Math 2150-02

1/22/25

Topic 0 – Calculus review

$$\frac{d}{dx} x^n = n x^{n-1}$$

$\frac{d}{dx} x^4 = 4x^3$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} e^x = e^x$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

$n \neq -1$

$\int x^5 dx = \frac{1}{6} x^6 + C$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int e^x dx = e^x + C$$

Chain rule $\left[f(g(x))\right]' = f'(g(x)) \cdot g'(x)$

Ex: $\left[\cos(x^5)\right]' = -\sin(x^5) \cdot 5x^4$

$= -5x^4 \sin(x^5)$

$\cos$ ← f outer; x^5 ← g inner

Ex: $\frac{d}{dx} e^{\tan(5x+1)}$

$\tan(5x+1)$ ← inner; e ← outer

derivative of $5x+1$

$= e^{\tan(5x+1)} \cdot \left[\sec^2(5x+1) \cdot 5\right]$

derivative of $\tan(5x+1)$

$\frac{d}{du}\tan(u) = \sec^2(u)$

Product rule $(fg)' = f'g + fg'$

Ex:

$$\frac{d}{dx}\, 3\underbrace{x^{10}}_{f}\underbrace{\sin(x)}_{g}$$

$$= 3\left[\underbrace{10x^{9}}_{f'}\underbrace{\sin(x)}_{g} + \underbrace{x^{10}}_{f}\underbrace{\cos(x)}_{g'}\right]$$

$$= 30x^{9}\sin(x) + 3x^{10}\cos(x)$$

Substitution rule:

$$\int g(\underbrace{h(x)}_{u})\cdot \underbrace{h'(x)\,dx}_{du} = \int g(u)\,du$$

Ex:

$$\int 3\sin(\overbrace{3x}^{u})\,dx = \int \sin(u)\,du$$

du

$u = 3x$

$du = 3dx$

$$= -\cos(u) + C$$

$$= -\cos(3x) + C$$

Ex: $\displaystyle\int e^{10x}\,dx = \int e^{u}\cdot\frac{1}{10}\,du$

$u = 10x$

$du = 10\,dx$

$\frac{1}{10}\,du = dx$

$$= \frac{1}{10}\int e^{u}\,du = \frac{1}{10}e^{u} + C$$

$$= \frac{1}{10}e^{10x} + C$$

Ex: $\displaystyle\int \frac{1}{x(\ln(x))^2}\,dx$

$$\frac{1}{(\ln(x))^2}\cdot\frac{1}{x}\,dx$$

$$= \int \frac{1}{u^2}\,du = \int u^{-2}\,du$$

$u = \ln(x)$
$du = \frac{1}{x}\,dx$

$$= \frac{u^{-1}}{-1} + C$$

$$= -\frac{1}{u} + C$$

$$= -\frac{1}{\ln(x)} + C$$

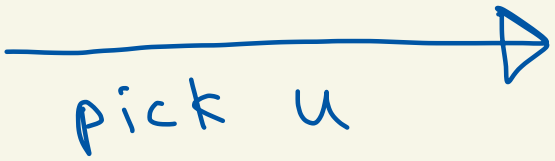
Integration by parts

$$\int u\,dv = uv - \int v\,du$$

Picking u:

L I A T E

- L: log
- I: inverse trig
- A: algebraic
- T: trig
- E: exponential



Ex:

algebraic

exponential

$$\int \underbrace{x}_{u} \underbrace{e^x\,dx}_{dv}$$

$u = x$ $\quad dv = e^x dx$

$du = dx$ $\quad v = e^x$

$$= x e^x - \int e^x dx$$

$$= x e^x - e^x + C$$