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HW 6

Part 1

① $V = \mathbb{R}^3$, $F = \mathbb{R}$

(a) four elements of V:

$\langle 0,0,0 \rangle$, $\langle -1, \pi, 2 \rangle$, $\langle \frac{1}{2}, e, -7 \rangle$, $\langle 3, 2, 1 \rangle$

four elements of F: 1, π, $\frac{1}{2}$, -17

(b) Let $\vec{v}, \vec{w}$ be in $V = \mathbb{R}^3$.

Then $\vec{v} = \langle a, b, c \rangle$ and $\vec{w} = \langle x, y, z \rangle$ where a, b, c, x, y, z are real numbers.

We have that

$$\begin{aligned}
\vec{v} + \vec{w} &= \langle a, b, c \rangle + \langle x, y, z \rangle \\
&= \langle a + x, b + y, c + z \rangle \\
&= \langle x + a, y + b, z + c \rangle \\
&= \langle x, y, z \rangle + \langle a, b, c \rangle \\
&= \vec{w} + \vec{v}
\end{aligned}$$

real numbers satisfy $m + n = n + m$

(c) Let $\vec{v}, \vec{w}$ be in $V = \mathbb{R}^3$ and α be in $F = \mathbb{R}$

Then $\vec{v} = \langle a, b, c \rangle$ and $\vec{w} = \langle x, y, z \rangle$

where a, b, c, x, y, z are real numbers.

We have that

$$
\begin{aligned}
\alpha \cdot (\vec{v} + \vec{w}) &= \alpha \overbrace{\left(\langle a, b, c \rangle + \langle x, y, z \rangle \right)}^{\text{because of the parentheses we have to compute this first}} \\
&= \alpha \langle a + x, b + y, c + z \rangle \\
&= \langle \alpha(a + x), \alpha(b + y), \alpha(c + z) \rangle \\
&= \langle \alpha a + \alpha x, \alpha b + \alpha y, \alpha c + \alpha z \rangle \\
&= \langle \alpha a, \alpha b, \alpha c \rangle + \langle \alpha x, \alpha y, \alpha z \rangle \\
&= \alpha \langle a, b, c \rangle + \alpha \langle x, y, z \rangle \\
&= \alpha \cdot \vec{v} + \alpha \cdot \vec{w}
\end{aligned}
$$

real numbers satisfy $m(n + t) = mn + mt$

② $V = P_2 = \{a + bx + cx^2 \mid a, b, c \in \mathbb{R}\}$

$F = \mathbb{R}$

(a) four elements from $V = P_2$:

$1 + x - x^2$

$1 = 1 + 0 \cdot x + 0 \cdot x^2$

$-\pi + \frac{1}{2}x + 10{,}032\, x^2$

$-13 + x^2$

four elements from $F = \mathbb{R}$:

$3, -\pi, \frac{1}{3}, 1.57932$

(b) Let $\vec{v}, \vec{w}, \vec{z}$ be in $V = P_2$.

Then, $\vec{v} = a_1 + b_1 x + c_1 x^2$,

$\vec{w} = a_2 + b_2 x + c_2 x^2$,

and $\vec{z} = a_3 + b_3 x + c_3 x^3$,

where $a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3$ are real numbers.

We have that

compute this first because of the parentheses

$$(\vec{v} + \vec{w}) + \vec{z} =$$

$$= \left((a_1 + b_1 x + c_1 x^2) + (a_2 + b_2 x + c_2 x^2) \right) + (a_3 + b_3 x + c_3 x^2)$$

$$= \left((a_1 + a_2) + (b_1 + b_2) x + (c_1 + c_2) x^2 \right) + (a_3 + b_3 x + c_3 x^2)$$

$$= \left[(a_1 + a_2) + a_3 \right] + \left[(b_1 + b_2) + b_3 \right] x + \left[(c_1 + c_2) + c_3 \right] x^3$$

real numbers satisfy: $(m+n)+t = m+(n+t)$

$$= \left[a_1 + (a_2 + a_3) \right] + \left[b_1 + (b_2 + b_3) \right] x + \left[c_1 + (c_2 + c_3) \right] x^3$$

(c) Let $\vec{v}$ be in $V = P_2$ and α, β be in $F = \mathbb{R}$.

Then $\vec{v} = a + bx + cx^2$ where a, b, c are real numbers

We have that

$$
\begin{aligned}
(\alpha\beta)\cdot\vec{v} &= (\alpha\beta)\left[a + bx + cx^2\right] \\
&= (\alpha\beta)a + (\alpha\beta)bx + (\alpha\beta)cx^2 \\
&= \alpha(\beta a) + \alpha(\beta b)x + \alpha(\beta c)x^2 \\
&= \alpha\left[\beta a + \beta b x + \beta c x^2\right] \\
&= \alpha\left[\beta\left(a + bx + cx^2\right)\right] \\
&= \alpha\cdot\left[\beta\cdot\vec{v}\right]
\end{aligned}
$$

③ $V = \left\{ \begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix} \,\middle|\, a, b \text{ are real numbers} \right\}, F = \mathbb{R}$

(a) Four elements from V:

$$\begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}, \begin{pmatrix} \pi & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & 1 \\ 1 & -\sqrt{2} \end{pmatrix}$$

(b) V is not closed under addition,

To be a vector space we must have that if $\vec{v}, \vec{w}$ are in V, then $\vec{v} + \vec{w}$ is in V.

This is not true for this V.

For example,

$\vec{v} = \begin{pmatrix} 2 & 1 \\ 1 & -5 \end{pmatrix}$ and $\vec{w} = \begin{pmatrix} \pi & 1 \\ 1 & 0 \end{pmatrix}$ are in V

but $\vec{v} + \vec{w} = \begin{pmatrix} 2+\pi & 2 \\ 2 & -5 \end{pmatrix}$ is not in V

since it isn't of the form $\begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix}$.

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(a) Four elements of $V = \mathbb{R}^2$:

$\langle 1, -1 \rangle$, $\langle 0, 0 \rangle$, $\langle \pi, -\frac{1}{2} \rangle$, $\langle 7, -1 \rangle$

(b) We defined a new scalar multiplication: $\boxed{\alpha \langle a, b \rangle = \langle 2\alpha a, 2\alpha b \rangle}$

$$3 \cdot \langle 1, -2 \rangle = \langle 2 \cdot 3 \cdot 1, 2 \cdot 3 \cdot (-2) \rangle$$
$$= \langle 6, -12 \rangle$$

(c) $V = \mathbb{R}^2$ is not a vector space using regular vector addition and this new scalar multiplication. For example, part 7 of being a vector space is not true: $1 \cdot \vec{v} = \vec{v}$

For example, if $\vec{v} = \langle 4, 3 \rangle$, then

$$1 \cdot \vec{v} = 1 \cdot \langle 4, 3 \rangle = \langle 2 \cdot 1 \cdot 4, 2 \cdot 1 \cdot 3 \rangle = \langle 8, 6 \rangle \neq \vec{v}$$