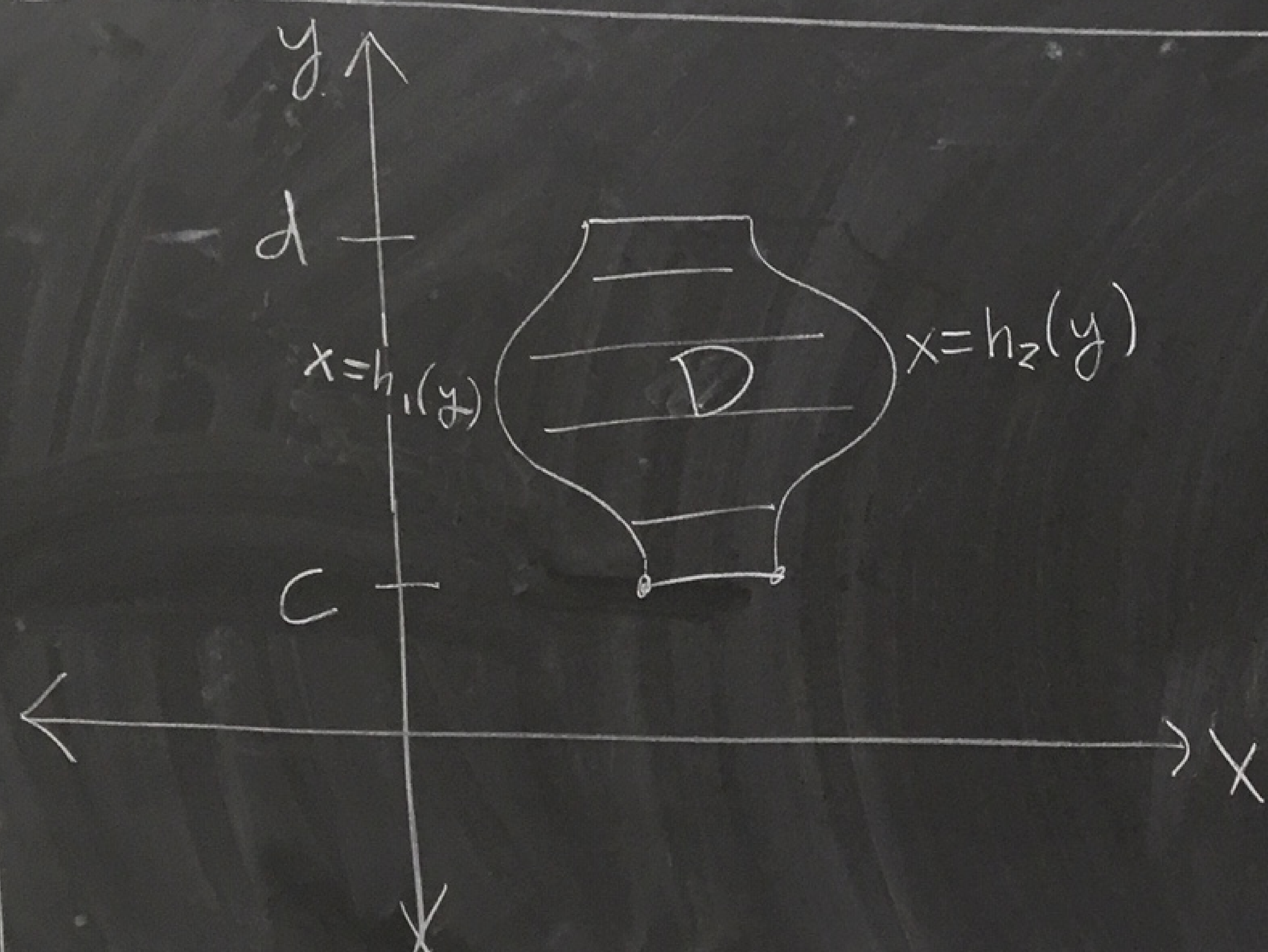
$$= \left[\left(\frac{-3}{5}(1)^5 - \frac{(1)^4}{4} + \frac{2}{3}(1)^3 + \frac{(1)^2}{2} + (1)\right) - \left(\frac{-3}{5}(-1)^5 - \frac{(-1)^4}{4} + \frac{2}{3}(-1)^3 + \frac{(-1)^2}{2} + (-1)\right)\right]$$

cancel

$$= \left[\frac{-3}{5} - \frac{3}{5} + \frac{2}{3} + \frac{2}{3} + 1 + 1\right] = \frac{-6}{5} + \frac{4}{3} + 2 = \frac{-18+20+30}{15} = \frac{32}{15}$$

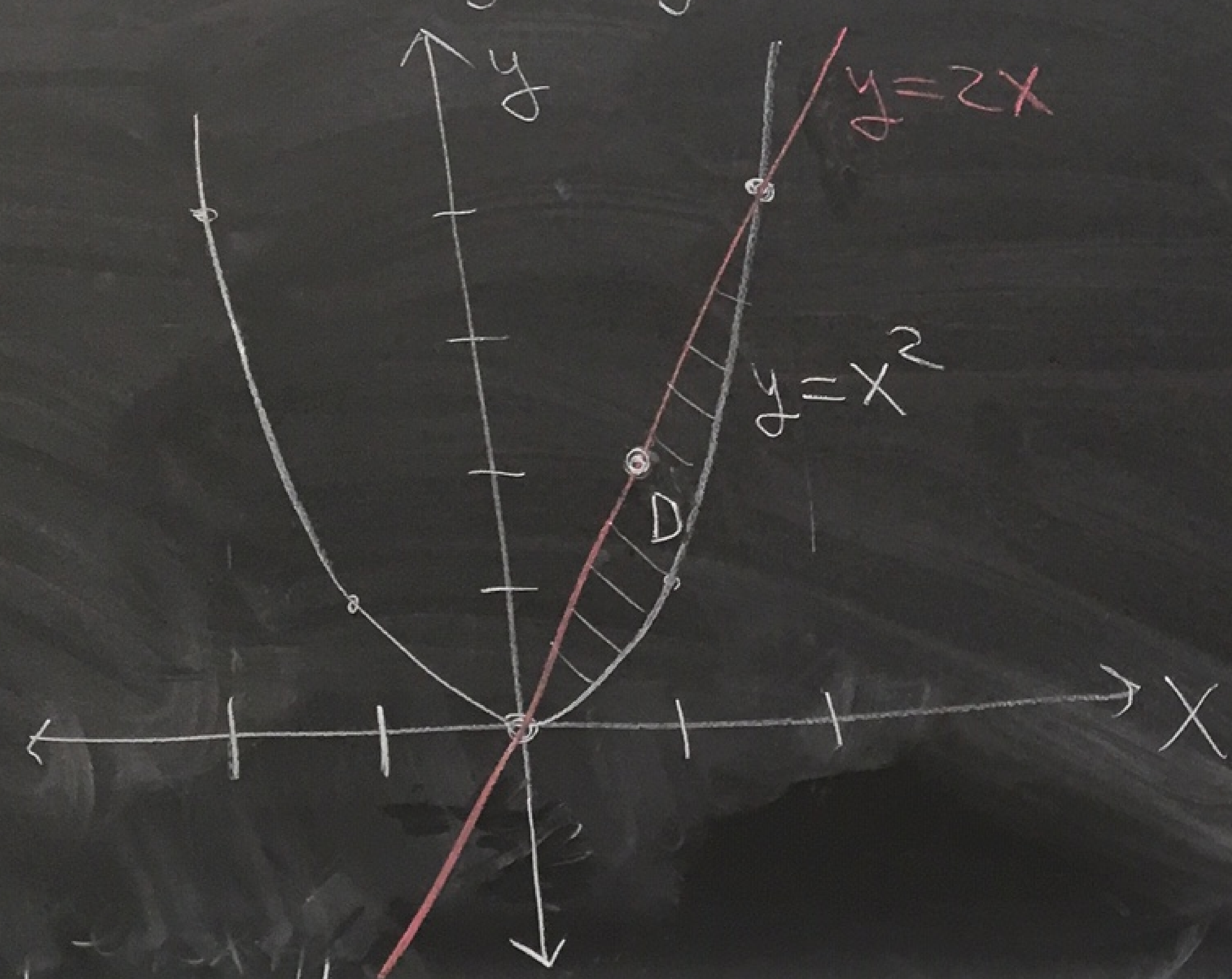
$$D = \left\{ (x,y) \;\middle|\; \begin{array}{l} c \le y \le d \\ h_1(y) \le x \le h_2(y) \end{array} \right\}$$

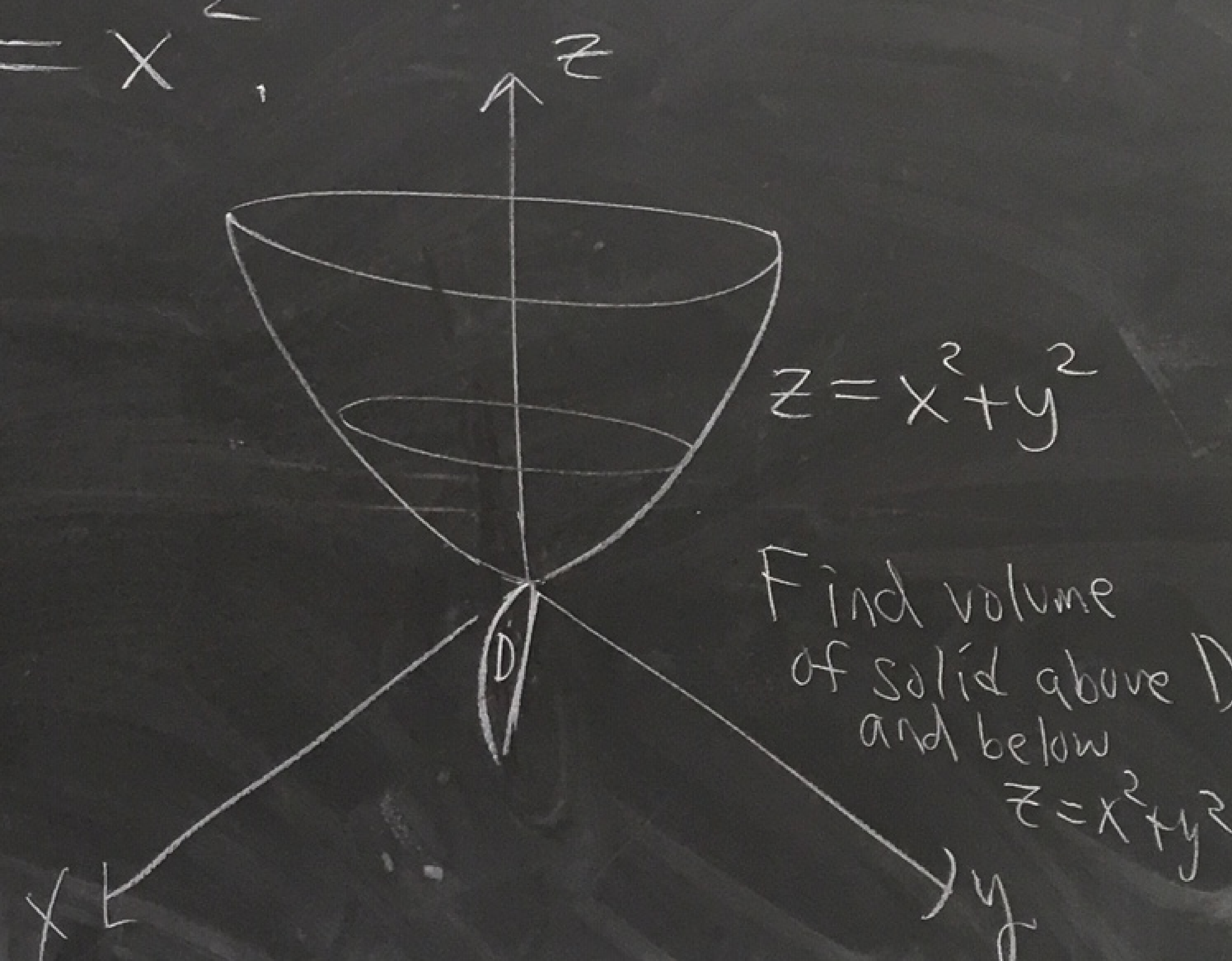
where $h_1(y)$ and $h_2(y)$ are continuous for $c \le y \le d$.



$$\iint_D f(x,y)\,dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\,dx\,dy$$

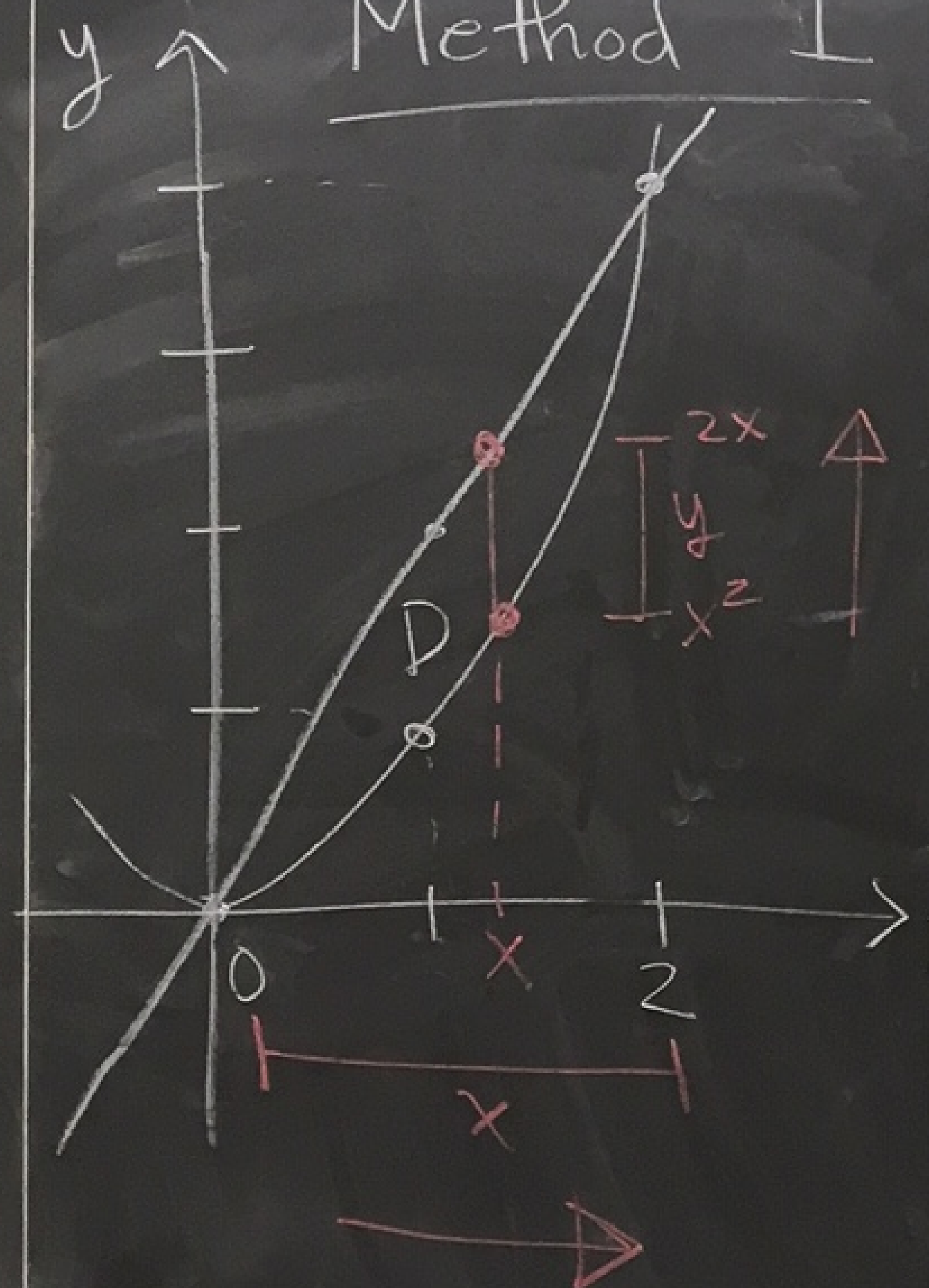
Ex: Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy-plane bounded by $y = 2x$ and $y = x^2$.





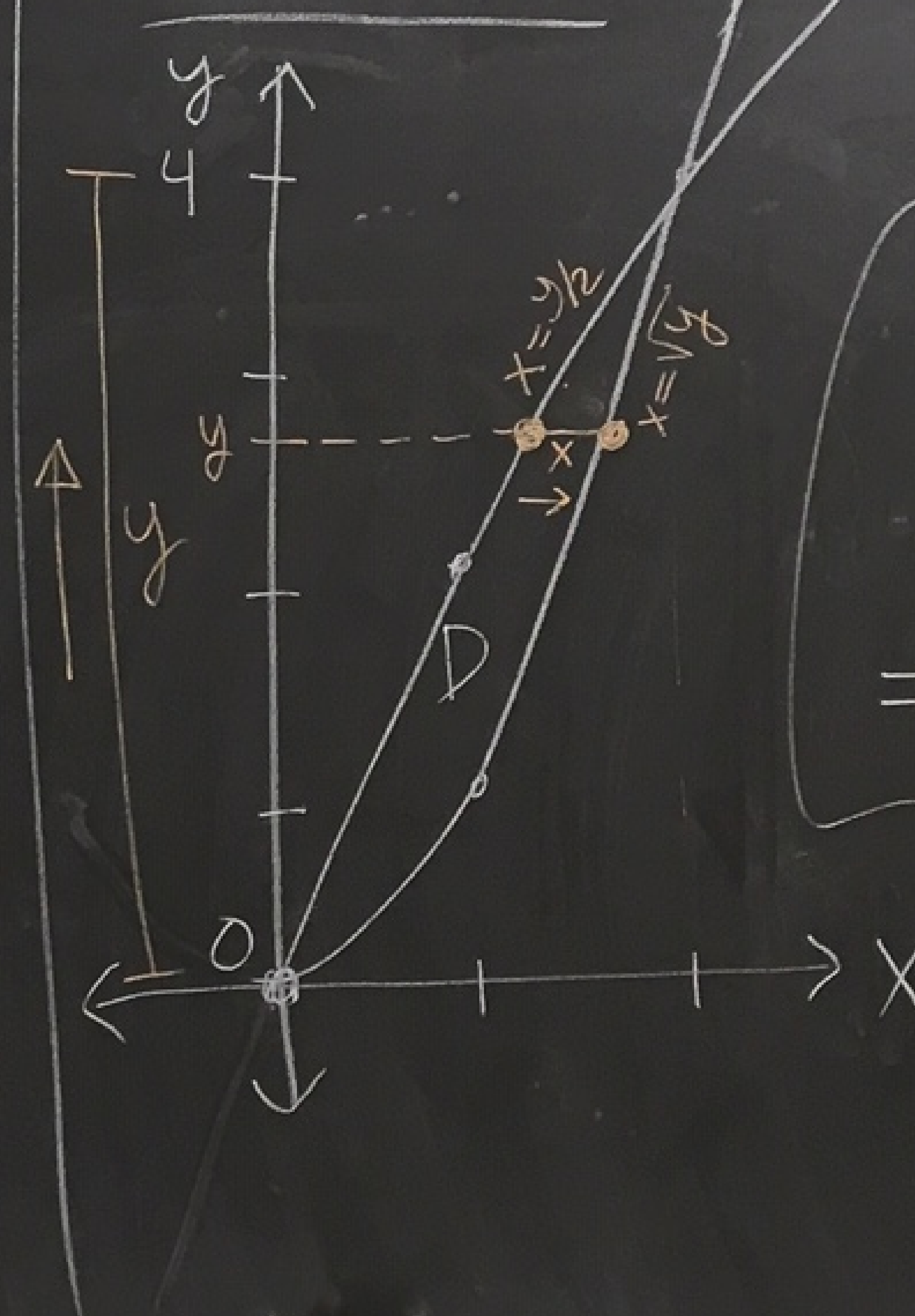
he
bove D
w
$=x^2+y^2$

Method 1



$$\iint_D (x^2+y^2)\,dA = \int_0^2 \int_{x^2}^{2x} (x^2+y^2)\,dy\,dx$$

Method 2



$$\iint_D (x^2+y^2)\,dA = \int_0^4 \int_{y/2}^{\sqrt{y}} (x^2+y^2)\,dx\,dy$$

Test 2

currently =
Tues 11/5

move to
Thurs 11/14