Math 2150
8/21/24

Topic 0 - Calculus review

Some derivatives and integrals

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = -\sin(x)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} e^x = e^x$$

$$\frac{d}{dx} \tan(x) = \sec^2(x)$$

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$$

if $n \neq -1$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int \sin(x) \, dx = -\cos(x) + C$$

$$\int \cos(x) \, dx = \sin(x) + C$$

$$\int e^x \, dx = e^x + C$$

Chain rule:

$$\big(f(g(x))\big)' = f'(g(x)) \cdot g'(x)$$

Ex: $\frac{d}{dx} e^{x^2} = e^{x^2} \cdot 2x$

$= 2x e^{x^2}$

Ex:

$$\big(\sin(2x)\big)' = \cos(2x)\cdot 2$$

$$= 2\cos(2x)$$

Substitution examples

$$\int f(g(x))\cdot g'(x)\,dx = \int f(u)\,du$$

$u = g(x)$

$du = g'(x)\,dx$

Ex:

$$\int \sin(3x)\,dx = \int \tfrac{1}{3}\sin(u)\,du$$

$u = 3x$

$du = 3\,dx$

$\tfrac{1}{3}du = dx$

$$= \tfrac{-1}{3}\cos(u) + C$$

$$= -\tfrac{1}{3}\cos(3x) + C$$

Ex: $\int e^{5x}\,dx = \frac{1}{5}e^{5x} + C$

You could do $u = 5x$ sub if you want to

Ex: $\int \frac{1}{x(\ln(x))^2}\,dx = \int \frac{1}{(\ln(x))^2} \cdot \frac{1}{x}\,dx$

$= \int \frac{1}{u^2}\,du = \int u^{-2}\,du$

$= \frac{1}{-1}u^{-1} + C$

$= -\frac{1}{\ln(x)} + C$

$u = \ln(x)$

$du = \frac{1}{x}dx$

Integration by parts

$$\int u\,dv = uv - \int v\,du$$

$$\int \underbrace{x}_{u}\underbrace{e^x\,dx}_{dv} = xe^x - \int e^x\,dx =$$

$u = x \qquad du = dx$

$dv = e^x dx \qquad v = e^x$

$$= xe^x - e^x + C$$

LIATE

for u

L = log

I = inverse trig

A = algebraic

T = trig

E = exponential

Ex:

$$\int x^2 \sin(x)\,dx = -x^2\cos(x) + \int 2x\cos(x)\,dx$$

u dv

$u = x^2$ $\quad du = 2x\,dx$

$dv = \sin(x)\,dx$ $\quad v = -\cos(x)$

$$= -x^2\cos(x) + 2\int x\cos(x)\,dx$$

u dv

$$= -x^2\cos(x) + 2\left[x\sin(x) - \int \sin(x)\,dx\right]$$

$u = x$ $\quad du = dx$

$dv = \cos(x)\,dx$ $\quad v = \sin(x)$

$$= -x^2\cos(x) + 2x\sin(x) + 2\cos(x) + C$$

Topic 1 - What is a differential equation?

Def:

- An equation relating an unknown function and one or more of its derivatives is called a differential equation.
- If the unknown function depends on only a single independent variable then the equation is called an ordinary differential equation (ODE)
- If the unknown function depends on two or more variables and contains partial derivatives then the equation is called a partial differential equation. (PDE)

- The <u>order</u> of a differential equation is the order of the highest derivative that appears in the equation.

Ex: $\boxed{\frac{dy}{dx} = y^2}$

ODE

$y \leftarrow$ $y = y(x)$ is a function of x

$x \leftarrow$ independent variable

$y \leftarrow$ dependent variable

Order is 1

Ex:

$\boxed{4x^2y'' + y = 0}$

ODE

$y = y(x)$ is a function of x

order is 2

Ex:

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$$

PDE

$u = u(x,t)$

is a function of x and t

order is 2