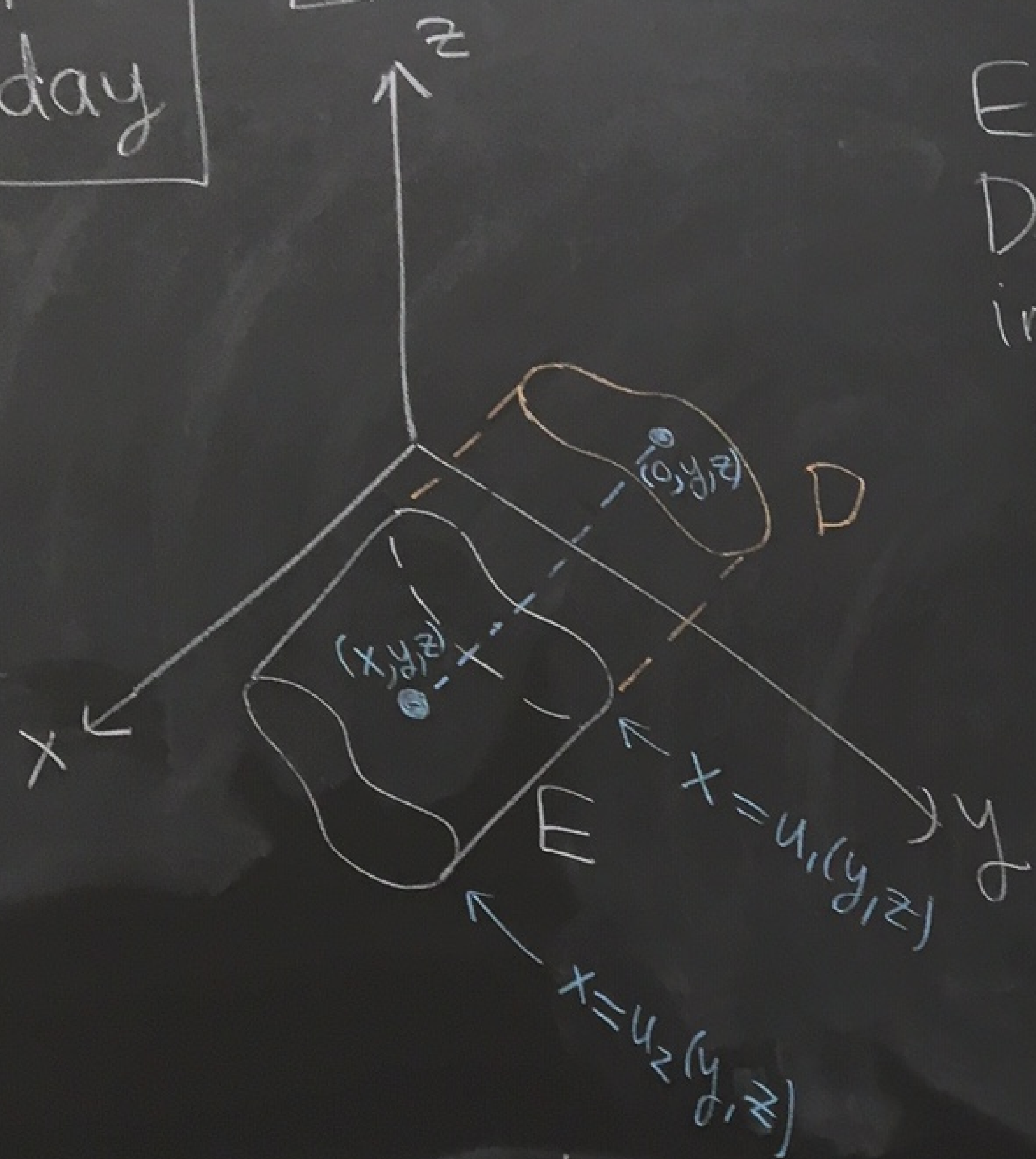
10/31
Thursday

13.4 continued...




E is a solid

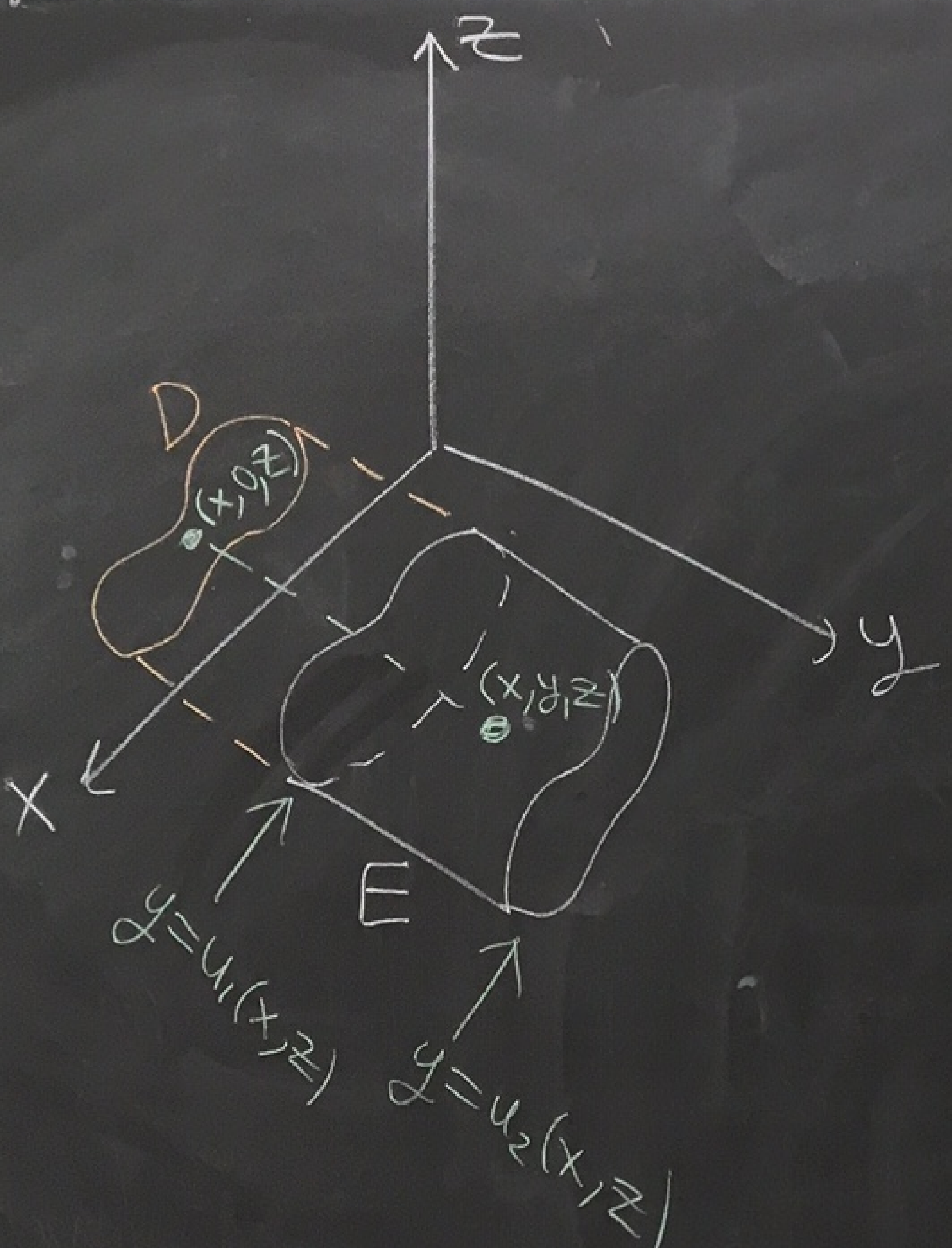
D is E's projection into the yz-plane.

E consists of all $(x,y,z)$ with $(0,y,z)$ in D and $u_1(y,z) \le x \le u_2(y,z)$.

If $f(x,y,z)$ is continuous on E, then

$$\iiint_E f(x,y,z)\,dV = \iint_D \left[ \int_{u_1(y,z)}^{u_2(y,z)} f(x,y,z)\,dx \right] dA$$

$=$




E is a solid

D is E's projection into the xz-plane.

E consists of all $(x,y,z)$ with

$(x,0,z)$ in D and $u_1(x,z) \le y \le u_2(x,z)$.

If $f(x,y,z)$ is continuous on E then

$$\iiint_E f(x,y,z)\,dV = \iint_D \left[ \int_{u_1(x,z)}^{u_2(x,z)} f(x,y,z)\,dy \right] dA$$

dA

If $E$ is a solid, then the volume of $E$ is $\iiint_E 1\, dV = \iiint_E dV$

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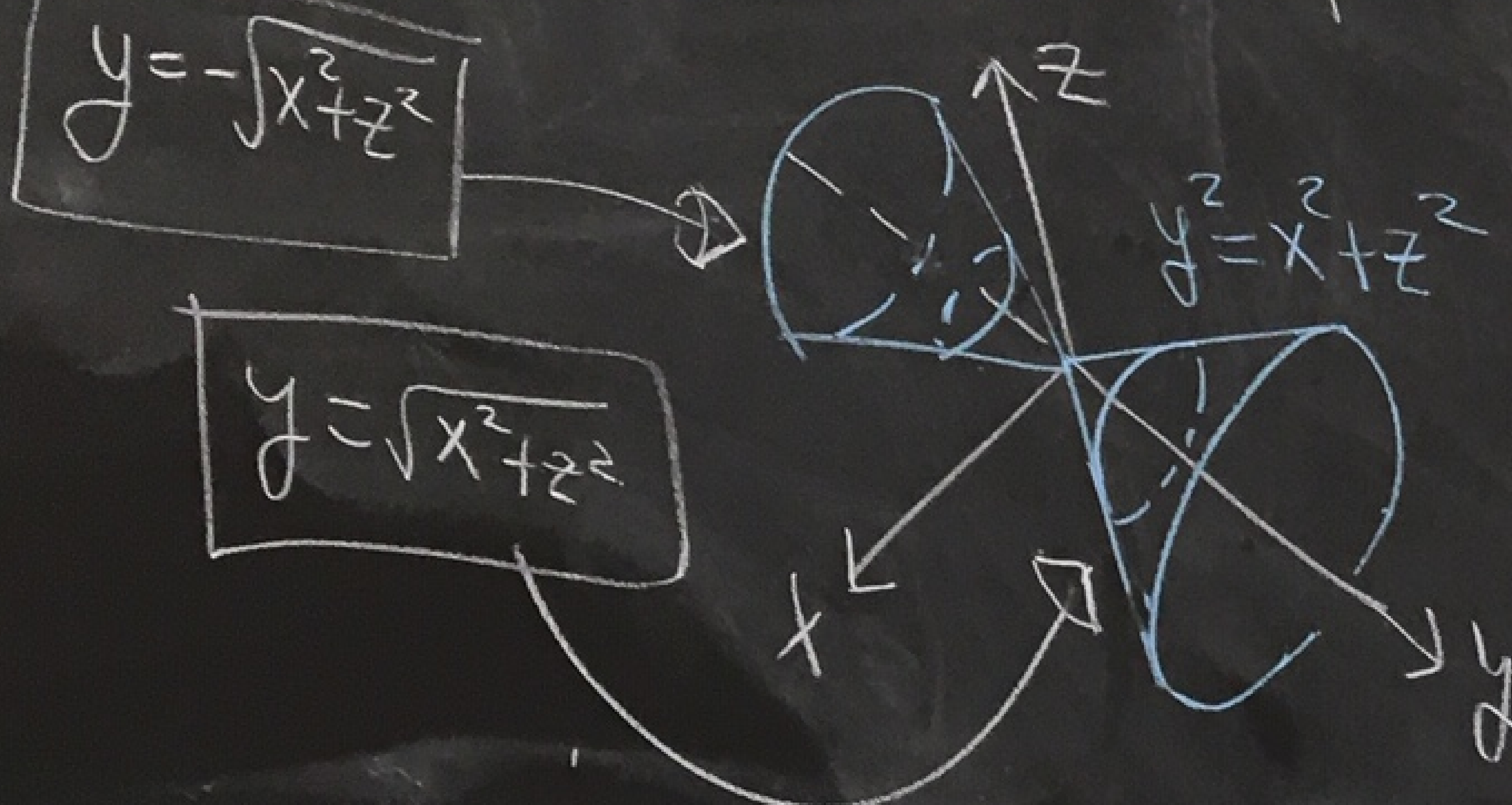
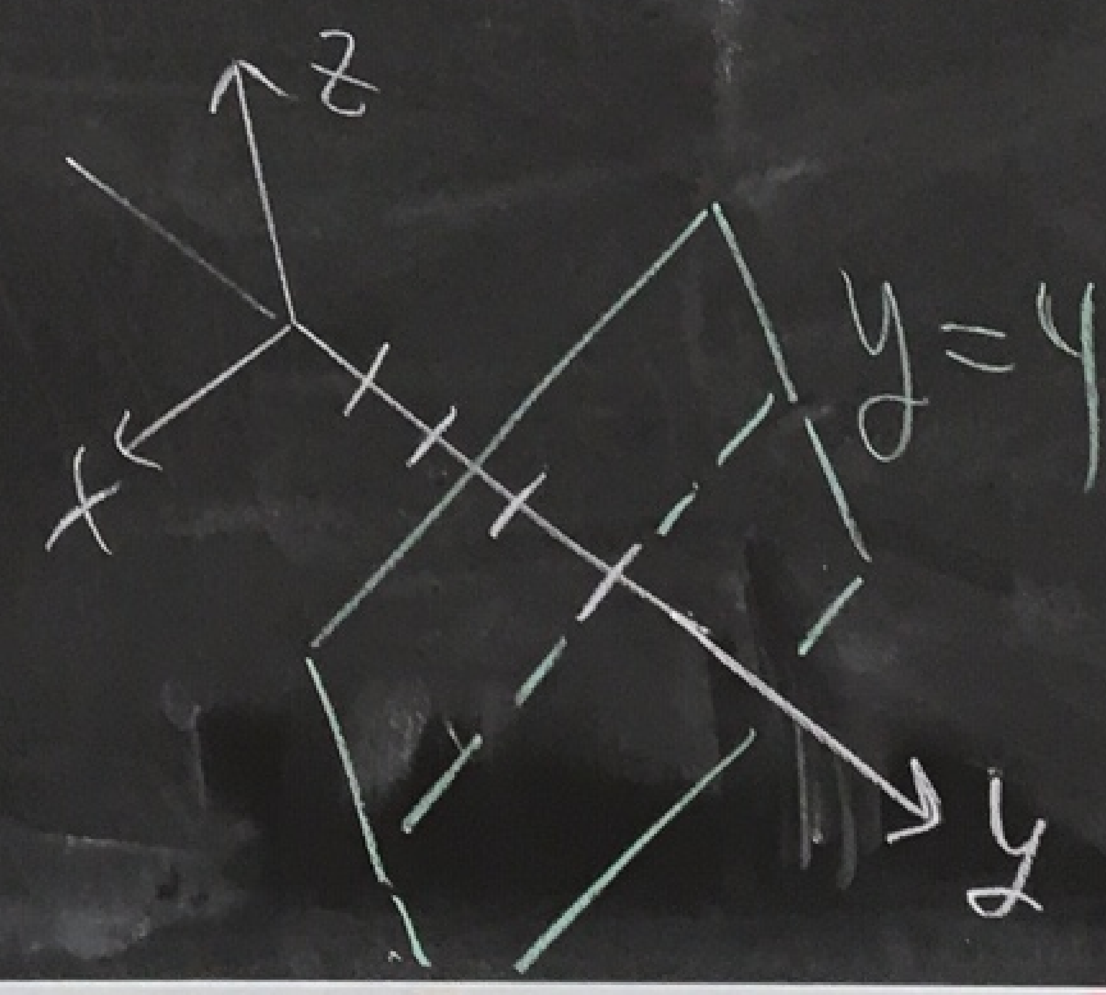
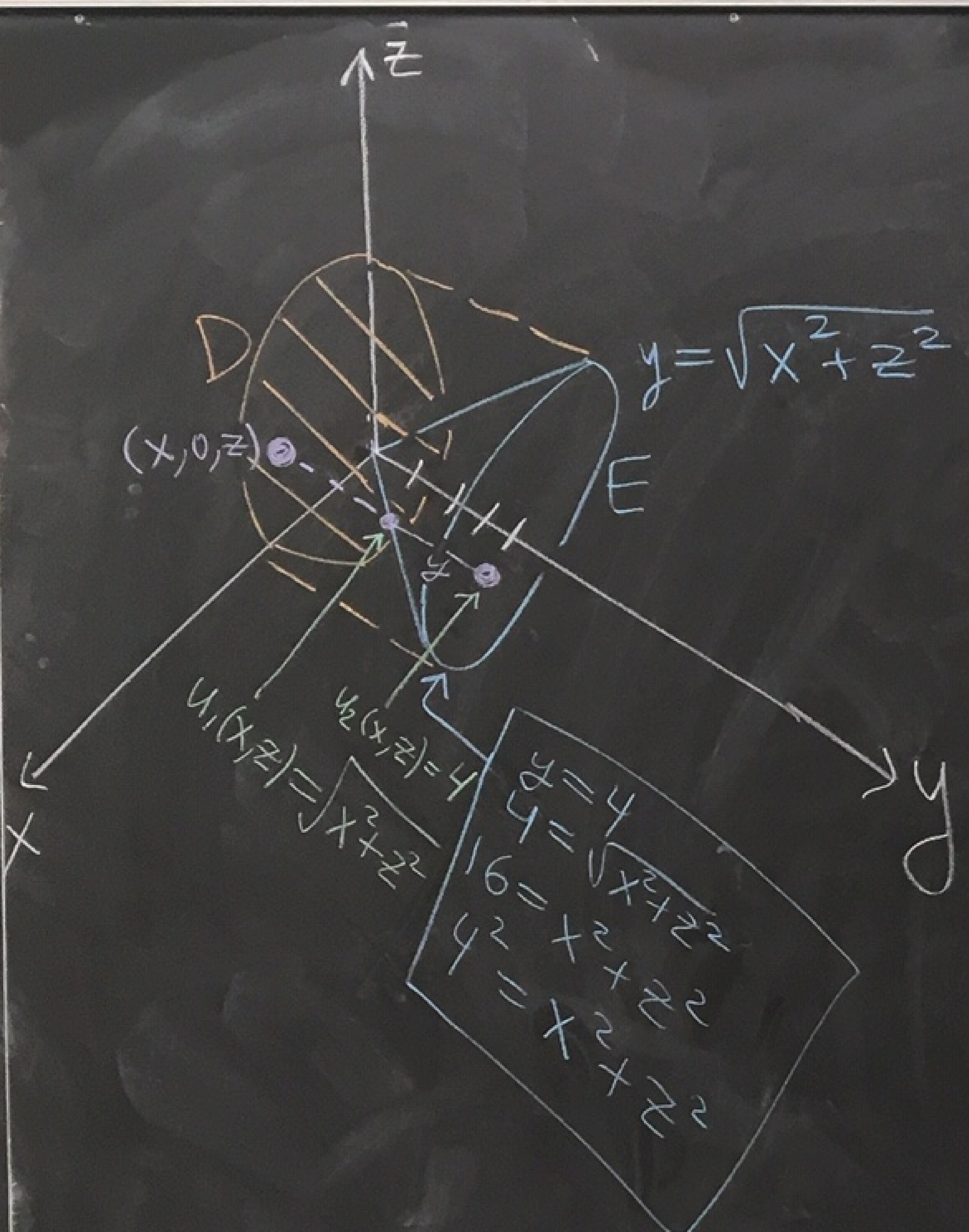
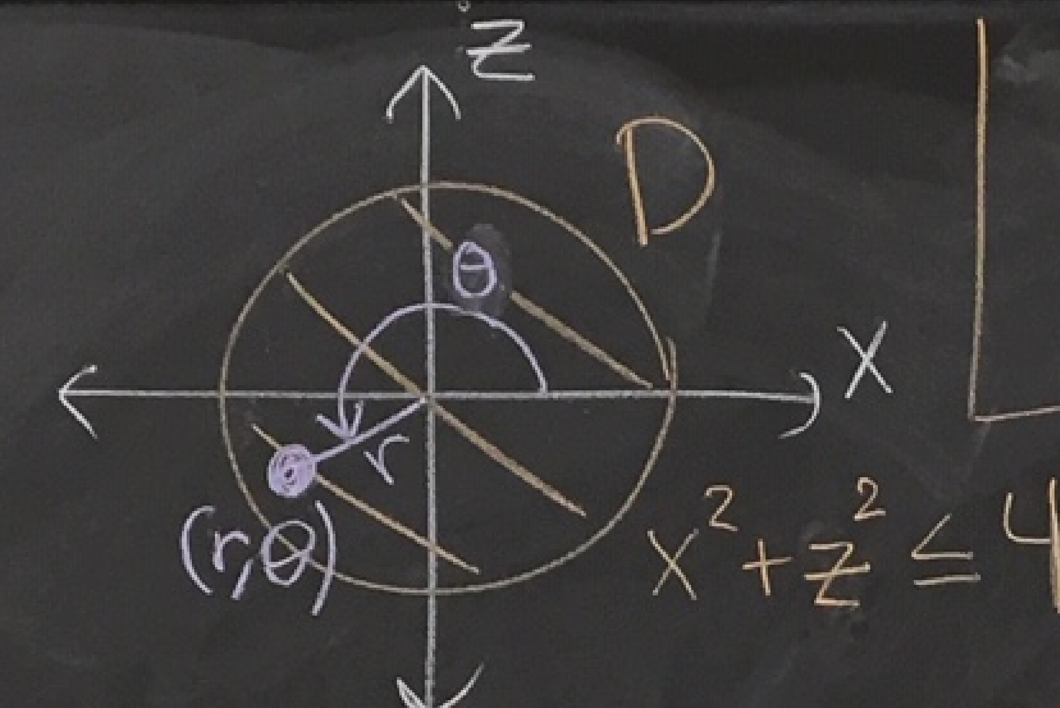
<u>Ex:</u> Consider the solid $E$ that lies between the surface $x^2+z^2=y^2$ and the plane $y=4$.






$x=r\cos(\theta)$
$z=r\sin(\theta)$
$r^2=x^2+z^2$

D consists of all $(r,\theta)$ with $0\le r\le 4$, $0\le\theta\le 2\pi$

$$\text{Volume}(E)=\iint_D\left[\int_{\sqrt{x^2+z^2}}^{4}1\,dy\right]dA$$

$$=\iint_D\left[y\Big|_{\sqrt{x^2+z^2}}^{4}\right]dA=\iint_D\left[4-\sqrt{x^2+z^2}\right]dA$$

$$
\begin{array}{|c|}
\hline
x^2+z^2=r^2 \\
\hline
0 \le r \le 4 \\
0 \le \theta \le 2\pi \\
\hline
x = r\cos\theta \\
z = r\sin\theta \\
\hline
dA = r\,dr\,d\theta \\
\hline
\end{array}
$$

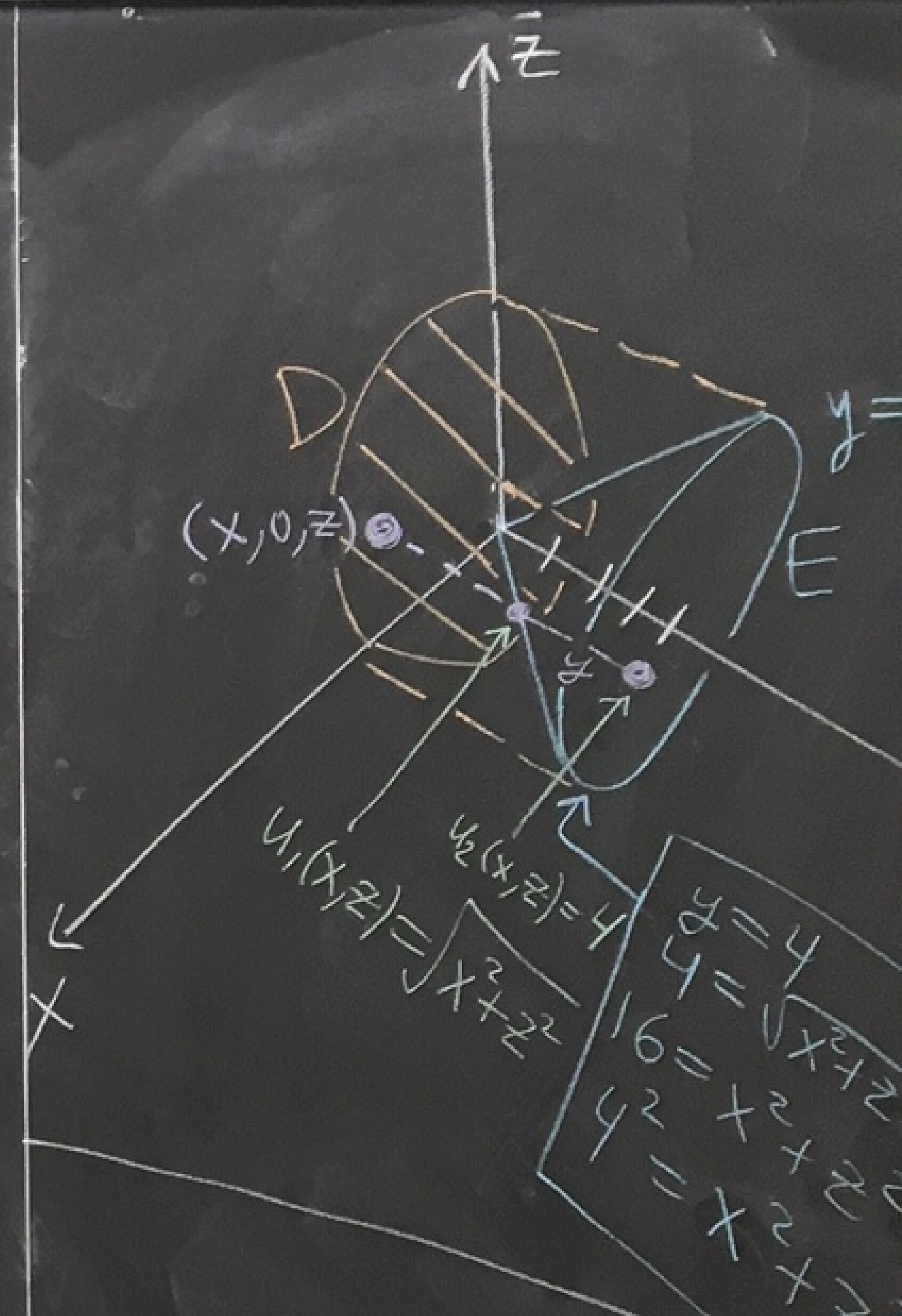
$$\int_0^{2\pi}\int_0^4 \underbrace{\left(4-\sqrt{r^2}\right)}_{4-r} r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^4 (4r - r^2)\,dr\,d\theta$$

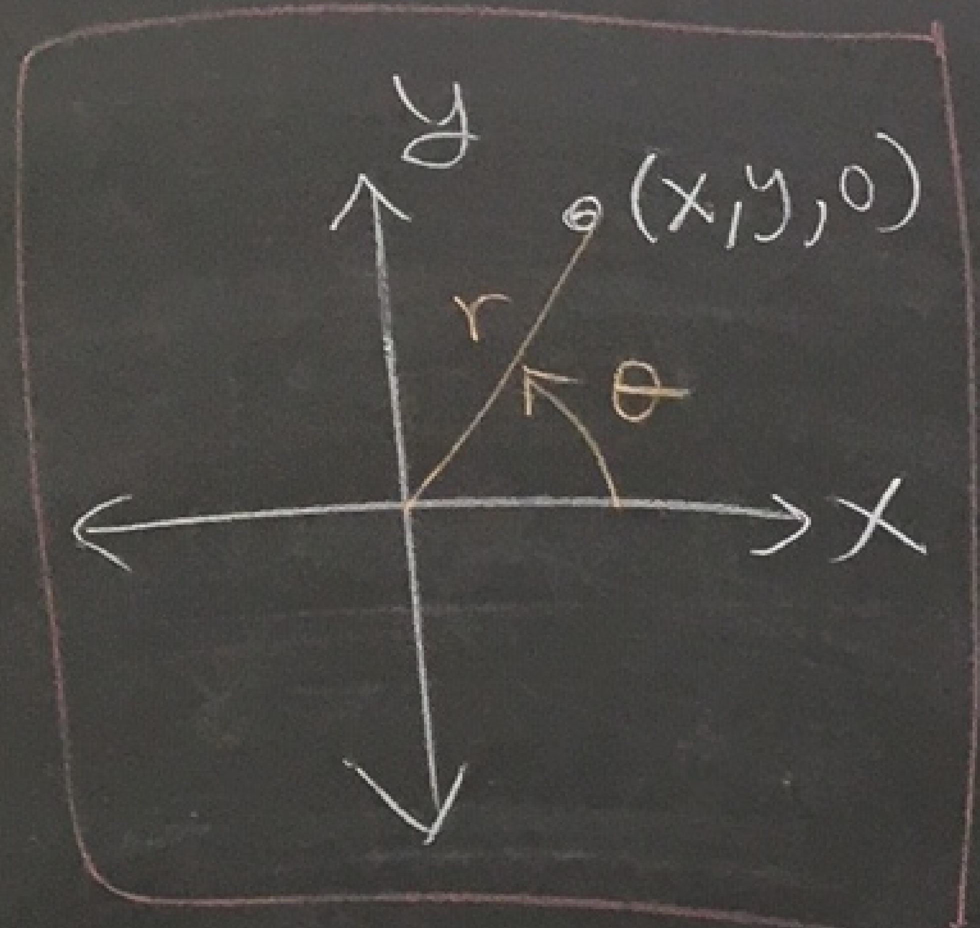
$$= \int_0^{2\pi} \underbrace{\left(4\frac{r^2}{2} - \frac{r^3}{3}\right)}_{\left(2r^2-\frac{r^3}{3}\right)}\Bigg|_{r=0}^{4} d\theta = \int_0^{2\pi}\left(32 - \frac{64}{3}\right)d\theta$$

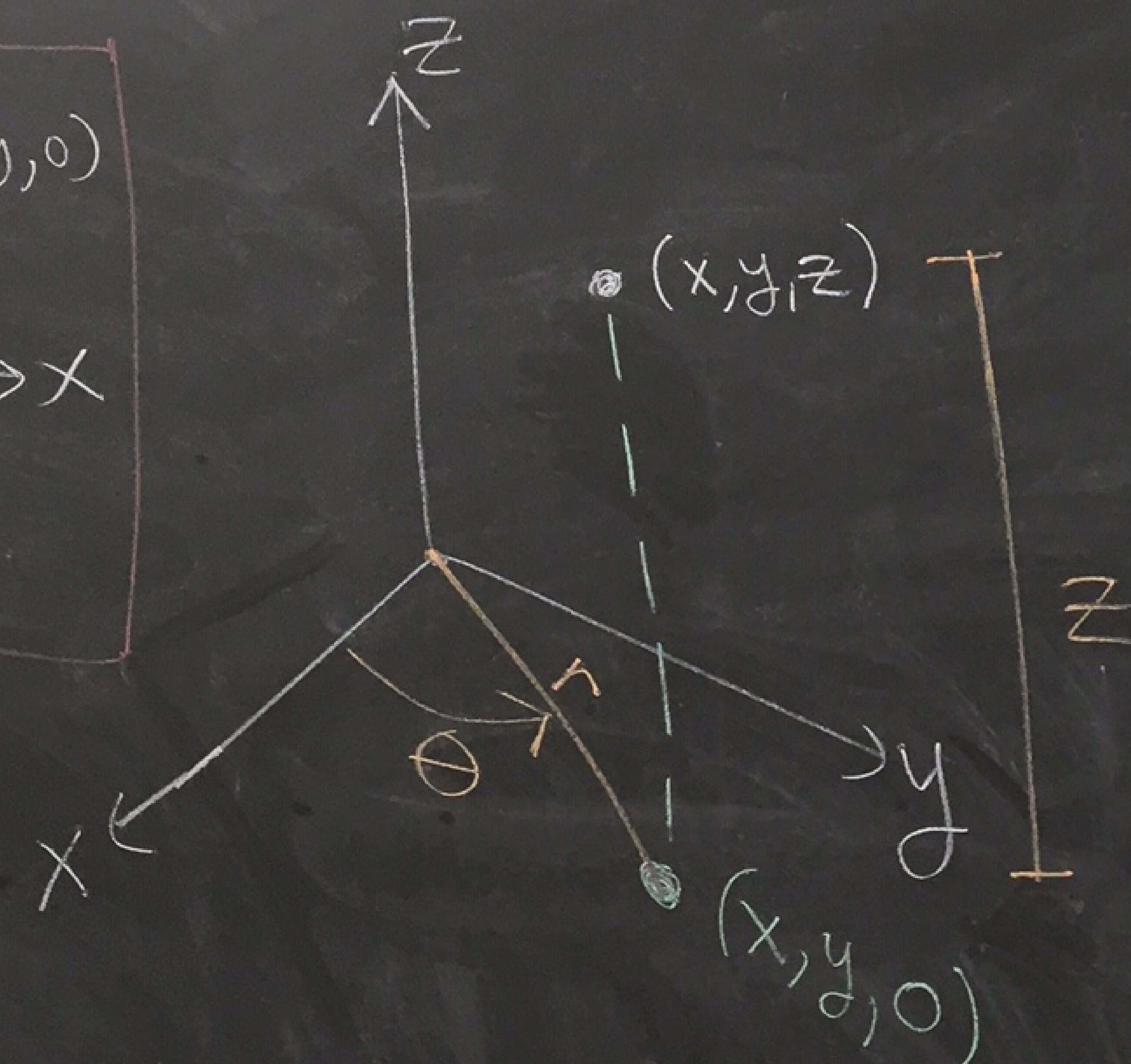
$$\begin{array}{r} 96 \\ -64 \\ \hline 32 \end{array}$$

$$= \int_0^{2\pi}\left(\frac{32}{3}\right)d\theta = \frac{32}{3}\theta\Big|_0^{2\pi} = \frac{32}{3}(2\pi - 0) = \frac{64}{3}\pi \approx 67.0206\ldots$$



# 13.5 — Cylindrical / Spherical Coordinates




## Cylindrical coordinates of $(x,y,z)$

- Find the polar coordinates $r$ & $\theta$ for $(x,y,0)$.
- The cylindrical coordinates of $(x,y,z)$ are

$$(r,\theta,z).$$

## Cylindrical coordinates formulas

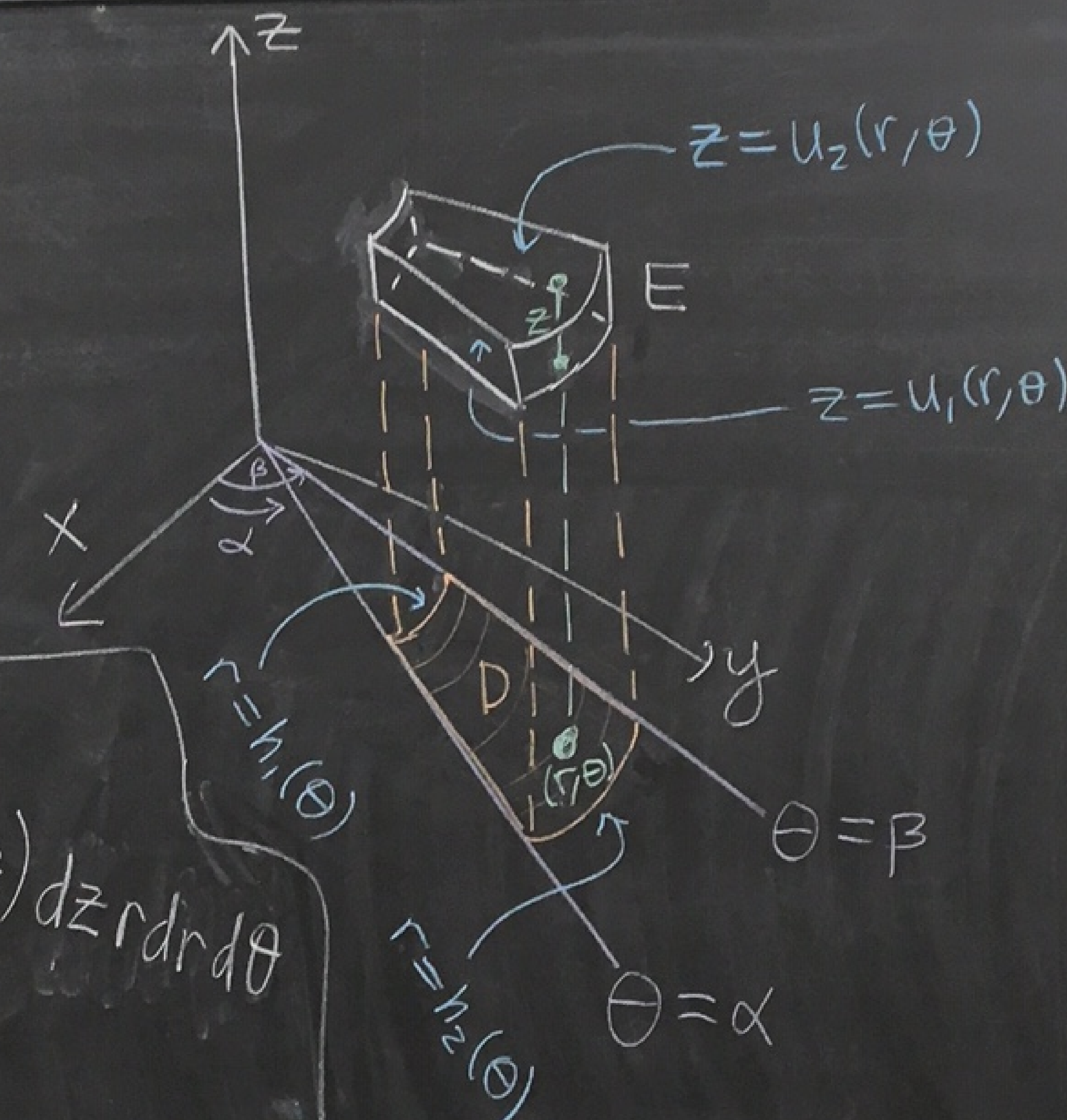
$x = r\cos(\theta)$

$y = r\sin(\theta)$

$z = z$

If $f(x,y,z)$ is continuous on $E$, then

$$\iiint_E f(x,y,z)\,dV = \int_{\alpha}^{\beta}\int_{h_1(\theta)}^{h_2(\theta)}\int_{u_1(r,\theta)}^{u_2(r,\theta)} f(\underbrace{r\cos(\theta)}_{x}, \underbrace{r\sin(\theta)}_{y}, \underbrace{z}_{z})\,dz\,r\,dr\,d\theta$$




E is a solid

D is the projection of E into the xy-plane.

D consists of all points $(r,\theta)$ with $\alpha \le \theta \le \beta$ and $h_1(\theta) \le r \le h_2(\theta)$.

E consists of all $(r,\theta,z)$ with $(r,\theta)$ in D and $u_1(r,\theta) \le z \le u_2(r,\theta)$.