Math 2550-04

9/18/24

Ex: Solve

$$\boxed{\begin{aligned} x+y+2z&=9\\ 2x+4y-3z&=1\\ 3x+6y-5z&=0 \end{aligned}}$$

want a 1 here

We get:

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9\\ 2 & 4 & -3 & 1\\ 3 & 6 & -5 & 0 \end{array}\right) \quad -2R_1+R_2\to R_2$$

Use the 1 to make these 0

$$\begin{array}{rrrr|r} & (-2 & -2 & -4 & -18)\\ + & (2 & 4 & -3 & 1)\\ \hline & (0 & 2 & -7 & -17) \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9\\ 0 & 2 & -7 & -17\\ 3 & 6 & -5 & 0 \end{array}\right) \quad -3R_1+R_3\to R_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 2 & -7 & -17 \\ 0 & 3 & -11 & -27 \end{array}\right)$$

$$\begin{array}{r} (-3 \quad -3 \quad -6 \mid -27) \\ +(3 \quad 6 \quad -5 \mid 0) \\ \hline (0 \quad 3 \quad -11 \mid -27) \end{array}$$

make this 1

$$\xrightarrow{\frac{1}{2}R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} \\ 0 & 3 & -11 & -27 \end{array}\right)$$

use the 1 to make this 0

$$\xrightarrow{-3R_2 + R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & -\frac{7}{2} & -\frac{17}{2} \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} \end{array}\right)$$

$$\frac{21}{2} - 11 = \frac{21-22}{2} = \frac{-1}{2}$$

$$\frac{51}{2} - 27 = \frac{51-54}{2} = \frac{-3}{2}$$

make this 1

$$-2R_3 \to R_3 \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 1 & -7/2 & -17/2 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

this is in row echelon form

Write down equations:

$$\boxed{\begin{aligned} x + y + 2z &= 9 \\ y - \tfrac{7}{2} z &= -17/2 \\ z &= 3 \end{aligned}}$$

leading variables: x, y, z

free variables: none

Solve for leading:

$$\boxed{\begin{aligned} x &= 9 - y - 2z \quad &(1) \\ y &= -17/2 + \tfrac{7}{2} z \quad &(2) \\ z &= 3 \quad &(3) \end{aligned}}$$

Back substitution:

③ $z = 3$

② $y = -\frac{17}{2} + \frac{7}{2}z = -\frac{17}{2} + \frac{7}{2}(3) = 2$

① $x = 9 - y - 2z = 9 - 2 - 2(3) = 1$

$$\boxed{\begin{array}{l}\text{Answer is} \\ x = 1,\ y = 2,\ z = 3\end{array}}$$

Ex: Solve

$$\boxed{\begin{aligned} -2b + 3c &= 1 \\ 3a + 6b - 3c &= -2 \\ 6a + 6b + 3c &= 5 \end{aligned}}$$

We get

Want a 1 here

$$\left(\begin{array}{ccc|c} 0 & -2 & 3 & 1 \\ 3 & 6 & -3 & -2 \\ 6 & 6 & 3 & 5 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} 3 & 6 & -3 & -2 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{array}\right)$$

$$\xrightarrow{\frac{1}{3}R_1 \to R_1} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 6 & 6 & 3 & 5 \end{array}\right)$$

make these 0

$$\xrightarrow{-6R_1 + R_3 \to R_3} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & -2 & 3 & 1 \\ 0 & -6 & 9 & 9 \end{array}\right)$$

make this 1

$$\xrightarrow{-\frac{1}{2}R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -\frac{3}{2} & -\frac{1}{2} \\ 0 & -6 & 9 & 9 \end{array}\right)$$

use the 1 to make this 0

$$6R_2+R_3 \to R_3 \quad \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2/3 \\ 0 & 1 & -3/2 & -1/2 \\ 0 & 0 & 0 & 6 \end{array}\right)$$

in row echelon form

Write down the equations:

$$\boxed{\begin{aligned} a+2b-c &= -2/3 \\ b-\tfrac{3}{2}c &= -1/2 \\ 0 &= 6 \end{aligned}}$$

this tells us that the system has no solutions

Answer:

No solutions

Ex: Solve

$$\boxed{\begin{aligned} 5x_1 - 2x_2 + 6x_3 &= 0 \\ -2x_1 + x_2 + 3x_3 &= 1 \end{aligned}}$$

We get

make this 1

$$\left(\begin{array}{ccc|c} 5 & -2 & 6 & 0 \\ -2 & 1 & 3 & 1 \end{array}\right)$$

$$\xrightarrow{\frac{1}{5}R_1 \to R_1} \left(\begin{array}{ccc|c} 1 & -2/5 & 6/5 & 0 \\ -2 & 1 & 3 & 1 \end{array}\right)$$

make this 0

$$\xrightarrow{2R_1 + R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & -2/5 & 6/5 & 0 \\ 0 & 1/5 & 27/5 & 1 \end{array}\right)$$

$\frac{12}{5} + 3 = \frac{27}{5}$

make this 1

$$\xrightarrow{5R_2 \to R_2} \left(\begin{array}{ccc|c} 1 & -2/5 & 6/5 & 0 \\ 0 & 1 & 27 & 5 \end{array}\right)$$

in row echelon form

Write down equations:

$$\boxed{\begin{aligned} x_1 - \tfrac{2}{5}x_2 + \tfrac{6}{5}x_3 &= 0 \quad (1) \\ x_2 + 27x_3 &= 5 \quad (2) \end{aligned}}$$

leading variables: x_1, x_2

free variables: x_3

Give free variable a new name and solve for leading variables:

$$x_1 = \tfrac{2}{5}x_2 - \tfrac{6}{5}x_3 \quad (1)$$

$$x_2 = 5 - 27x_3 \quad (2)$$

$$x_3 = t$$ ③

Back substitute:

③ $x_3 = t$

② $x_2 = 5 - 27x_3 = 5 - 27t$

① $x_1 = \frac{2}{5}x_2 - \frac{6}{5}x_3$

$$= \frac{2}{5}(5 - 27t) - \frac{6}{5}t$$

$$= 2 - \frac{54}{5}t - \frac{6}{5}t$$

$$= 2 - 12t$$

So,

$x_1 = 2 - 12t$

$x_2 = 5 - 27t$

$x_3 = t$

where t can be any number

What does this mean?

There are infinitely many solutions, one for each value of t.

Here are some of them:

t	$x_1 = 2-12t$	$x_2 = 5-27t$	$x_3 = t$
0	2	5	0
1	−10	−22	1
−10	122	275	−10
⋮	⋮	⋮	⋮