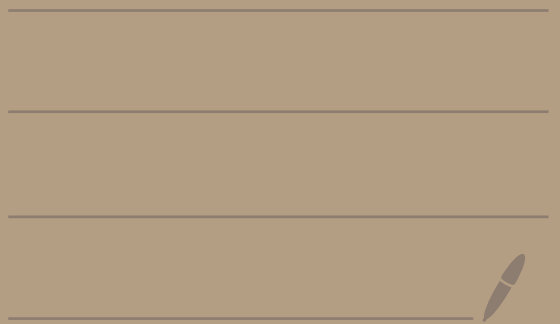


Math 2150
8/21/24



Topic 0 - Calculus review

Some derivatives and integrals

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\int x^n dx = \frac{1}{n+1} x^{n+1} + C$$

if $n \neq -1$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int \sin(x) dx = -\cos(x) + C$$

$$\int \cos(x) dx = \sin(x) + C$$

$$\int e^x dx = e^x + C$$

Chain rule:

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

Ex: $\frac{d}{dx} e^{x^2} = e^{x^2} \cdot 2x$
 $= 2x e^{x^2}$

Ex:

$$\begin{aligned}(\sin(2x))' &= \cos(2x) \cdot 2 \\ &= 2\cos(2x)\end{aligned}$$

Substitution examples

$$\int f(g(x)) \cdot g'(x) dx = \int f(u) du$$

$$\begin{aligned}u &= g(x) \\ du &= g'(x) dx\end{aligned}$$

Ex:

$$\int \sin(3x) dx = \int \frac{1}{3} \sin(u) du$$

$$\begin{aligned}u &= 3x \\ du &= 3 dx \\ \frac{1}{3} du &= dx\end{aligned}$$

$$= -\frac{1}{3} \cos(u) + C$$

$$= -\frac{1}{3} \cos(3x) + C$$

Ex: $\int e^{5x} dx = \frac{1}{5} e^{5x} + C$

↑
you could do $u=5x$
sub if you want to

Ex: $\int \frac{1}{x(\ln(x))^2} dx = \int \frac{1}{(\ln(x))^2} \cdot \frac{1}{x} dx$

$= \int \frac{1}{u^2} du = \int u^{-2} du$

$= \frac{1}{-1} u^{-1} + C$

$= -\frac{1}{\ln(x)} + C$

↑
 $u = \ln(x)$
 $du = \frac{1}{x} dx$

Integration by parts

$$\int u dv = uv - \int v du$$

$$\int \underbrace{x}_u \underbrace{e^x dx}_{dv} = x e^x - \int e^x dx =$$

$$\begin{array}{ll} u = x & du = dx \\ dv = e^x dx & v = e^x \end{array}$$

$$\rightarrow = x e^x - e^x + C$$

LIATE

for u

L = log

I = inverse trig

A = algebraic

T = trig

E = exponential

Ex:

$$\int \underbrace{x^2}_u \underbrace{\sin(x)}_{dv} dx = -x^2 \cos(x) + \int 2x \cos(x) dx$$

$$\begin{aligned} u &= x^2 & du &= 2x dx \\ dv &= \sin(x) dx & v &= -\cos(x) \end{aligned}$$

$$= -x^2 \cos(x) + 2 \int \underbrace{x}_u \underbrace{\cos(x)}_{dv} dx$$

$$= -x^2 \cos(x) + 2 \left[x \sin(x) - \int \sin(x) dx \right]$$

$$\begin{aligned} u &= x & du &= dx \\ dv &= \cos(x) dx & v &= \sin(x) \end{aligned}$$

$$= -x^2 \cos(x) + 2x \sin(x) + 2 \cos(x) + C$$

Topic 1 - What is a differential equation?

Def.

- An equation relating an unknown function and one or more of its derivatives is called a differential equation.
- If the unknown function depends on only a single independent variable then the equation is called an ordinary differential equation (ODE).
- If the unknown function depends on two or more variables and contains partial derivatives then the equation is called a partial differential equation (PDE).

- The order of a differential equation is the order of the highest derivative that appears in the equation.
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Ex:

$$\frac{dy}{dx} = y^2$$

ODE

$y \leftarrow y = y(x)$ is a function of x

$x \leftarrow$ independent variable

$y \leftarrow$ dependent variable

order is 1

Ex:

$$4x^2 y'' + y = 0$$

ODE

$y = y(x)$ is a function of x

order is 2

Ex:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

PDE

$$u = u(x, t)$$

is a function of
 x and t

order is 2