11/21
Thursday

Schedule

T	R
	14.2
HOLIDAY	
≥14.2	review
	Final 12-2 12/12

Final

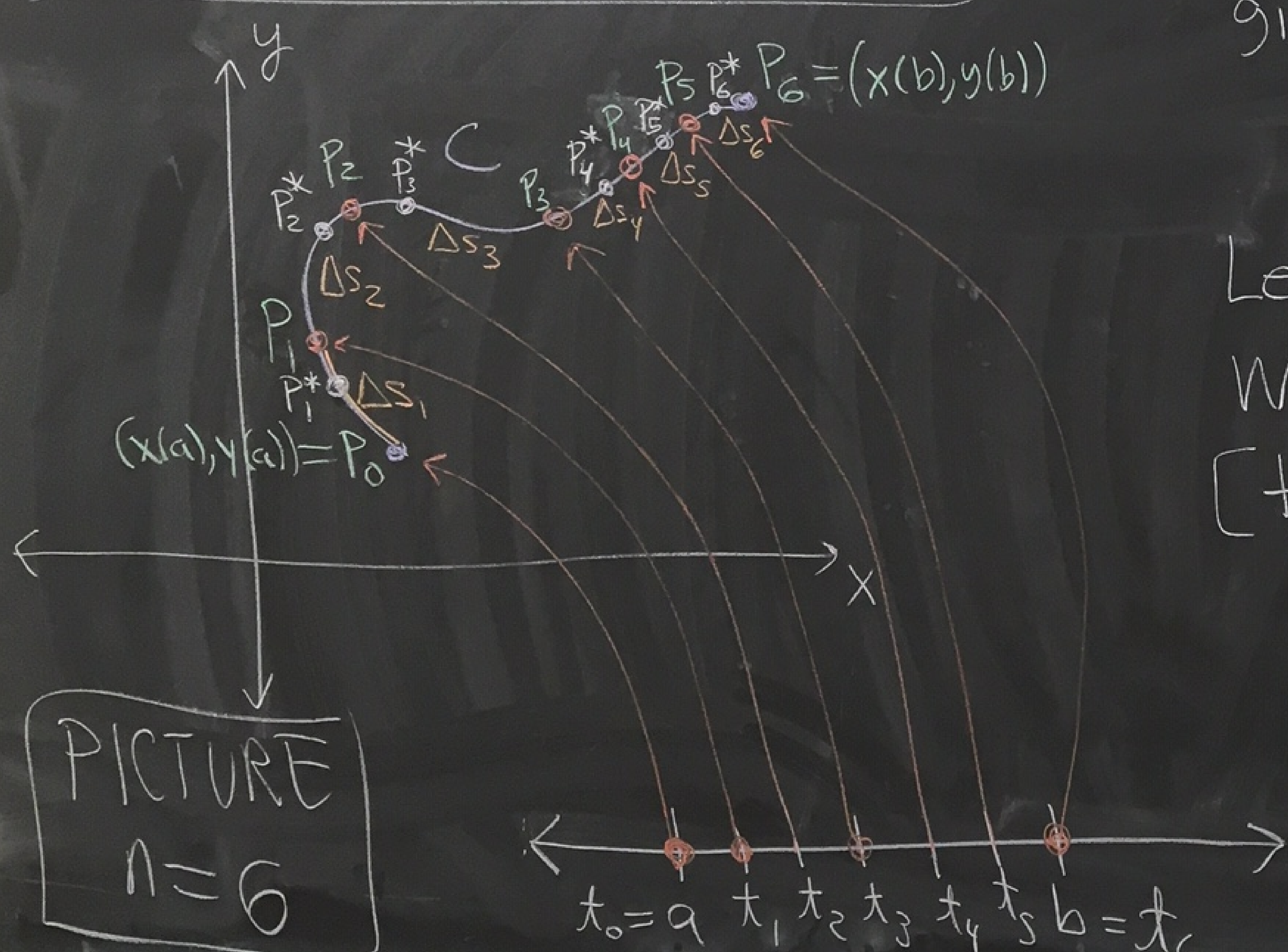
Test 1 (12.1–12.7)
Test 2 (12.8–12.9, 13.1–13.3)

New: 13.4, 13.5, 14.1

Test 2 didn't have stuff like
12.8 # 45, 46, 48
12.9 # 46–49

Maybe one of these

14.2 – Line Integrals



Let C be a curve in the plane given by the parametric equations

$$x = x(t), \quad y = y(t), \quad a \le t \le b$$

Let $\vec{r}(t) = \langle x(t), y(t) \rangle$.

We assume C is a smooth curve [that means, $\vec{r}\,'(t) \neq \vec{0}$ and $\vec{r}\,'(t)$ is continuous.]

Subdivide $[a,b]$ into n sub-intervals $[t_{i-1}, t_i]$ of equal width. Let

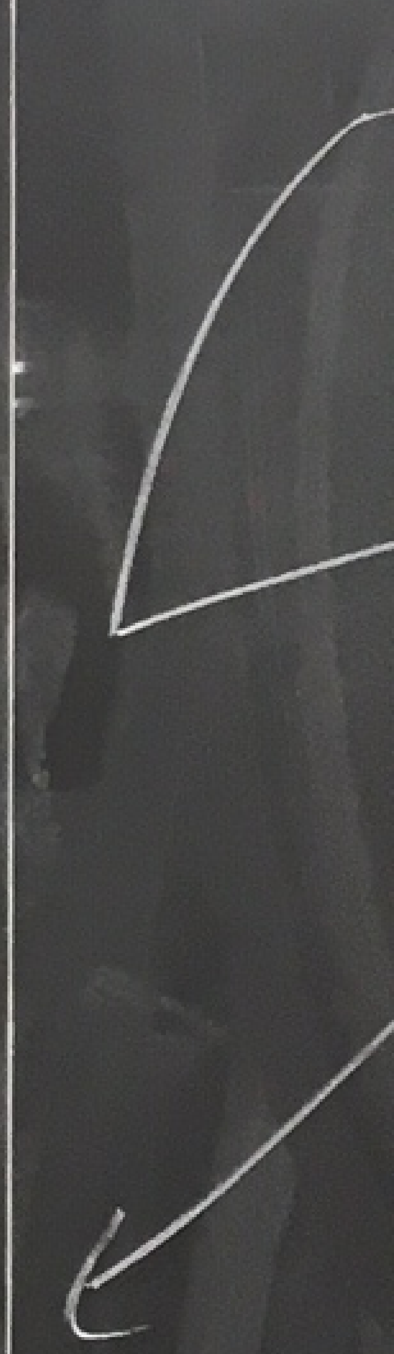
$$P_i = (x(t_i), y(t_i)) \text{ on } C.$$

Then $P_0, P_1, \ldots, P_n$ subdivide C
into n subarcs with arclengths
$\Delta s_1, \Delta s_2, \ldots, \Delta s_n$.
Choose any point $P_i^*(x_i^*, y_i^*)$
in the i-th subarc.
Now suppose we have a function $f(x,y)$
whose domain includes C. Let's integrate f over C.
We define the <u>integral of f along C</u> to be

ds is because of arclength

$$\int_C f(x,y)\,ds = \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{f(x_i^*, y_i^*)}_{f(P_i^*)}\,\Delta s_i$$

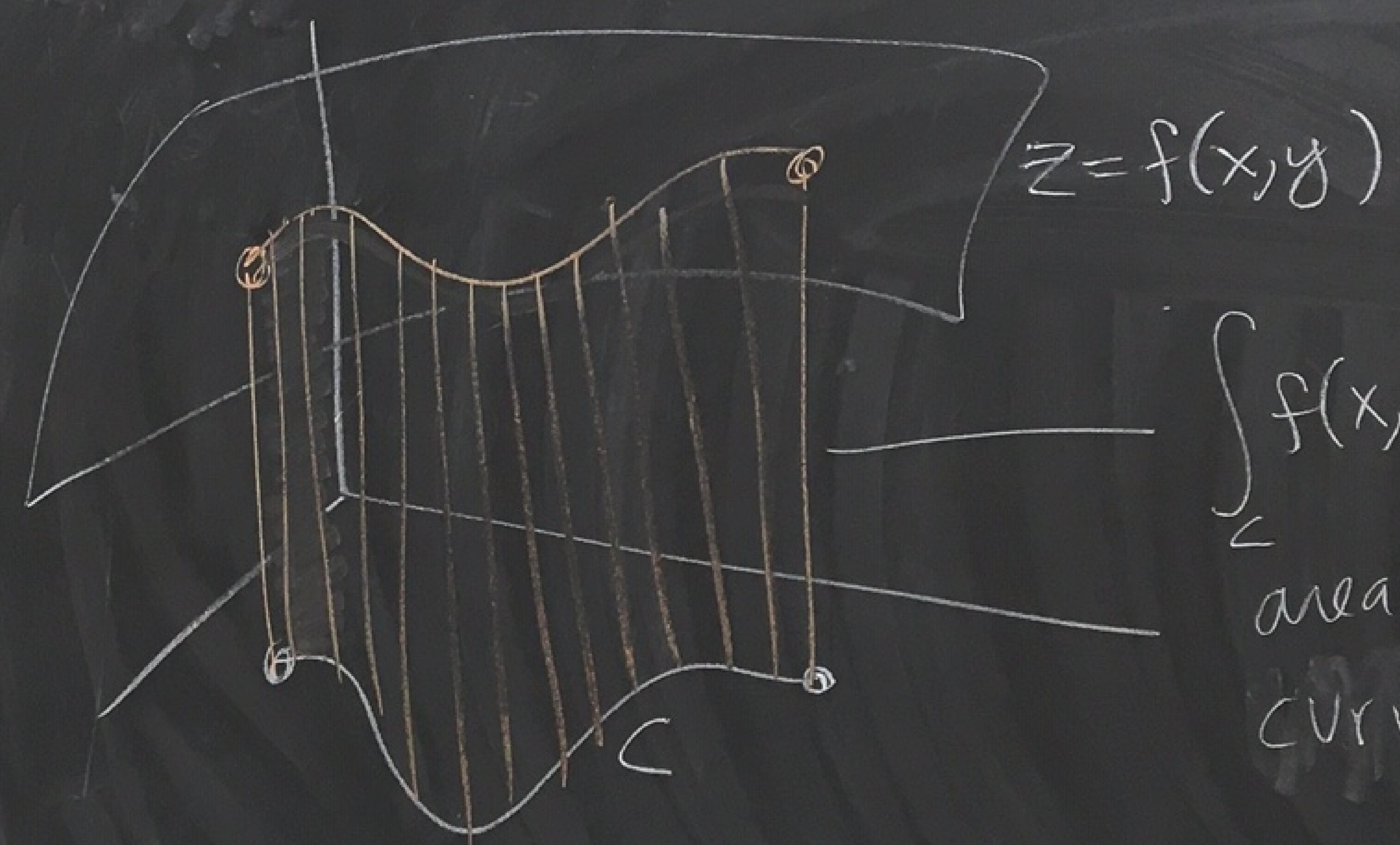
if this limit exists.



area of all the 4 boxes is

$$f(P_1^*)\Delta s_1 + f(P_2^*)\Delta s_2 + f(P_3^*)\Delta s_3 + f(P_4^*)\Delta s_4 = \sum_{i=1}^{4} f(P_i^*)\Delta s_i$$

area is
$f(P_1^*)\,\Delta s_1$
(height)*(base length)



$\int_C f(x,y)\,ds$ gives the area of this curvy wall

If $f(x,y)$ is continuous, then the limit defining $\int_C f(x,y)\,ds$ exists and

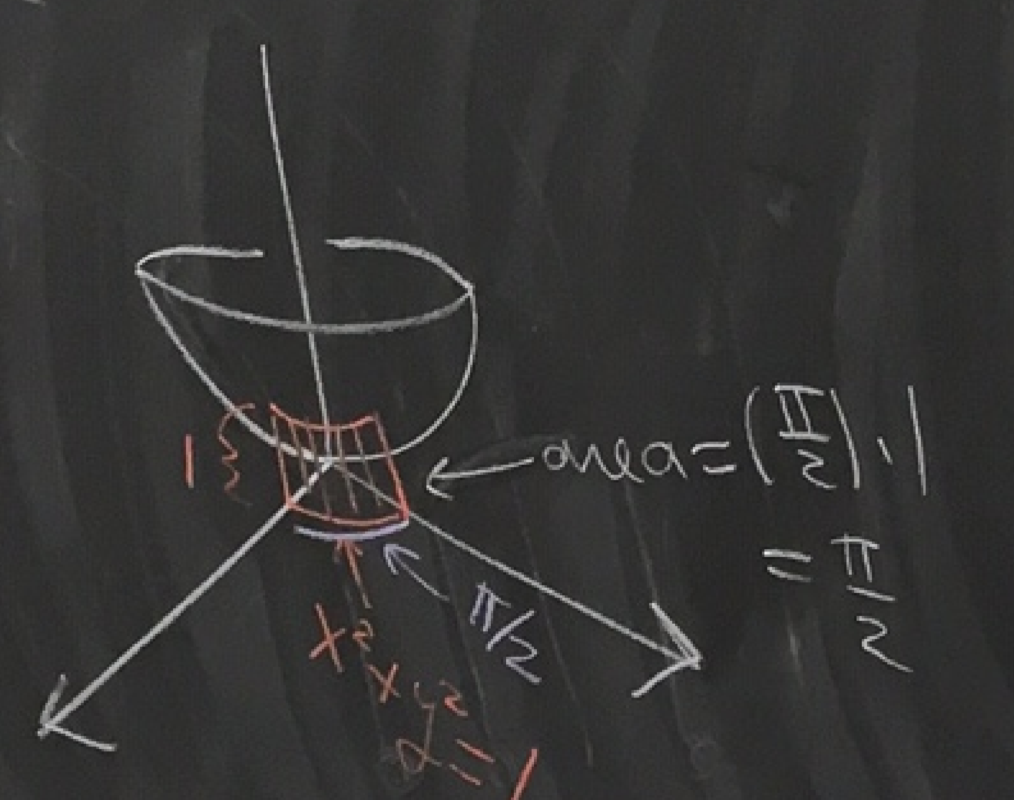
$$\int_C f(x,y)\,ds = \int_a^b f(x(t),y(t))\sqrt{(x'(t))^2+(y'(t))^2}\,dt$$

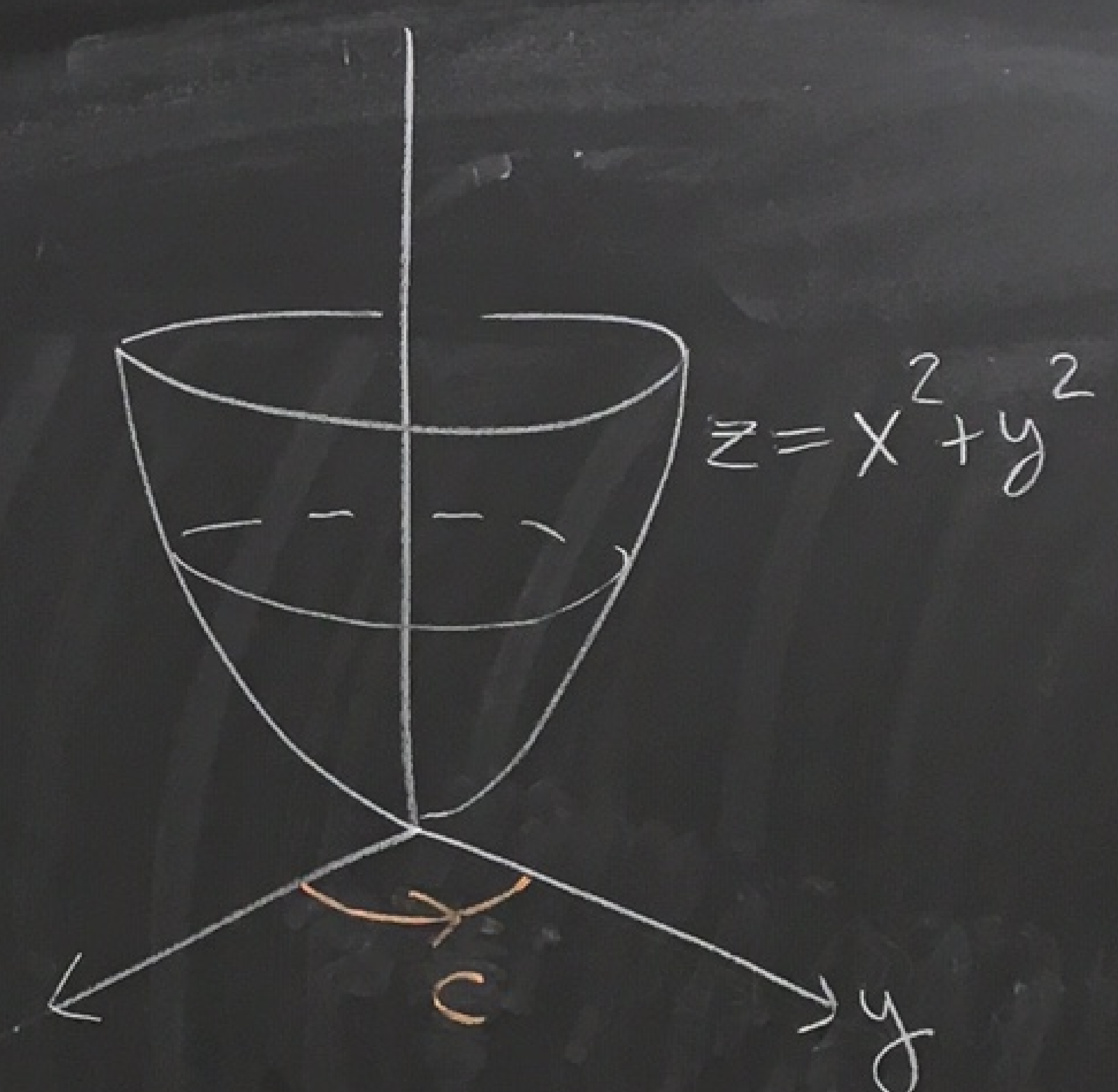
<u>Note:</u> The value of $\int_C f(x,y)\,ds$ does not depend on the parameterization of C that you choose.

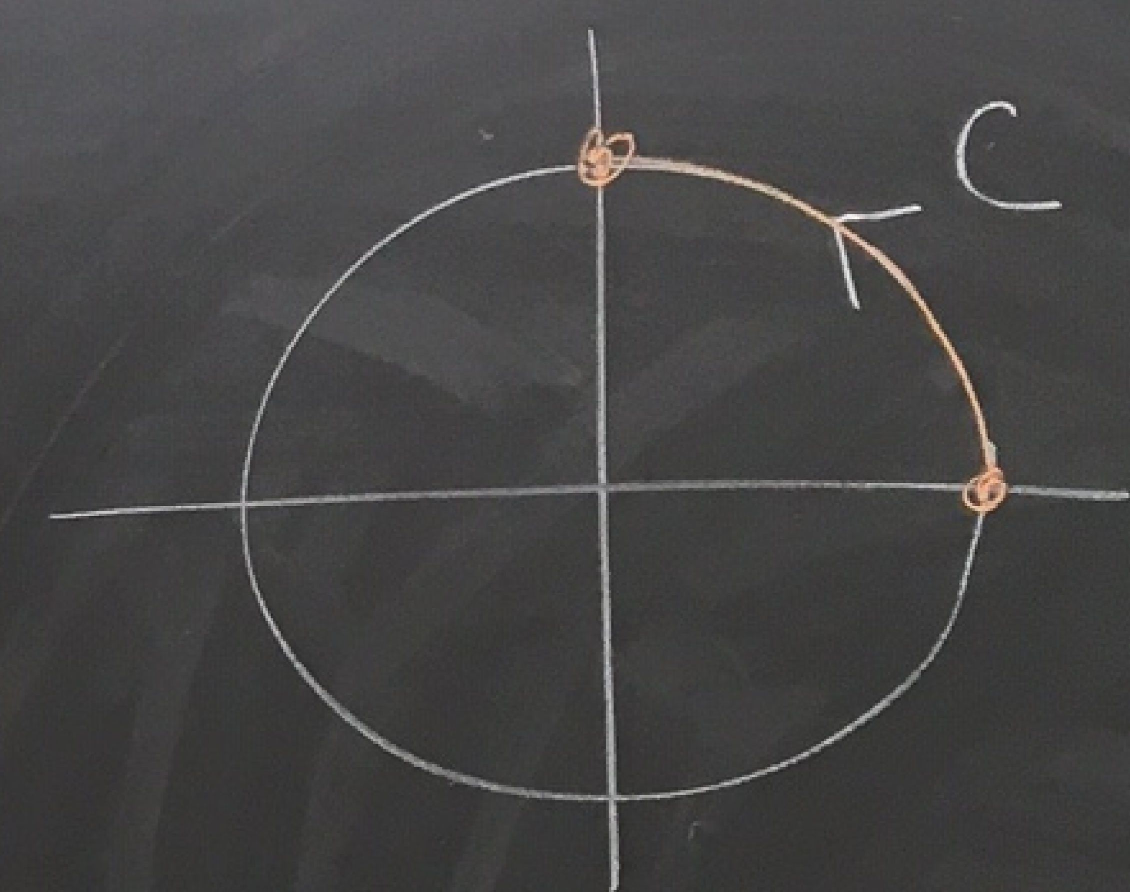
Ex: Calculate $\int_C (x^2+y^2)\,ds$

where C is the part of the unit circle in the first quadrant.

guess:







$x(t)=\cos(t)$

$y(t)=\sin(t)$

$0 \le t \le \pi/2$

$x'(t)=-\sin(t)$

$y'(t)=\cos(t)$

$$\int_C (x^2+y^2)\,ds = \int_0^{\pi/2} \underbrace{\left(\cos^2(t)+\sin^2(t)\right)}_{x^2+y^2} \underbrace{\sqrt{(-\sin(t))^2+(\cos(t))^2}}_{\sqrt{(x')^2+(y')^2}}\,dt$$

$$= \int_0^{\pi/2} (1)\sqrt{1}\,dt$$

$$= \int_0^{\pi/2} 1\,dt = t\Big|_0^{\pi/2} = \frac{\pi}{2}$$

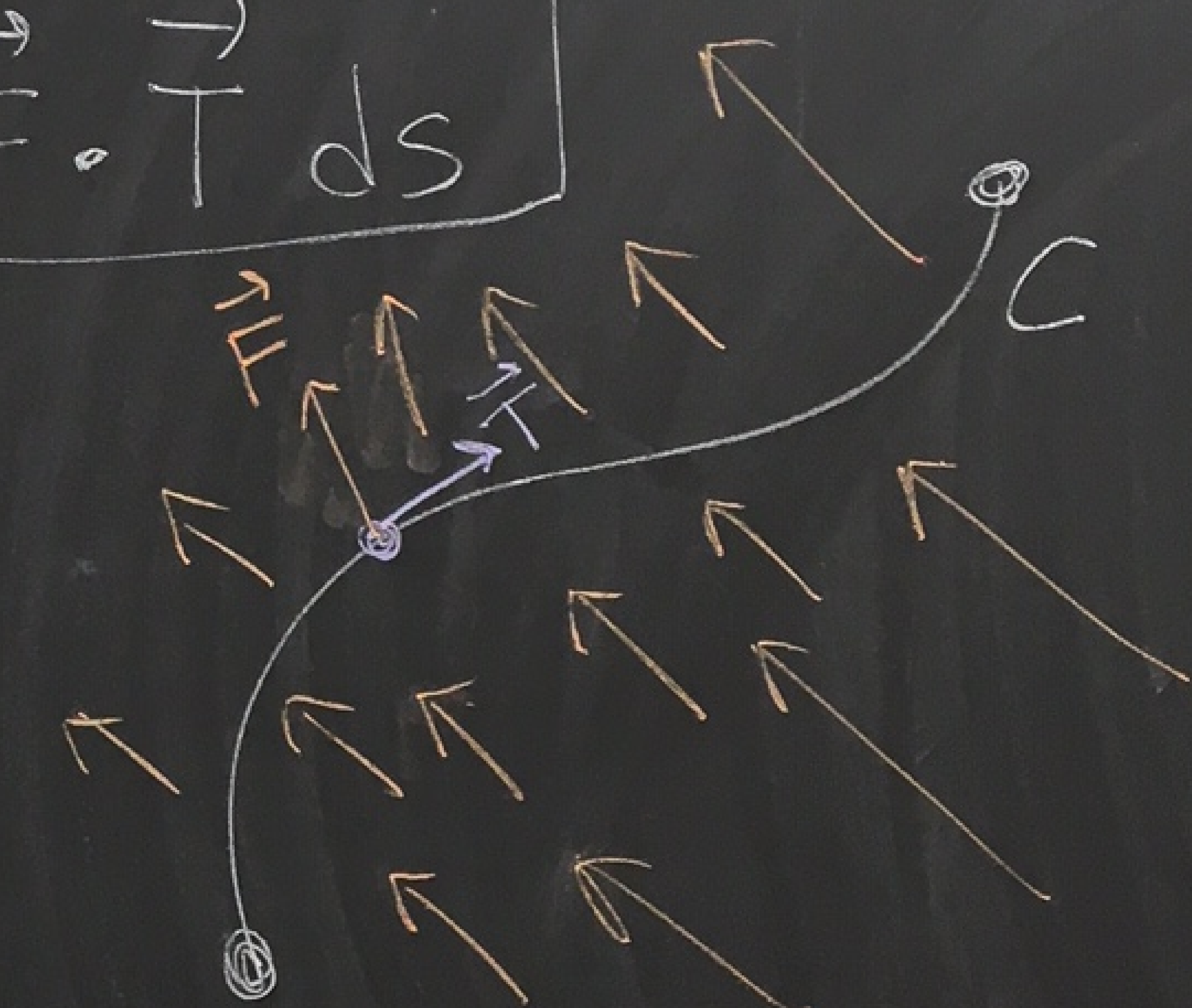
You can also integrate over a vector field.

Let $\vec{F}$ is a vector field that is continuous in a region containg a curve C.

Let $\vec{T}$ be the unit tangent vector at each point along C.

The line integral of $\vec{F}$ along C is defined to be

$$\int_C \vec{F} \cdot \vec{T}\, ds$$



$\vec{F}$ is orange

$\vec{T}$ has length 1

If C is given by $\vec{r}(t) = \langle x(t), y(t) \rangle$
with $a \leq t \leq b$ then

$$\int_C \vec{F} \cdot \vec{T}\, ds = \int_a^b \vec{F}(\vec{r}(t)) \cdot \vec{r}\,'(t)\, dt$$

Also notated by

$$\int_C \vec{F} \cdot d\vec{r}$$

We say that this is the <u>work</u> done by $\vec{F}$ along C.