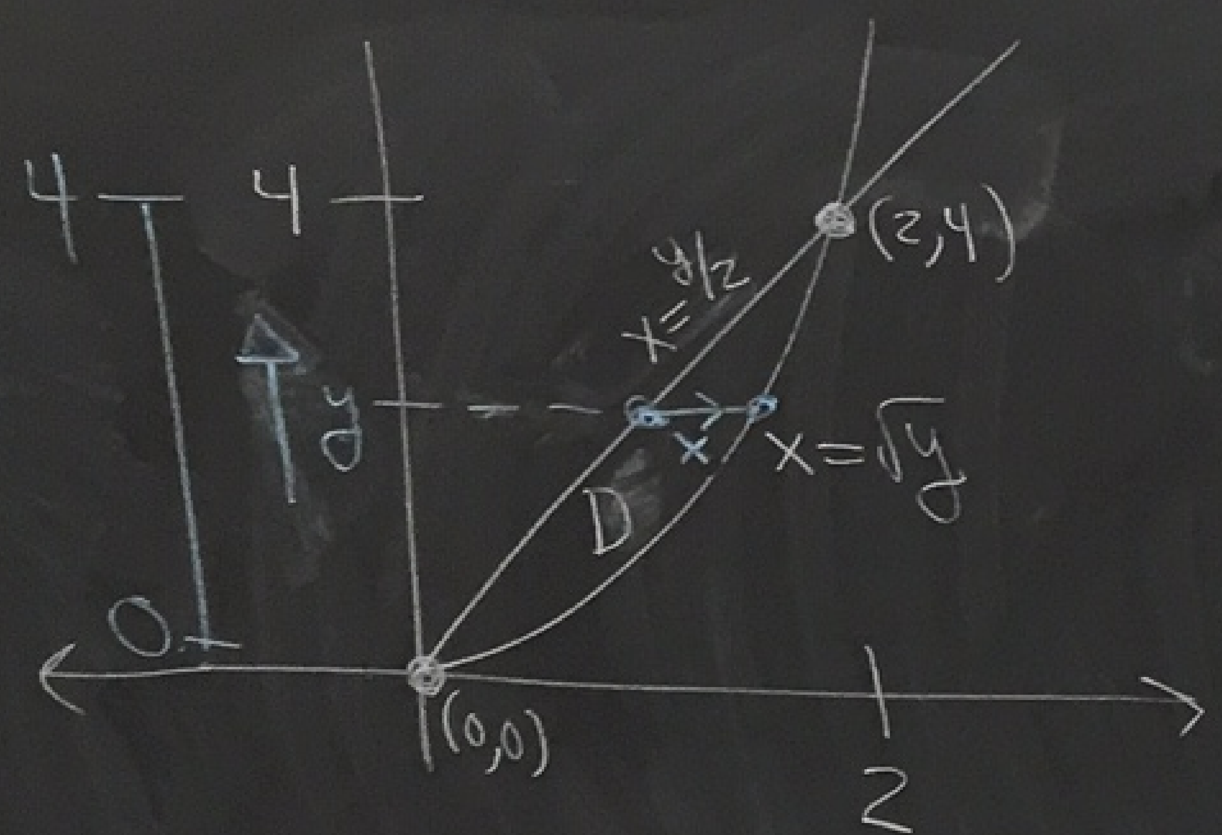
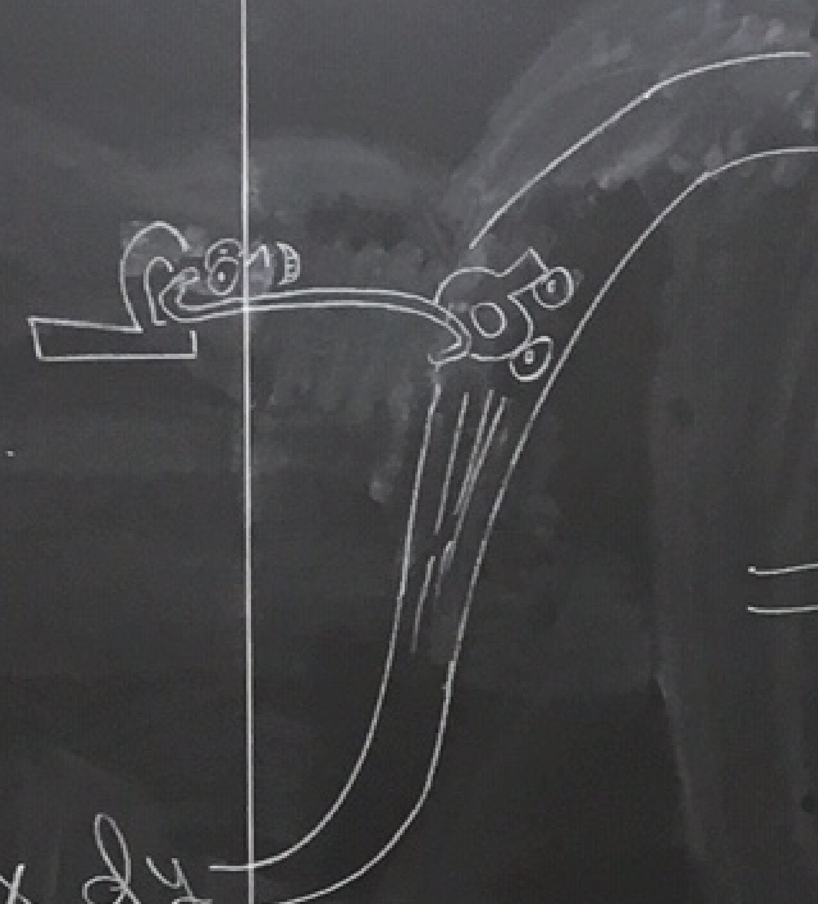
Thursday 10/17

From last time



$$\iint_D (x^2+y^2)\,dA$$

$$= \int_0^4 \int_{y/2}^{\sqrt{y}} (x^2+y^2)\,dx\,dy$$



$$\int_0^4 \left[\frac{x^3}{3} + y^2 x \right]_{x=\frac{y}{2}}^{y^{1/2}} dy$$

$$= \int_0^4 \Bigg[\underbrace{\left(\frac{(y^{1/2})^3}{3} + y^2\left(y^{1/2}\right)\right)}_{x=y^{1/2}} - \underbrace{\left(\frac{\left(\frac{y}{2}\right)^3}{3} + y^2\left(\frac{y}{2}\right)\right)}_{x=y/2} \Bigg] dx$$

$$= \int_0^4 \Big(\frac{1}{3} y^{3/2} + y^{5/2} \underbrace{- \frac{1}{24} y^3 - \frac{1}{2} y^3}_{-\frac{13}{24} y^3}\Big) dy = \left(\frac{1}{3} \frac{y^{5/2}}{(5/2)} + \right.$$

$$\int_0^4 \left[\frac{x^3}{3} + y^2 x \right]_{x=\frac{y}{2}}^{y^{1/2}} dy$$

$$= \int_0^4 \left[\underbrace{\left(\frac{(y^{1/2})^3}{3} + y^2 (y^{1/2}) \right)}_{x = y^{1/2}} - \underbrace{\left(\frac{\left(\frac{y}{2}\right)^3}{3} + y^2 \left(\frac{y}{2}\right) \right)}_{x = y/2} \right] dx$$

$$= \int_0^4 \left(\frac{1}{3} y^{3/2} + y^{5/2} \underbrace{- \frac{1}{24} y^3 - \frac{1}{2} y^3}_{-\frac{13}{24} y^3} \right) dy = \left(\frac{1}{3} \frac{y^{5/2}}{(5/2)} + \frac{y^{7/2}}{(7/2)} - \frac{13}{24} \cdot \frac{y^4}{4} \right)_0^4$$

$$\left(\frac{2}{15} y^{5/2} + \frac{2}{7} y^{7/2} - \frac{13}{96} y^4 \right)_0^4$$

$$= \left(\frac{2}{15}(4^{5/2}) + \frac{2}{7}(4^{7/2}) - \frac{13}{96} 4^4 \right)$$

$$= \frac{2}{15}(2^5) + \frac{2}{7}(2^7) - \frac{13}{96}(256)$$

$$= \frac{64}{15} + \frac{256}{7} - \frac{13}{96}(256)$$

$$= \frac{64}{15} + 256\left(\frac{1}{7} - \frac{13}{96} \right)$$

$$= \frac{64}{15} + 256\left(\frac{96 - 91}{672} \right)$$

$$= \frac{64}{15} + \frac{1280}{672}$$

$$= \frac{43008 + 19200}{10080} = \boxed{\frac{62208}{10080}}$$

Ex: Evaluate the integral

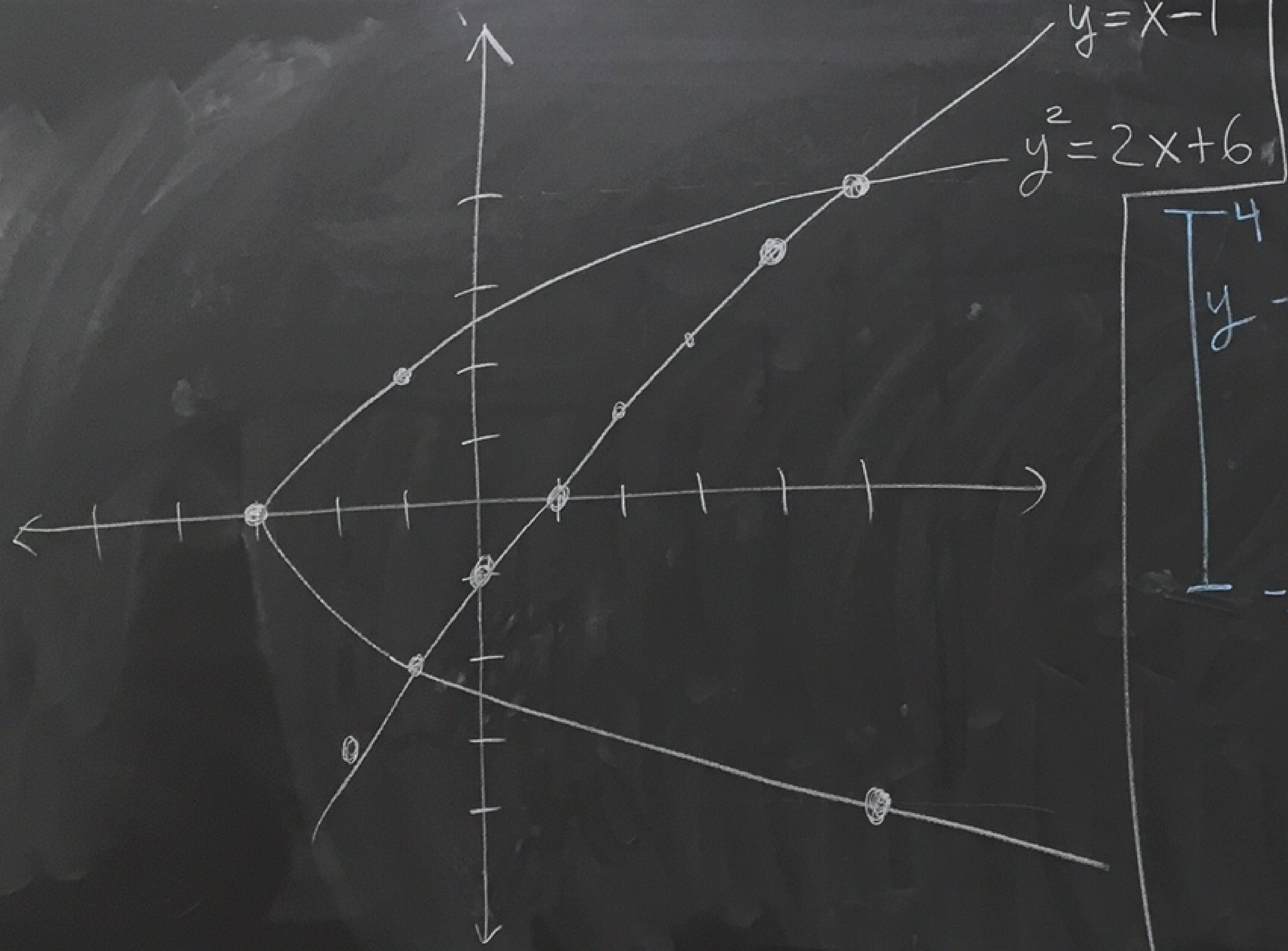
$\iint_D (xy)\,dA$ where D is the region

bounded by the line $y = x - 1$ and

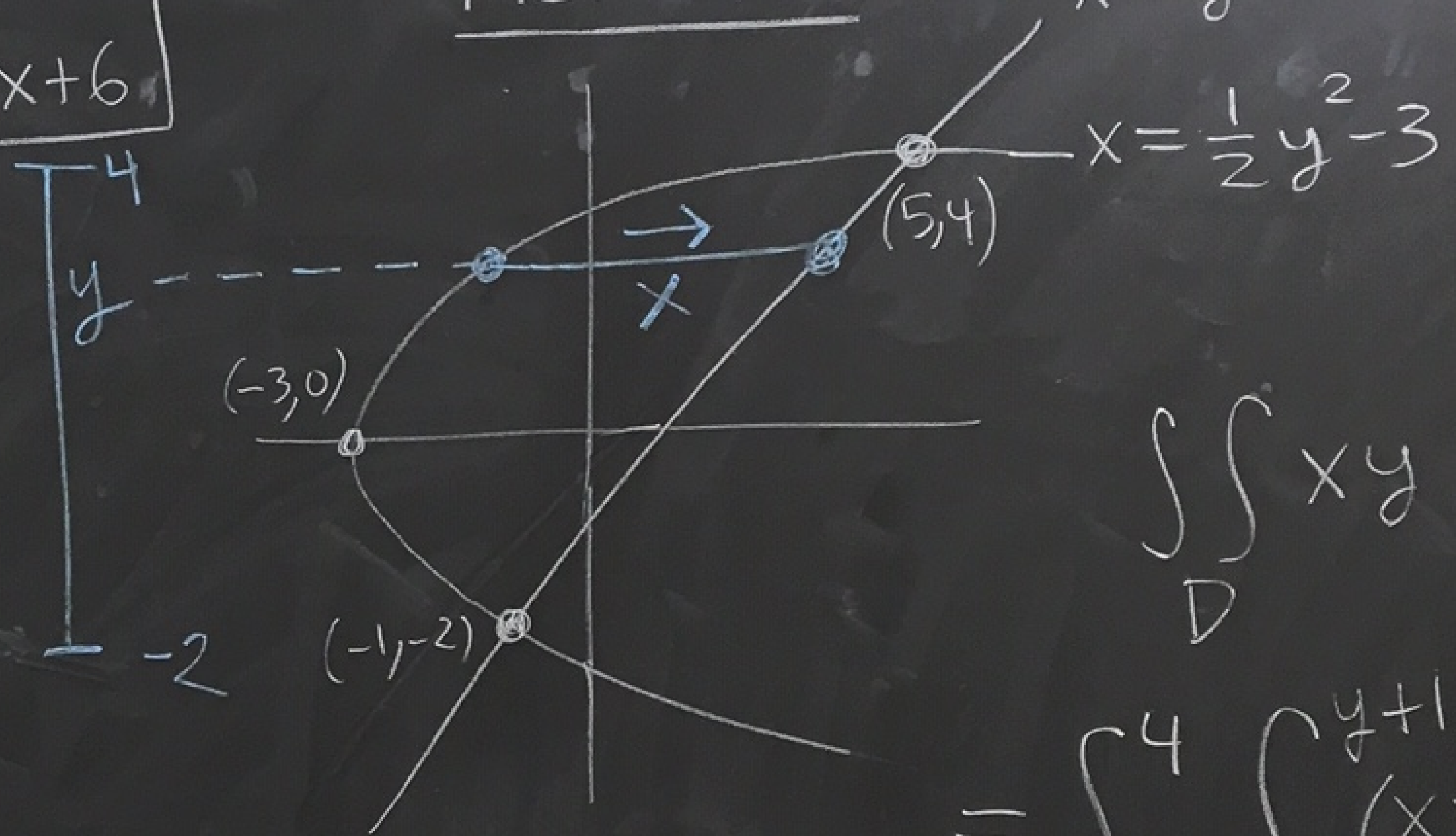
the parabola $y^2 = 2x + 6$.

$$\boxed{x = \frac{1}{2}y^2 - 3}$$

y	$x = \frac{1}{2}y^2 - 3$
0	-3
± 2	-1
± 4	5



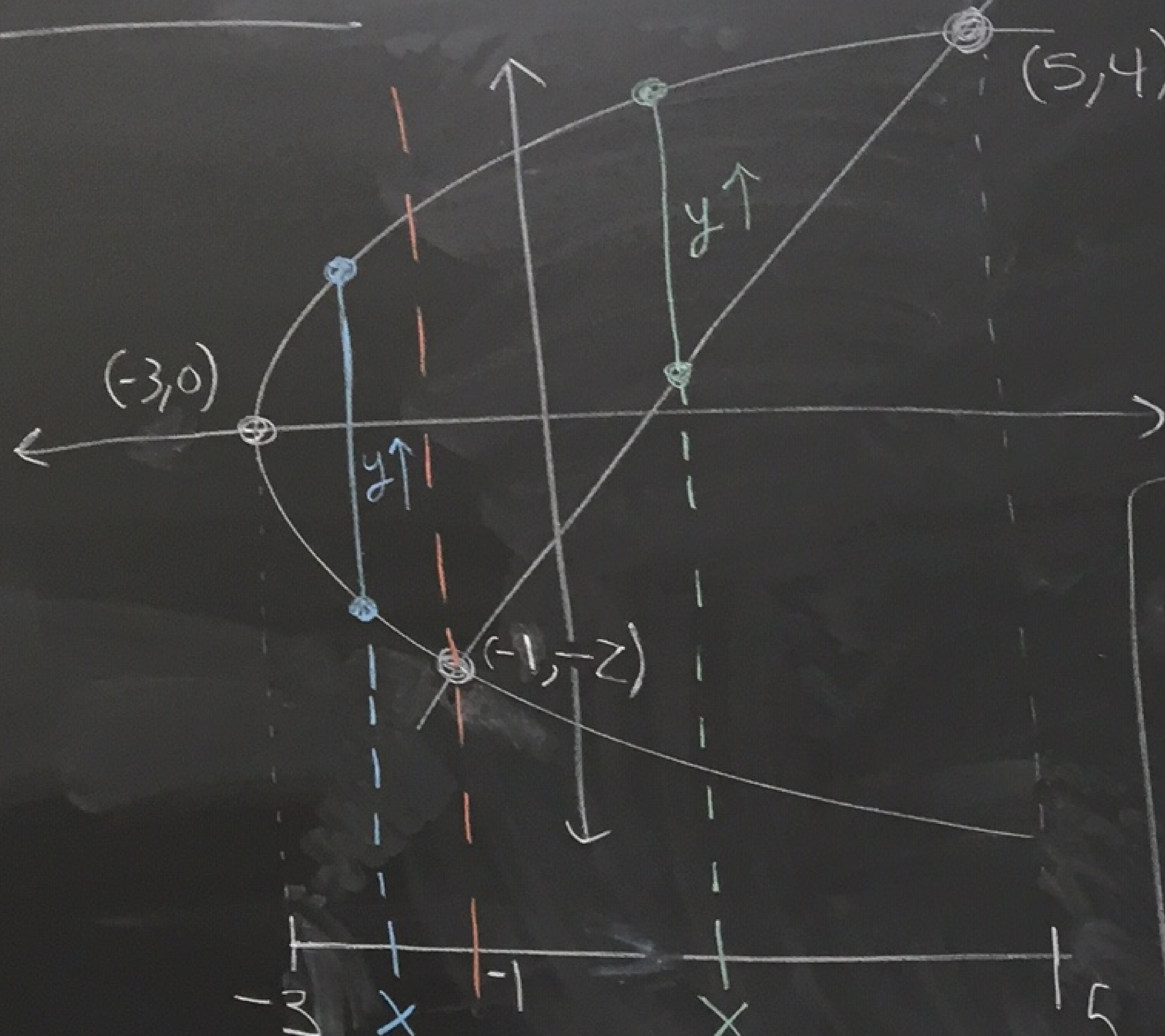
Method 1

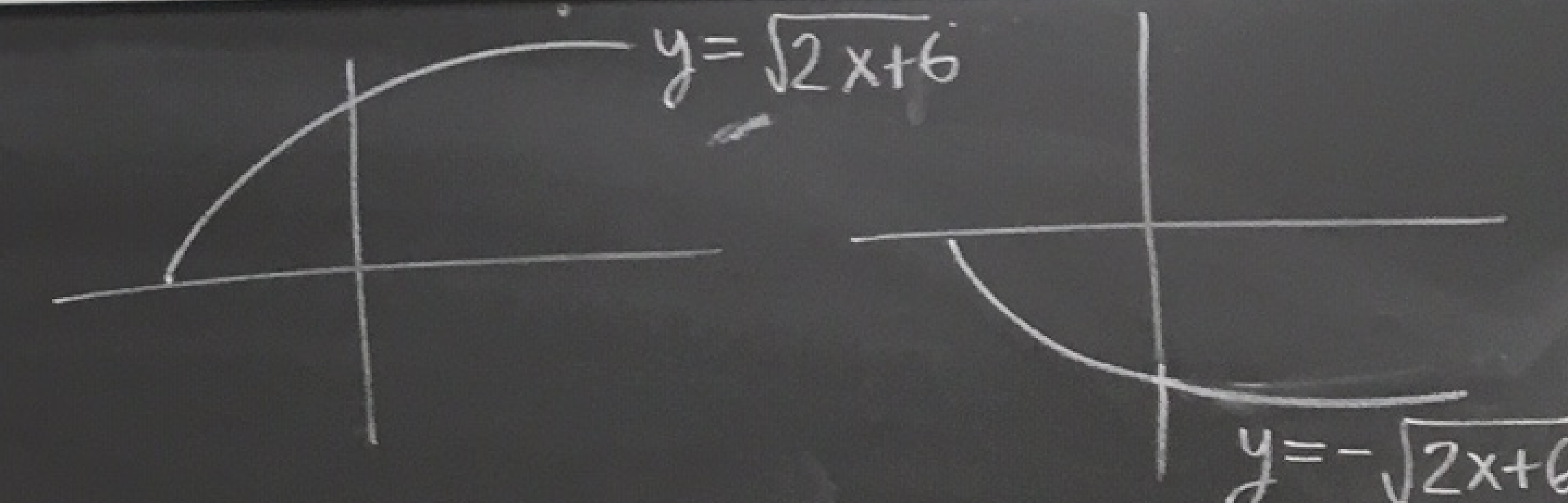


$$\iint_D xy\, dA$$

$$= \int_{-2}^{4} \int_{\frac{1}{2}y^2-3}^{y+1} (xy)\, dx\, dy$$

Method 2





$$y^2 = 2x+6$$
$$y = \pm\sqrt{2x+6}$$

$$\iint_D (xy)\,dA = \int_{-3}^{-1}\int_{-\sqrt{2x+6}}^{\sqrt{2x+6}} (xy)\,dy\,dx + \int_{-1}^{5}\int_{x-1}^{\sqrt{2x+6}} (xy)\,dy\,dx$$

$$= \int_{-3}^{-1} \left(x \frac{y^2}{2} \right)\Bigg|_{y=-\sqrt{2x+6}}^{\sqrt{2x+6}} dx + \int_{-1}^{5} \left(x \frac{y^2}{2} \right)\Bigg|_{y=x-1}^{\sqrt{2x+6}} dx = \boxed{36}$$

$$= \int_{-3}^{-1} \left(\frac{x}{2}\left(\sqrt{2x+6}\right)^2 - \frac{x}{2}\left(-\sqrt{2x+6}\right)^2 \right) dx + \int_{-1}^{5} \left(\frac{x}{2}\left(\sqrt{2x+6}\right)^2 - \frac{x}{2}\overbrace{(x-1)^2}^{x^2-2x+1} \right) dx$$

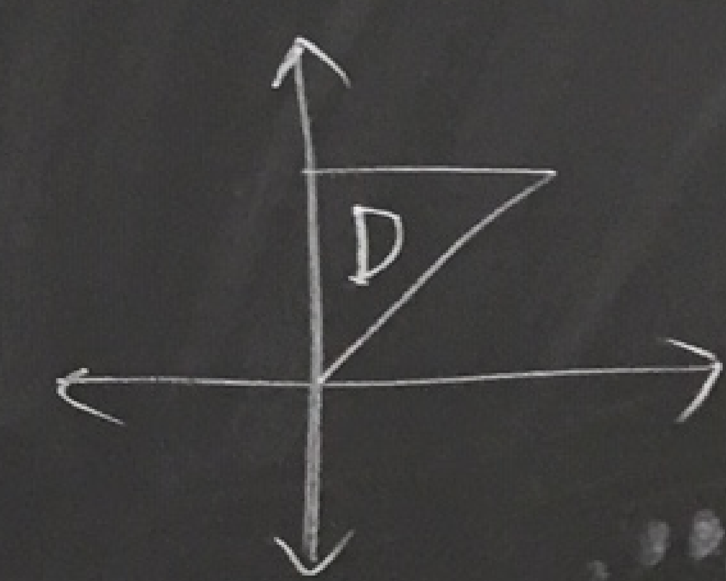
$$= \underbrace{\int_{-3}^{-1} \left(\frac{x}{2}(2x+6) - \frac{x}{2}(2x+6) \right) dx}_{0} + \int_{-1}^{5} \left(\underbrace{\frac{x}{2}(2x+6)}_{x^2+3x} - \frac{x^3}{2} + x^2 - \frac{x}{2} \right) dx = \int_{-1}^{5} \left(-\frac{x^3}{2} + 2x^2 + \frac{5}{2}x \right) dx$$

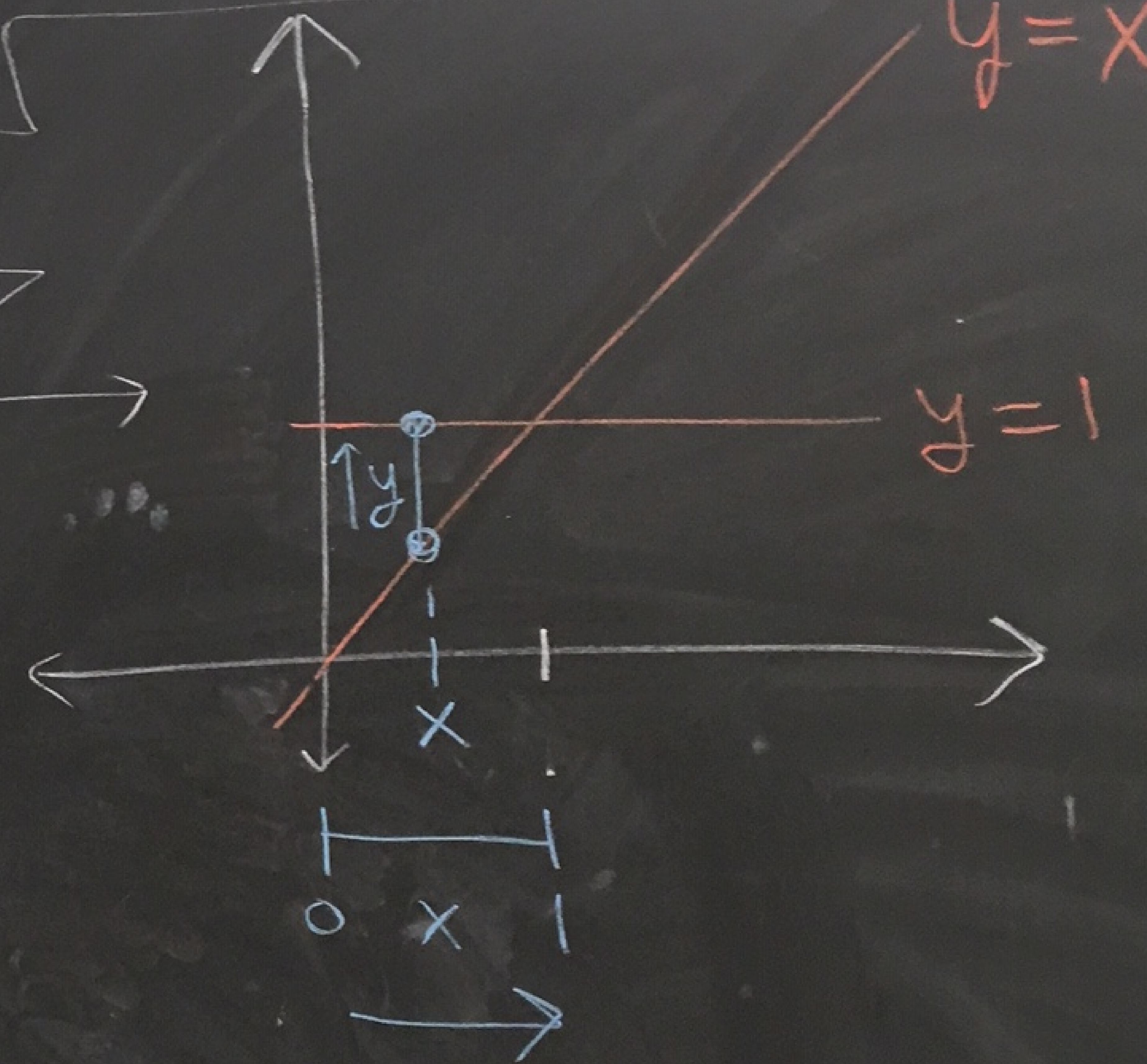
$$= \int_{-3}^{-1} 0\, dx = 0$$

$$= \left(-\frac{1}{2}\cdot\frac{x^4}{4} + 2\frac{x^3}{3} + \frac{5}{2}\frac{x^2}{2} \right)\Bigg|_{-1}^{5} = \left(-\frac{(5^4)}{8} + \frac{2}{3}(5^3) + \frac{5}{4}(5^2) \right) - \left(-\frac{1}{2}\cdot\frac{(-1)^4}{4} + \frac{2}{3}(-1)^3 + \frac{5}{4}(-1)^2 \right)$$

Ex: Evaluate $\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx$

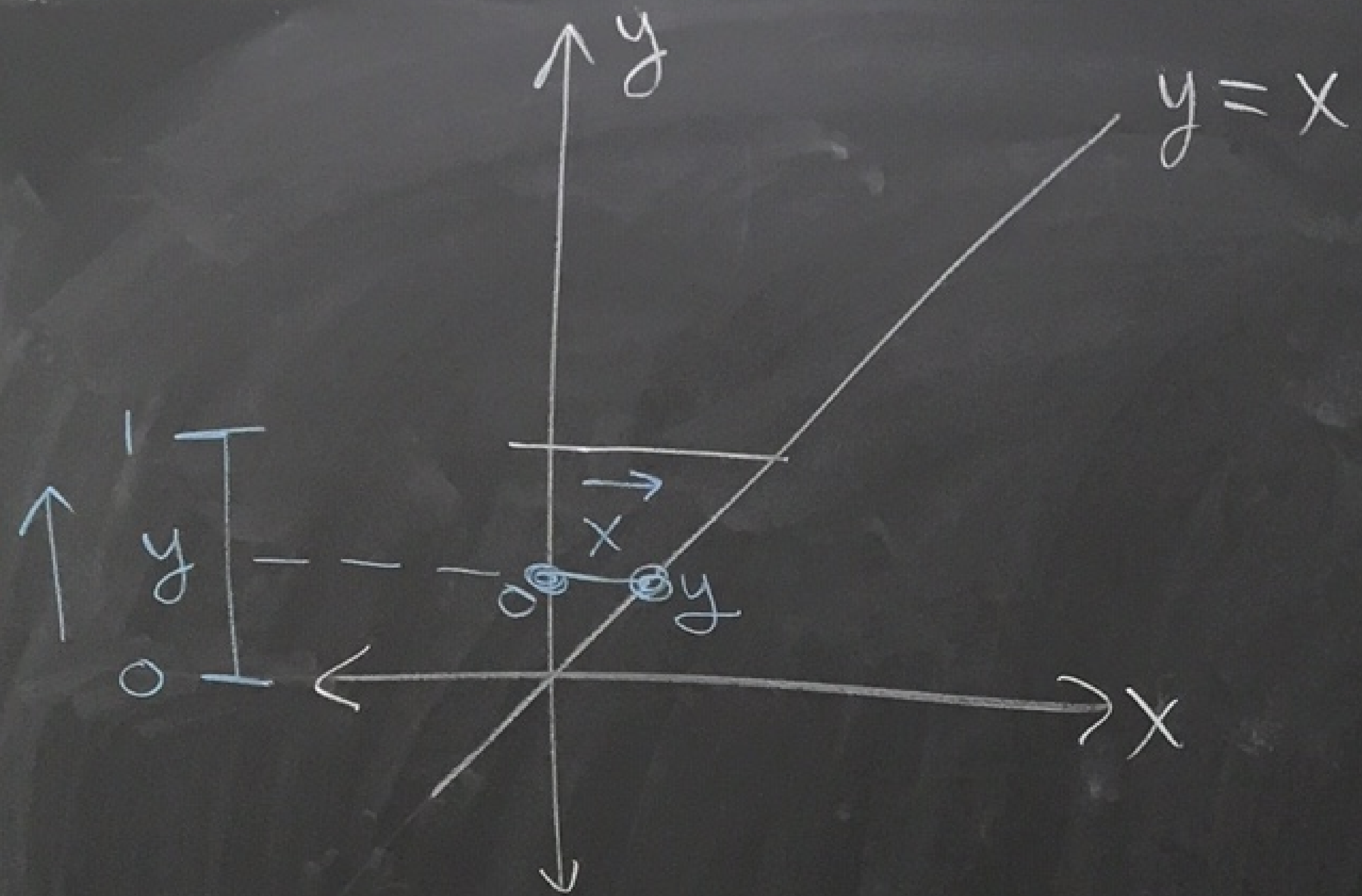
$$\int_0^1 \Big(\underbrace{\int_x^1 \sin(y^2)\,dy}_{\text{what's the antiderivate of } \sin(y^2)\text{?}}\Big)\,dx$$





$= x$

$= 1$



$$\int_0^1\int_x^1 \sin(y^2)\,dy\,dx = \int_0^1\left(\int_0^y \sin(y^2)\,dx\right)dy$$

$$= \int_0^1 \left(x\sin(y^2)\right)\Big|_{x=0}^{y} dy = \int_0^1 \left(y\sin(y^2) - 0\right)dy$$

$$= \int_0^1 \frac{1}{2}\sin(u)\,du = -\frac{1}{2}\cos(u)\Big|_0^1 = -\frac{1}{2}\cos(1) + \frac{1}{2}\underbrace{\cos(0)}_{1}$$

$$= \boxed{\frac{1}{2} - \frac{1}{2}\cos(1)}$$

change bounds

$y=0$

$u=y^2=0^2=0$

$y=1$

$u=y^2=1^2=1$

$u=y^2$

$du=2y\,dy$

$\frac{1}{2}du=y\,dy$