

Math 2550-04

9/4/24



Ex:

$$\begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix} + \begin{pmatrix} 0 & 7 \\ -\frac{1}{2} & 6 \end{pmatrix} = \begin{pmatrix} 5+0 & 1+7 \\ 2-\frac{1}{2} & -3+6 \end{pmatrix}$$
$$= \begin{pmatrix} 5 & 8 \\ \frac{3}{2} & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 3 \\ -1 & -5 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ -1 & 7 \\ 6 & -3 \end{pmatrix} = \begin{pmatrix} 1-2 & 0-3 \\ 2+1 & 3-7 \\ -1-6 & -5+3 \end{pmatrix}$$
$$= \begin{pmatrix} -1 & -3 \\ 3 & -4 \\ -7 & -2 \end{pmatrix}$$

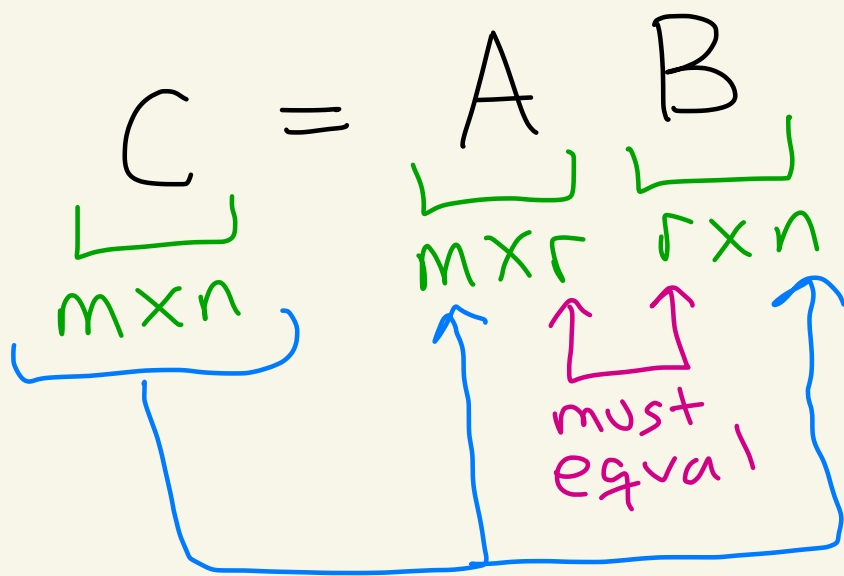
$$(-3) \cdot \begin{pmatrix} 2 & 6 \\ 7 & -2 \end{pmatrix} = \begin{pmatrix} (-3)(2) & (-3)(6) \\ (-3)(7) & (-3)(-2) \end{pmatrix}$$

$$= \begin{pmatrix} -6 & -18 \\ -21 & 6 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 6 \\ 2 & 0 \end{pmatrix}}_{2 \times 2} + \underbrace{\begin{pmatrix} 1 & 2 \\ 7 & 3 \\ 1 & 2 \end{pmatrix}}_{3 \times 2}$$

← this addition is undefined since the matrices aren't the same size

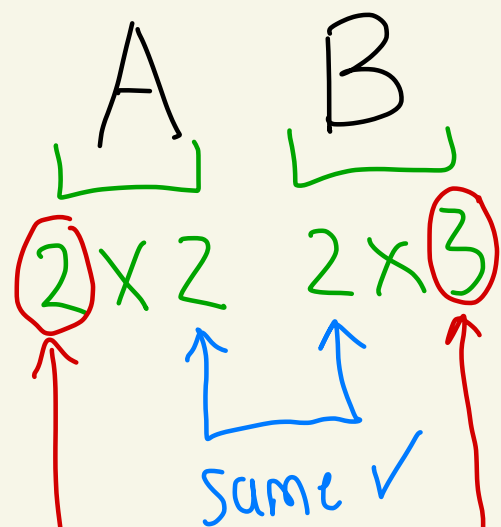
Def: Let A be an $m \times r$ matrix and let B be an $r \times n$ matrix. We define the product of A and B , denoted by AB , as the $m \times n$ matrix C whose entry in row i and column j is defined as the dot product of row i of A and column j of B .



Ex: Let

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

Calculate AB if possible.



answer will
be 2×3

$$AB =$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} =$$

(row 1 of A) ·
(col 1 of B)

(row 1 of A) ·
(col 2 of B)

(row 1 of A) ·
(col 3 of B)

$$= \begin{pmatrix} (1 \ 2) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\ (-1 \ 0) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (-1 \ 0) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} & (-1 \ 0) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{pmatrix}$$

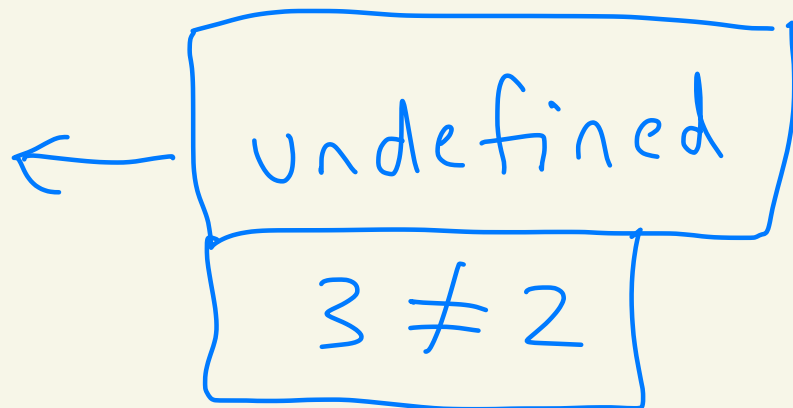
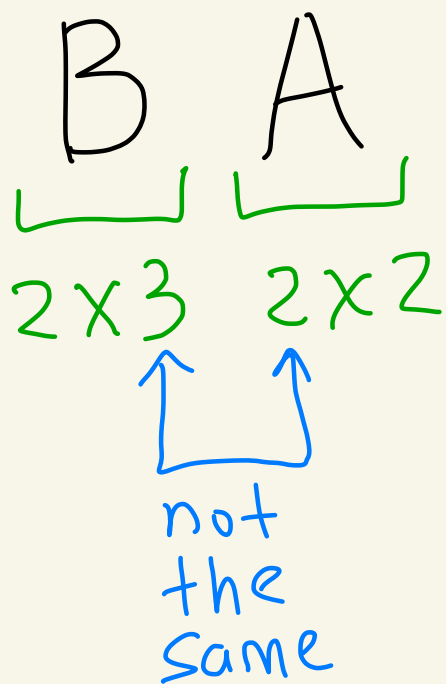
(row 2 of A) · (col 1 of B) (row 2 of A) · (col 2 of B) (row 2 of A) · (col 3 of B)

$$= \begin{pmatrix} (1)(1) + (2)(0) & (1)(2) + (2)(1) & (1)(-1) + (2)(0) \\ (-1)(1) + (0)(0) & (-1)(2) + (0)(1) & (-1)(-1) + (0)(0) \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

Ex: Using the same matrices

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

can we calculate BA ?



You can see why you can't do this product by trying to do it.

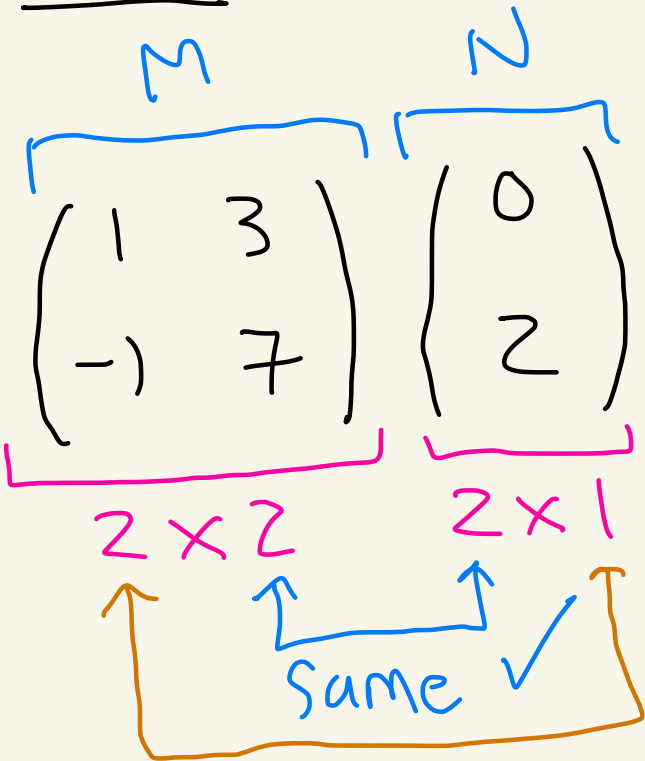
$$BA = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$$

(row 1 of B) ·
(col 1 of A)

$$(1 \ 2 \ -1) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

this dot product isn't defined. That's why you can't calculate BA.

Ex: Calculate



answer is
2x1

(row 1 of M) ·
(col 1 of N)

$$= \begin{pmatrix} (1 \ 3) \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix} \\ (-1 \ 7) \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} (1)(0) + (3)(2) \\ (-1)(0) + (7)(2) \end{pmatrix}$$

(row 2 of M) ·
(col 1 of N)

$$= \begin{pmatrix} 6 \\ 14 \end{pmatrix}$$

Note: For matrices

$AB = BA$ is not always true. For example above we saw AB was defined but BA wasn't.

Try for practice:

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_A \underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_B \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_A = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

not
equal
 $AB \neq BA$

Def: Let A be an $m \times n$ matrix. The transpose of A , denoted by A^T , is defined to be the $n \times m$ matrix that results by interchanging the rows and columns of A .

That is, the i -th row of A^T is the i -th column of A .
Or the j -th column of A^T is the j -th row of A .

Ex:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

A is
3 x 2

$$A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

A^T is
2 x 3

OR

$$A = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

Def: The $m \times n$ zero matrix is the $m \times n$ matrix where every entry is zero.

We will denote this matrix by $O_{m \times n}$ or just O if we don't want to mention the size.

Ex: $O_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$O_{3 \times 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$O_{5 \times 1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Ex: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$A + O_{2 \times 2} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

$$O_{2 \times 2} + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

So,

$$A + O_{2 \times 2} = A$$

$$O_{2 \times 2} + A = A$$

Def: The $n \times n$ identity matrix denoted by I_n or sometimes just I is the $n \times n$ matrix with 1's along the main diagonal and 0's everywhere else.

Ex:

$$I_1 = (1)$$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and so on...