① Use the ratio test.

$$L = \lim_{k\to\infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k\to\infty} \left| \frac{(-1)^{k+1}(x-4)^{k+1}}{2^{k+1}} \cdot \frac{2^k}{(-1)^k (x-4)^k} \right|$$

$$= \lim_{k\to\infty} \left| \frac{(-1)(x-4)}{2} \right| = |x-4| \cdot \frac{1}{2}$$

Note that

$$|x-4| \cdot \frac{1}{2} < 1$$

when

$$|x-4| < 2.$$

That is when $-2 < x-4 < 2$

or $2 < x < 6.$

<u>Checking endpoints:</u>

<u>$x=2$:</u> $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{(2-4)^k}{2^k} = \sum_{k=1}^{\infty} (-1)^k \frac{(-2)^k}{2^k}$

$$= \sum_{k=1}^{\infty} (-1)^k \frac{(-1)^k 2^k}{2^k} = \sum_{k=1}^{\infty} (-1)^{2k} = \sum_{k=1}^{\infty} 1$$

diverges by divergence test

$\underline{x = 6}$:

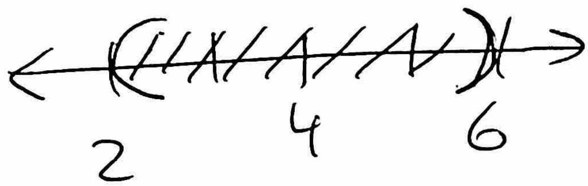
$$\sum_{k=1}^{\infty} (-1)^k \frac{(6-4)^k}{2^k} = \sum_{k=1}^{\infty} (-1)^k \frac{2^k}{2^k} = \sum_{k=1}^{\infty} (-1)^k$$

diverges by divergence test.

$\boxed{\text{Answer for \#1}}$

Interval of convergence is

$2 < x < 6$



#2 next page

(2) Let's differentiate $\dfrac{1}{(1+x^3)}$.

Note that
$$\frac{d}{dx}(1+x^3)^{-1} = -1\cdot(1+x^3)^{-2}\cdot 3x^2 = \frac{-3x^2}{(1+x^3)^2}.$$

Now
$$\frac{1}{1+x^3} = \frac{1}{1-(-x^3)} = \sum_{k=0}^{\infty}(-x^3)^k = \sum_{k=0}^{\infty}(-1)^k x^{3k}$$

$$\boxed{\begin{gathered}|-x^3|<1\\ |x|^3<1\\ |x|<1\end{gathered}}$$

So,
$$\frac{-3x^2}{(1+x^3)^2} = \frac{d}{dx}\frac{1}{1+x^3} = \frac{d}{dx}\sum_{k=0}^{\infty}(-1)^k x^{3k}$$

$$\boxed{|x|<1}$$

$$= \sum_{k=0}^{\infty}(-1)^k\, 3k\, x^{3k-1}$$

Answer

$$\boxed{= \sum_{k=1}^{\infty}(-1)^k\, 3k\, x^{3k-1} \qquad |x|<1}$$

radius of convergence is $R=1$

(3) $\frac{d}{dx} \ln(1+x) = \frac{1}{1+x}$

So,

$$\ln(1+x) = \int \frac{1}{1+x}\,dx = \int \frac{1}{1-(-x)}\,dx$$

$$= \int \sum_{k=0}^{\infty} (-x)^k\,dx = \int \sum_{k=0}^{\infty} (-1)^k x^k\,dx$$

$$\boxed{\begin{matrix} |-x|<1 \\ |x|<1 \end{matrix}}$$

$$= \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1} + C$$

When $x=0$,

$$0 = \ln(1) = \ln(1+0) = C + \sum_{k=0}^{\infty} (-1)^k \frac{0^{k+1}}{k+1}$$

$$= C.$$

So, $\ln(1+x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{k+1}}{k+1}$ where $|x|<1$

radius of convergence $R=1$

(next page for endpoint checking)

What about the endpoints?

Check $x = 1$:

$$\sum_{k=0}^{\infty} (-1)^k \frac{(1)^{k+1}}{k+1} = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$$ converges by alt. series test.

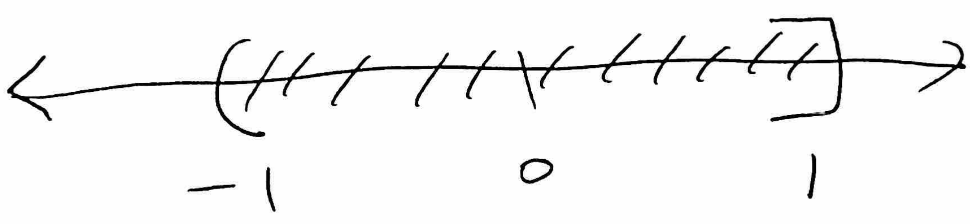
check $x = -1$:

$$\sum_{k=0}^{\infty} (-1)^k \frac{(-1)^{k+1}}{k+1} = \sum_{k=0}^{\infty} (-1)^k \frac{(-1)(-1)^k}{k+1}$$

$$= \sum_{k=0}^{\infty} (-1)^{2k} \cdot \frac{-1}{k+1}$$

$$= \sum_{k=0}^{\infty} \frac{-1}{k+1}$$ diverges by comparison to harmonic series

So interval of convergence is



$$\boxed{-1 < x \leq 1}$$

④ Recall that $\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

So,

$$\lim_{x \to 0} \frac{\sin(x)}{x} = \lim_{x \to \infty} \frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots}{x}$$

$$= \lim_{x \to 0} \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots\right)$$

$$= 1 - \frac{0^2}{3!} + \frac{0^4}{5!} - \frac{0^6}{7!} + \cdots$$

$$= 1$$

⑤ $\langle 2-1, -1-0, 3-1 \rangle$

(a) $\overrightarrow{PQ} =$ $= \langle 1, -1, 2 \rangle$

(b) $\dfrac{\vec{u}}{|\vec{u}|} = \dfrac{\langle 3, 0, 4 \rangle}{\sqrt{3^2+0^2+4^2}} = \dfrac{\langle 3, 0, 4 \rangle}{5}$

$= \left\langle \frac{3}{5}, 0, \frac{4}{5} \right\rangle$

(c) $\vec{u} \cdot \vec{v} = (3)(-3) + (0)(2) + (4)\left(\frac{9}{4}\right)$

$= -9 + 0 + 9 = 0$

So, $\vec{u}$ and $\vec{v}$ are orthogonal

(d) $\text{proj}_{\vec{w}}(\vec{u}) = \dfrac{\vec{u} \cdot \vec{w}}{|\vec{w}|^2} \vec{w}$

$= \dfrac{(3)(1)+(0)(2)+(4)(2)}{\left(\sqrt{1^2+2^2+2^2}\right)^2} \langle 1, 2, 2 \rangle = \dfrac{11}{9} \langle 1, 2, 2 \rangle$

$= \left\langle \frac{11}{9}, \frac{22}{9}, \frac{22}{9} \right\rangle$

6

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 6 \\ 2 & -5 & -3 \end{vmatrix}$$

$$= \vec{i} \begin{vmatrix} 0 & 6 \\ -5 & -3 \end{vmatrix} - \vec{j} \begin{vmatrix} -1 & 6 \\ 2 & -3 \end{vmatrix} + \vec{k} \begin{vmatrix} -1 & 0 \\ 2 & -5 \end{vmatrix}$$

$$= \vec{i}\left((0)(-3) - (6)(-5) \right)$$
$$- \vec{j}\left((-1)(-3) - (6)(2) \right)$$
$$+ \vec{k}\left((-1)(-5) - (0)(2) \right)$$

$$= 30\vec{i} + 9\vec{j} + 5\vec{k}$$

$$= \langle 30, 9, 5 \rangle$$