Math 2550-04
9/4/24

Ex:

$$\begin{pmatrix} 5 & 1 \\ 2 & -3 \end{pmatrix} + \begin{pmatrix} 0 & 7 \\ -\frac{1}{2} & 6 \end{pmatrix} = \begin{pmatrix} 5+0 & 1+7 \\ 2-\frac{1}{2} & -3+6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 8 \\ \frac{3}{2} & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 3 \\ -1 & -5 \end{pmatrix} - \begin{pmatrix} 2 & 3 \\ -1 & 7 \\ 6 & -3 \end{pmatrix} = \begin{pmatrix} 1-2 & 0-3 \\ 2+1 & 3-7 \\ -1-6 & -5+3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -3 \\ 3 & -4 \\ -7 & -2 \end{pmatrix}$$

$$(-3) \cdot \begin{pmatrix} 2 & 6 \\ 7 & -2 \end{pmatrix} = \begin{pmatrix} (-3)(2) & (-3)(6) \\ (-3)(7) & (-3)(-2) \end{pmatrix}$$

$$= \begin{pmatrix} -6 & -18 \\ -21 & 6 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 6 \\ 2 & 0 \end{pmatrix}}_{2\times 2} + \underbrace{\begin{pmatrix} 1 & 2 \\ 7 & 3 \\ 1 & 2 \end{pmatrix}}_{3\times 2}$$

this addition is undefined since the matrices aren't the same size

Def: Let A be an $m \times r$ matrix and let B be an $r \times n$ matrix. We define the product of A and B, denoted by AB, as the $m \times n$ matrix C whose entry in row i and column j is defined as the dot product of row i of A and column j of B.

$$\underbrace{C}_{m \times n} = \underbrace{A}_{m \times r}\ \underbrace{B}_{r \times n}$$

must equal

Ex: Let

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

Calculate AB if possible.

$$\underbrace{A}_{2 \times 2} \quad \underbrace{B}_{2 \times 3}$$

same ✓

answer will be 2×3

$$AB =$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} \overbrace{\begin{pmatrix}1 & 2\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}}^{\substack{\text{(row 1 of A)}\cdot\\ \text{(col 1 of B)}}} & \overbrace{\begin{pmatrix}1 & 2\end{pmatrix}\cdot\begin{pmatrix}2\\1\end{pmatrix}}^{\substack{\text{(row 1 of A)}\cdot\\ \text{(col 2 of B)}}} & \overbrace{\begin{pmatrix}1 & 2\end{pmatrix}\cdot\begin{pmatrix}-1\\0\end{pmatrix}}^{\substack{\text{(row 1 of A)}\cdot\\ \text{(col 3 of B)}}} \\ \underbrace{\begin{pmatrix}-1 & 0\end{pmatrix}\cdot\begin{pmatrix}1\\0\end{pmatrix}}_{\substack{\text{(row 2 of A)}\cdot\\ \text{(col 1 of B)}}} & \underbrace{\begin{pmatrix}-1 & 0\end{pmatrix}\cdot\begin{pmatrix}2\\1\end{pmatrix}}_{\substack{\text{(row 2 of A)}\cdot\\ \text{(col 2 of B)}}} & \underbrace{\begin{pmatrix}-1 & 0\end{pmatrix}\cdot\begin{pmatrix}-1\\0\end{pmatrix}}_{\substack{\text{(row 2 of A)}\cdot\\ \text{(col 3 of B)}}} \end{pmatrix}$$

$$= \begin{pmatrix} (1)(1)+(2)(0) & (1)(2)+(2)(1) & (1)(-1)+(2)(0) \\ (-1)(1)+(0)(0) & (-1)(2)+(0)(1) & (-1)(-1)+(0)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

Ex: Using the same matrices

$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

can we calculate BA?

$$\underbrace{B}_{2\times 3}\ \underbrace{A}_{2\times 2}$$

not the same

undefined

$3 \neq 2$

You can see why you can't do this product by trying to do it.

$$BA = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$$

(row 1 of B) ·
(col 1 of A)

$$= \begin{pmatrix} \begin{pmatrix} 1 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} & \\ & \end{pmatrix}$$

this dot product isn't defined. Thats why you can't calculate BA.

Ex: Calculate

$$\underbrace{\overbrace{\begin{pmatrix} 1 & 3 \\ -1 & 7 \end{pmatrix}}^{M}}_{2\times 2}\underbrace{\overbrace{\begin{pmatrix} 0 \\ 2 \end{pmatrix}}^{N}}_{2\times 1}$$

same ✓

answer is 2×1

$$= \begin{pmatrix} \overbrace{\begin{pmatrix} 1 & 3 \end{pmatrix}\cdot\begin{pmatrix} 0 \\ 2 \end{pmatrix}}^{\text{(row 1 of M)}\cdot\text{(col 1 of N)}} \\ \\ \underbrace{\begin{pmatrix} -1 & 7 \end{pmatrix}\cdot\begin{pmatrix} 0 \\ 2 \end{pmatrix}}_{\text{(row 2 of M)}\cdot\text{(col 1 of N)}} \end{pmatrix} = \begin{pmatrix} (1)(0)+(3)(2) \\ \\ (-1)(0)+(7)(2) \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 14 \end{pmatrix}$$

<u>Note:</u> For matrices $AB = BA$ is not always true. For example above we saw AB was defined but BA wasn't.

Try for practice:

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{B} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

not equal $AB \neq BA$

$$\underbrace{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}}_{B} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}}_{A} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

Def: Let A be an $m \times n$ matrix. The *transpose* of A, denoted by A^T, is defined to be the $n \times m$ matrix that results by interchanging the rows and columns of A.

That is, the i-th row of A^T is the i-th column of A.

Or the j-th column of A^T is the j-th row of A.

Ex:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

A is 3x2

$$A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

A^T is 2x3

OR

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

<u>Def:</u> The $m \times n$ <u>zero matrix</u> is the $m \times n$ matrix where every entry is zero.
We will denote this matrix by $0_{m \times n}$ or just 0 if we don't want to mention the size.

<u>Ex:</u>

$$0_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$0_{3 \times 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$0_{5 \times 1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Ex: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$A + O_{2\times 2} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

$$O_{2\times 2} + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

So,

$$A + O_{2\times 2} = A$$

$$O_{2\times 2} + A = A$$

Def: The $n \times n$ identity matrix denoted by I_n or sometimes just I is the $n \times n$ matrix with 1's along the main diagonal and 0's everywhere else.

Ex:

$$I_1 = \begin{pmatrix} 1 \end{pmatrix}$$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and so on...