Thurs,
Nov 7

$\sqrt{3} \approx 1.73$

Ex: Convert $P(x,y,z) = P(1,\sqrt{3},2)$ into spherical coordinates.

$$\rho^2 = x^2 + y^2 + z^2$$

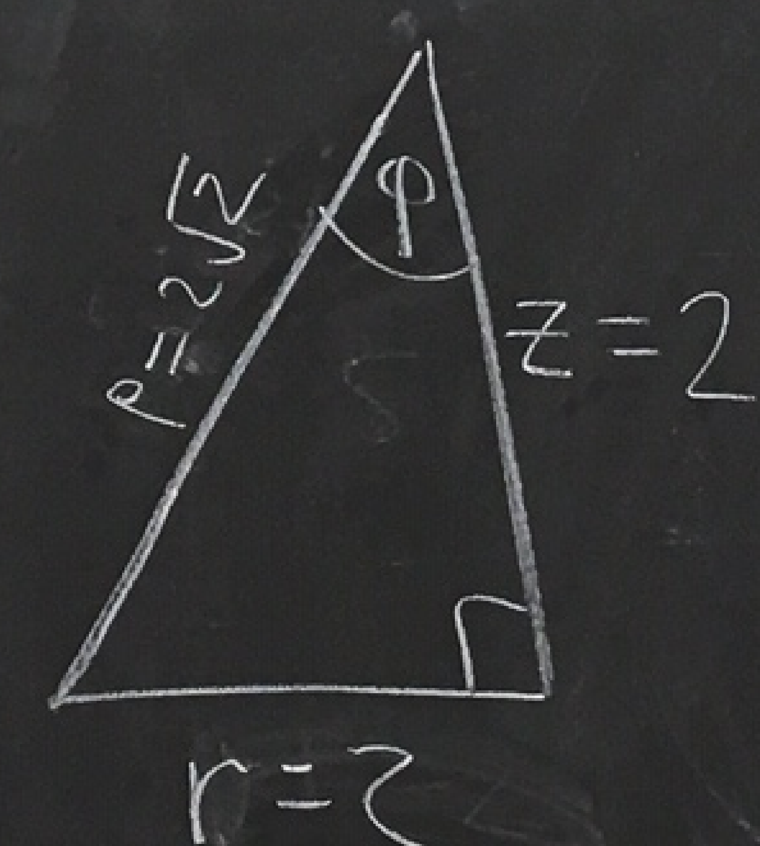
$$\rho^2 = 1^2 + \sqrt{3}^2 + 2^2 = 8$$

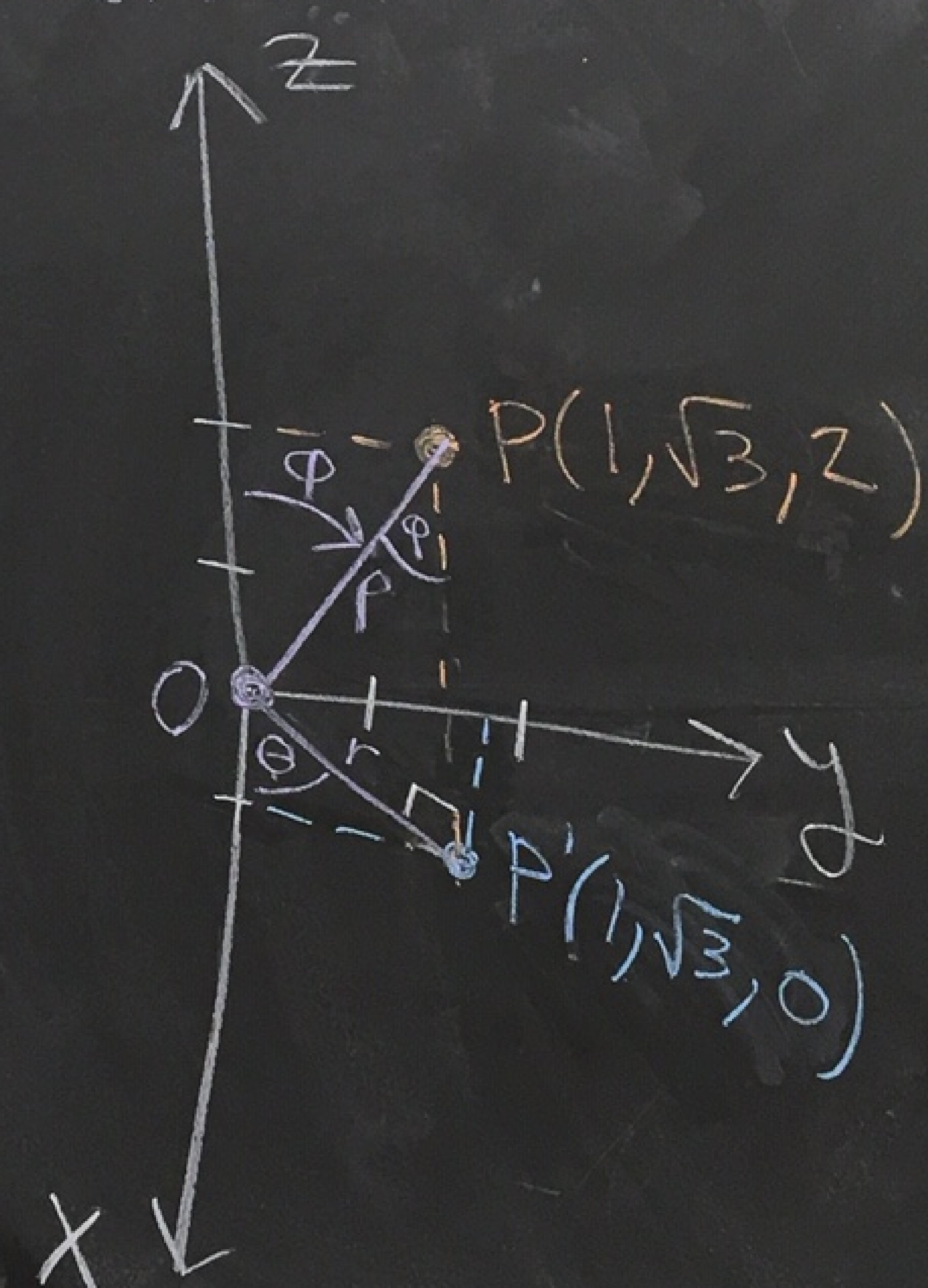
$$\boxed{\rho = \sqrt{8} = 2\sqrt{2}}$$

$$r^2 = x^2 + y^2 = 1^2 + \sqrt{3}^2$$

$$r^2 = 4$$

$$\boxed{r = 2}$$



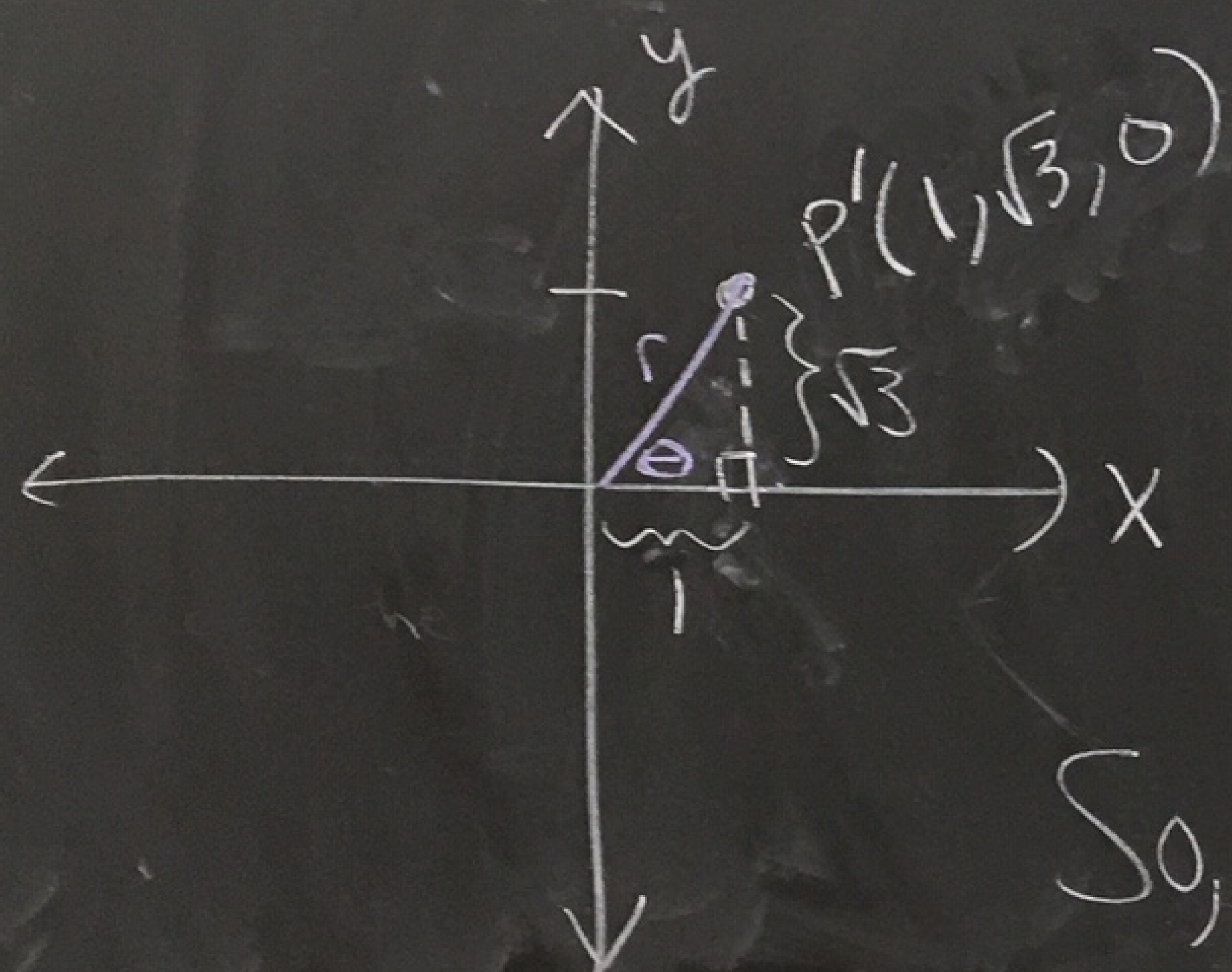


cos(

$$\cos(\varphi) = \frac{adj}{hyp} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{\varphi = \frac{\pi}{4}}$$

$$\tan(\theta) = \frac{opp}{adj} = \frac{\sqrt{3}}{1}$$

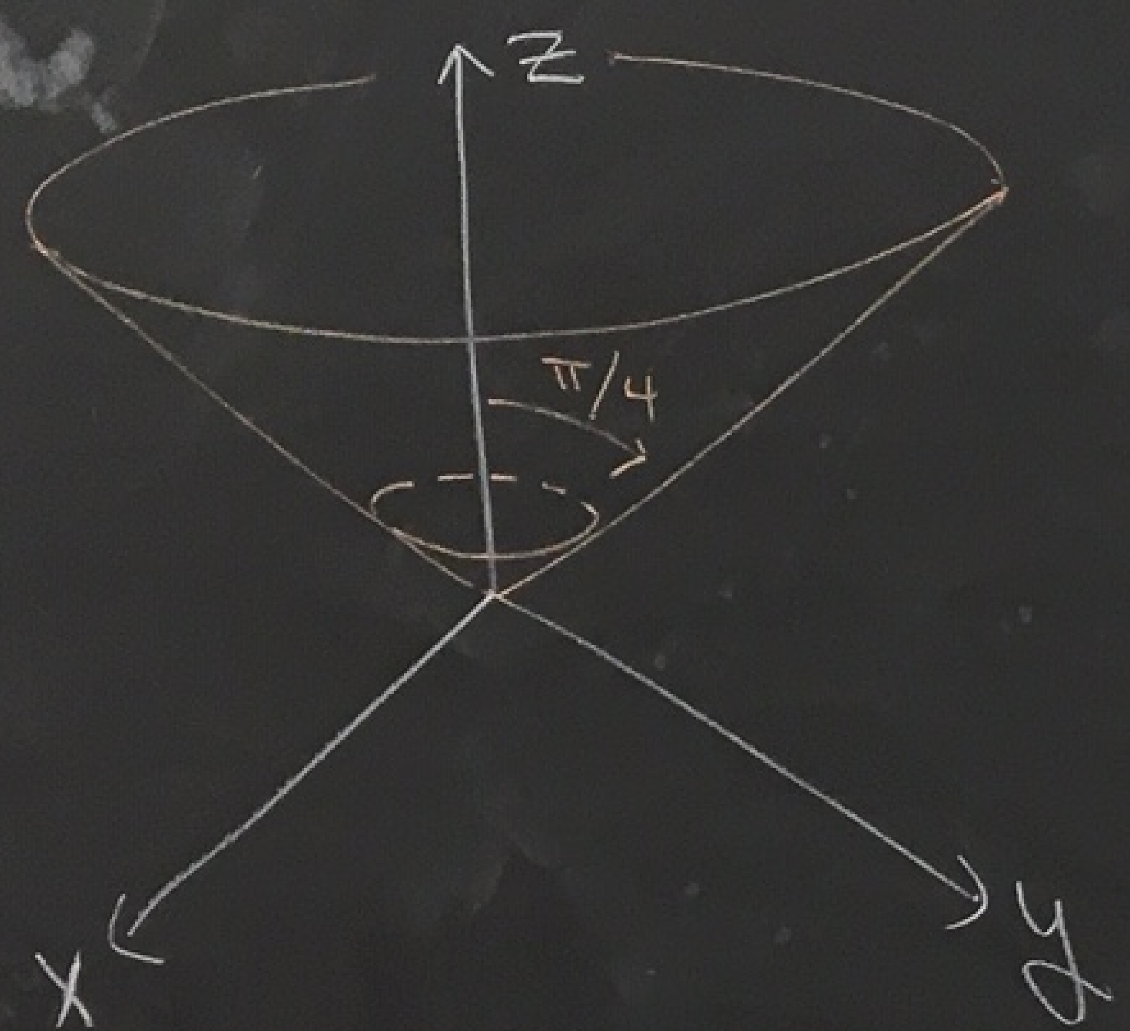


$$\boxed{\theta = \frac{\pi}{3}}$$

So, $(x,y,z) = (1,\sqrt{3},2)$

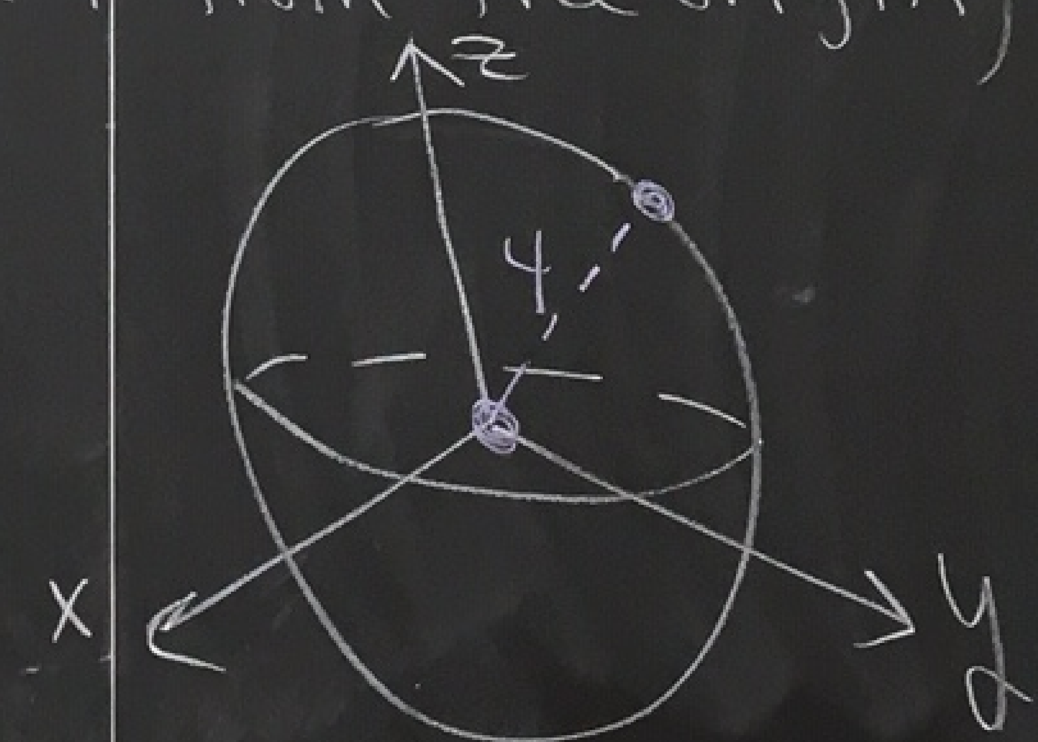
has speherical coordinates $(\rho,\varphi,\theta) = \left(2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{3}\right)$

Ex: Sketch the surface given by $\varphi = \frac{\pi}{4}$.



Ex: Sketch the surface given by $\rho = 4$

(ρ is the distance from the origin. So $\rho = 4$ consists of all points that are distance 4 from the origin)



$$\rho = 4$$
$$\rho^2 = 4^2$$
$$\underbrace{x^2 + y^2 + z^2 = 4^2}_{\text{sphere of radius 4 centered at } (0,0,0)}$$

Suppose E is a solid containing all points (ρ, φ, θ) where $a \leq \rho \leq b$, $c \leq \varphi \leq d$, and $f \leq \theta \leq g$

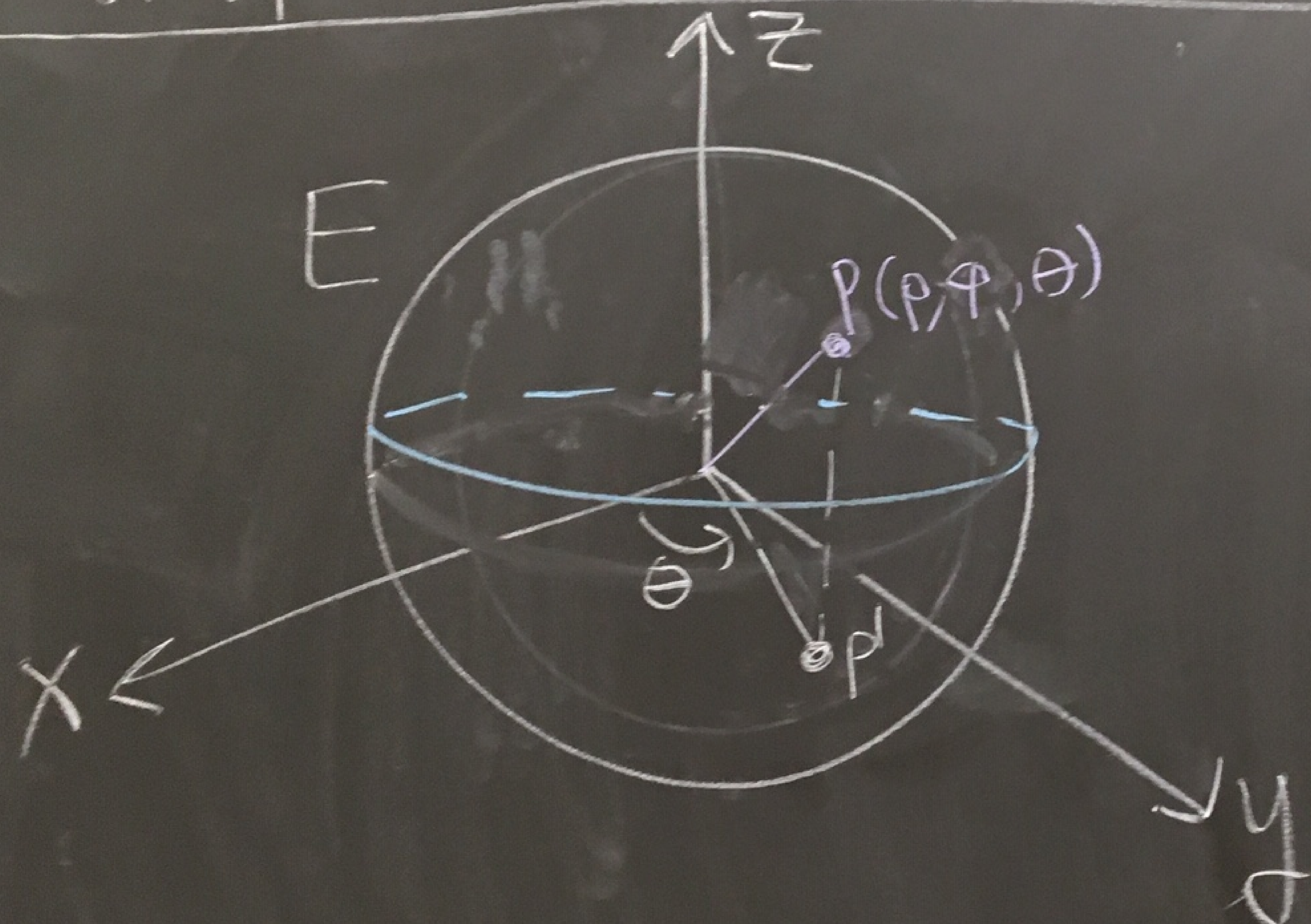
then

$$\iiint_E f(x,y,z)\,dV = \int_f^g \int_c^d \int_a^b f(\rho\sin(\varphi)\cos(\theta), \rho\sin(\varphi)\sin(\theta), \rho\cos(\varphi)) \cdot \rho^2 \sin(\varphi)\,d\rho\,d\varphi\,d\theta$$

$x = \rho\sin(\varphi)\cos(\theta)$
$y = \rho\sin(\varphi)\sin(\theta)$
$z = \rho\cos(\varphi)$

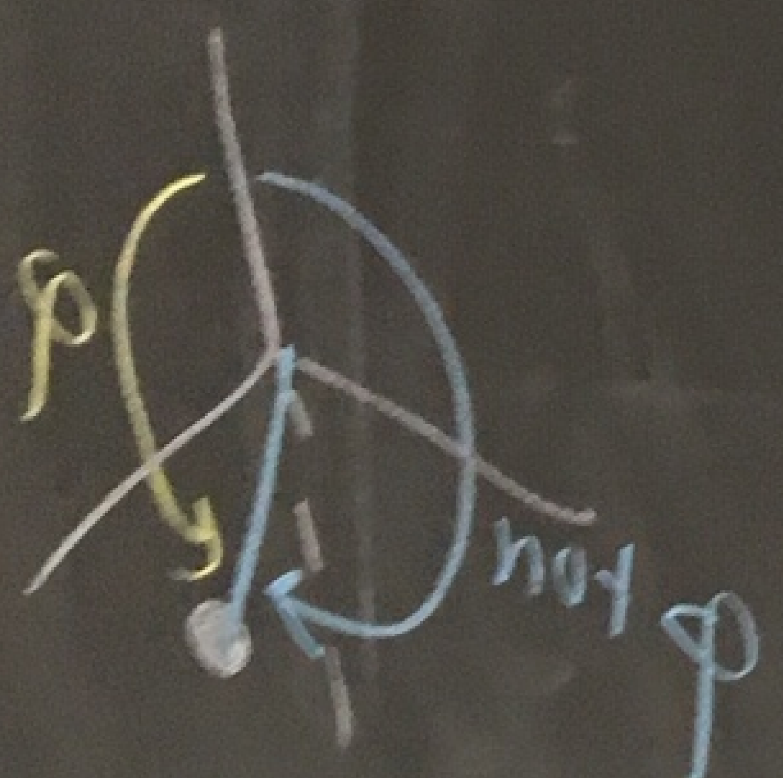
$dV = \rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta$

Ex: Find the volume of a sphere of radius $a > 0$.



parameterization of this filled in sphere

$$0 \le \rho \le a$$
$$0 \le \varphi \le \pi$$
$$0 \le \theta \le 2\pi$$



Volume of sphere $= \iiint_E 1\,dV$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{a} \underbrace{\rho^2\sin(\varphi)\,d\rho\,d\varphi\,d\theta}_{dV}$$

$$= \int_0^{2\pi}\int_0^{\pi}\left(\frac{\rho^3}{3}\sin(\varphi)\Big|_{\rho=0}^{a}\right)d\varphi\,d\theta$$

$$\int_0^{2\pi}\int_0^{\pi}\frac{a^3}{3}\sin(\varphi)\,d\varphi\,d\theta$$

$$= \frac{a^3}{3}\int_0^{2\pi}\Big[-\cos(\varphi)\Big]_{\varphi=0}^{\pi}d\theta$$

$$= \frac{a^3}{3}\int_0^{2\pi}\underbrace{\Big[\underbrace{-\cos(\pi)}_{-1}+\underbrace{\cos(0)}_{1}\Big]}_{2}d\theta$$

$$= \frac{2a^3}{3}\int_0^{2\pi}d\theta = \frac{2a^3}{3}\,\theta\Big|_0^{2\pi} = \frac{2a^3}{3}\cdot 2\pi = \boxed{\frac{4\pi a^3}{3}}$$

Ex

a

x

Ex: Find the volume of the solid E that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=z$.

$$x^2+y^2+z^2=z$$
$$x^2+y^2+z^2-z=0$$
$$x^2+y^2+z^2-z+\frac{1}{4}-\frac{1}{4}=0$$
$$x^2+y^2+\left(z-\frac{1}{2}\right)^2=\frac{1}{4}$$

$$\boxed{(x-0)^2+(y-0)^2+\left(z-\frac{1}{2}\right)^2=\left(\frac{1}{2}\right)^2}$$

sphere centered at $(0,0,\frac{1}{2})$ with radius $\frac{1}{2}$

$$z=\sqrt{x^2+y^2}$$
$$\rho\cos(\varphi)=\sqrt{\rho^2\sin^2(\varphi)\cos^2(\theta)+\rho^2\sin^2(\varphi)\sin^2(\theta)}$$
$$=\sqrt{(\rho^2\sin^2(\varphi))\underbrace{(\cos^2(\theta)+\sin^2(\theta))}_{1}}$$
$$=\sqrt{\rho^2\sin^2(\varphi)}=\rho\sin(\varphi)$$

$\rho\geq 0$, $0\leq\varphi\leq\pi$, $\sin(\varphi)\geq 0$

$$\boxed{\sqrt{t^2}=|t|=\begin{cases} t & \text{i} \\ -t & \text{i} \end{cases}}$$

$$\sqrt{(-1)^2}=1$$

$$\boxed{\sqrt{t^2} = |t| = \begin{cases} t & \text{if } t \geq 0 \\ -t & \text{if } t < 0 \end{cases}}$$

$$\boxed{\sqrt{(-1)^2} = 1 \leftarrow \boxed{t=-1}}$$

$(\varphi)\cos^2(\theta) + \rho^2\sin^2(\varphi)\sin^2(\theta)$

$(\varphi))\underbrace{(\cos^2(\theta)+\sin^2(\theta))}_{1}$

$\sin^2(\varphi)$ = $\rho\sin(\varphi)$

$\rho \geq 0, \quad 0 \leq \varphi \leq \pi, \quad \sin(\varphi) \geq 0$

$\rho\cos(\varphi) = \rho\sin(\varphi)$

$\cos(\varphi) = \sin(\varphi)$

$$\boxed{\varphi = \frac{\pi}{4}}$$