$Math 2550-01$ Math 2550-01 10/21/24

Ex: In R3, let $\vec{a} = (-1, 0, 1),$ $\vec{b} = \zeta - 5$, - 3 , $2\sum_{i=1}^{n}$ $\vec{c} = \langle 1,1,0 \rangle$ Then $E_X: I_n \R^3, let$
 $\vec{a} = \langle -1, 0, 1 \rangle, \vec{b} = \langle -5, -3, 2 \rangle, \vec{c} = \langle 1, 0, 1 \rangle$

Then,
 $\langle -5, -3, 2 \rangle = 2 \langle -1, 0, 1 \rangle - 3 \langle 1, 1, 0 \rangle$
 $\vec{b} = 2\vec{a} - 3\vec{c}$

Since \vec{b} is a linear combination

at a and \vec{c} we say that -)
رn 3, $\{2\} = 2 \langle -1, 0, 1 \rangle - 3 \langle 1, 1, 0 \rangle$ $\vec{b} = 2\vec{a} - 3\vec{c} \leq$ $Since$ b is a linear combination that $\frac{2}{\lambda^{2}}$ $\frac{1}{\lambda^{2}}$ $\frac{1}{\lambda^{2}}$, , b is a linear ent
and $\frac{1}{c}$ we say that
I are linearly dependent. Note we can rewrite the $\begin{array}{ccc} \text{volC} & \text{v} & \text$ $\begin{array}{ccc} 1 & 0 & 0 \\ 2 & 0 & -1 \cdot b & -3200 \\ 1 & 0 & 0 & 0 \end{array}$ $3\vec{c} = 0$

The vectors
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\vec{v}_1, \vec{v}_2, \dots, \vec{v}_r
$$
 are linearly
\n i_0 dependent if and only if
\nthe only solution to
\n $c_1\vec{v}_1 + c_2\vec{v}_2 + ... + c_r\vec{v}_r = 0$
\n $i_1 c_1 = 0, c_2 = 0, ..., c_r = 0$.
\nIf there are more solutions
\nthen the vectors are linearly
\ndependent.
\nEx: Are $\vec{v}_1 = \langle 1, -2, 1 \rangle$
\n $\vec{v}_2 = \langle 1, 0, 1 \rangle$, $\vec{v}_3 = \langle 0, 1, 0 \rangle$
\nlinearly dependent or linearly

independent ? We need to solve leper
neer
at i $+ c₂ v₂ + c₃ v₃ = 0$ that is $c_1 < 1, -2,$ $\begin{array}{l} 1 \setminus 1 \\ 1 \setminus 1 \setminus 1 \setminus 1 \end{array}$ 0 $\overline{\mathcal{L}}$ $17 + C_3 < 0,1$ $\bigg\langle$ $=\big<$ 0,0,0 $\big>$ ${\cal O}$ We need to solve
 $C_1V_1 + C_2V_2 + C_3V_3 = 0$

that is
 $C_1C_2 + C_2C_1 + C_2C_2 + C_3C_3 = 0$
 $\Rightarrow C_1C_2 + C_2C_1 + C_2C_1 + C_2C_2 + C_3C_3 = 0$

We get
 $\Rightarrow C_1C_2 + C_2C_1 + C_3C_1 + C_2 = 0.0$

This gives
 $\Rightarrow C_1 + C_2C_1 + C_3C_1 + C_2 = 0.0$

We get $\langle c_{1j} - 2c_{1j} \rangle$ c_1) + $(c_2, 0, 0)$ $(c_2) + (0, c_3, 0)$ $=$ $(0, 0)$, \circ \rangle

This gives gives
- - 7 c + c 2 , C + C > = (0) $\overline{\mathsf{O}}$, $0/$ C_2 = $(0,0,0)$

We get
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C_{1} + C_{2} = 0
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-2C_{1} + C_{2} = 0
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\nSolving we get
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C_{1} + C_{2} = 0
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C_{1} + C_{2} = 0
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C_{9} + C_{1} = 0
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$$
S = \frac{C_1}{C_2} = -\frac{C_2}{2}C_3
$$

 $\Big| C_3 = \pm \ \Big| (3)$ Back substitution gives $3c_3 = t$ 3 C₃ = \pm

(2) C₂ = $-\frac{1}{2}$ C₃ = - $\frac{1}{2}$ S = $\frac{1}{2}C$

(1) $C_1 = -C$

(1) $C_1 = -C$ $B = -\overline{2} \times (-\frac{1}{2} \pm) = \frac{1}{2} \pm \frac{1}{2}$ Plugging this back into
 $C_1V_1+C_2V_2+C_3V_3=0$ $c_2 = -\frac{1}{2}c_3 = -\frac{1}{2}\overline{t}$
 $c_1 = -c_2 = -(-\frac{1}{2}\overline{t})$
 $ygying this back in
\n $c_1\overline{y}_1 + c_2\overline{y}_2 + c_3\overline{y}_3 = 0$$ $g\vee\vee g$ $iues$
 $(\frac{1}{2}t) y (\frac{1}{2}t)\vec{v}_{2} + \vec{v}_{3} = 0$ for any t. $\frac{1}{t}$ For example if $t = 2$ we get
For example if $t = 2$ we get $(\frac{1}{2}t) V_1 - (\frac{1}{2}t) V_2$

any t .

(example if $t =$
 $\frac{1}{2} + 2V_3 = 0$
 $\frac{1}{2} + 2V_3 = 0$ $\vec{v}_1 - \vec{v}_2 + 2 \vec{v}_3 = 0$
So, $\vec{v}_1 = \vec{v}_2 - 2 \vec{v}_3$

The vectors are linearly dependent.
 $\overline{Ex: \underline{Let} \vec{x} = \langle 1, 0 \rangle \vec{j} = \langle 0, 1 \rangle}$

in \overline{R} . Are \vec{x} , linearly

dependent or linearly independent?
 \overline{R} ? Let's solve $Ex: Le+iz = \langle 1,0 \rangle, j = \langle 0,1 \rangle$
in \mathbb{R}^2 . Are izj linearly dependent or linearly independent? Let's solve $\frac{L \times L}{L}$ Let $x = \sqrt{107}$ $3 = \sqrt{9}$

in R². Are $\frac{1}{2}$ linearly

dependent or linearly indep
 $L = \frac{1}{2}$ solve
 $L = \frac{1}{2}$ solve
 $L = \frac{1}{2}$ we get We get $\frac{1}{C_1} < 1,$ $0) + C_{2} 0,$ $|0\rangle = \langle 0, 0 \rangle$ This gives 5 gives
 $2 < 0,0$) + 20 $(2) = 60,$ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ Then $\begin{array}{c} \diagup \\ \diagdown \\ \diagdown \end{array}$ \langle \langle \rangle , \langle \rangle \rangle $=$ \langle \rangle , $\sqrt[3]{\sqrt[3]{}}$

$$
S_{0, C_{1} = 0, C_{2} = 0.
$$

Thus, $\frac{1}{2}, \frac{1}{3}$ are linearly independent

E,I - - Coordinate system theorem Let s...., In be ⁿ linearly independent vectors in IRV. Then , Visi , ..., In are called a ordinatesystem oris for IR". Moreover , if " is any rector in I then " can be expressed

Uniquely in the form

\n
$$
\vec{v} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + \dots + c_n \vec{v}_n
$$
\nIf we write $\vec{p} = [\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n]$

\nThis means \vec{p} is the name

\nof our coordinates system and

\nThe brackets mean we've fixed

\nan ordering on $\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n$

\n $\pm f$ $\vec{v} = c_1 \vec{v}_1 + c_2 \vec{v}_2 + \dots + c_n \vec{v}_n$

\nthen c_1, c_2, \dots, c_n are called

\nThe coordinates of \vec{v} with

\nrespect to \vec{p} and we write

\n
$$
[\vec{v}]_{\vec{p}} = \langle c_1, c_2, \dots, c_n \rangle
$$

 $EX: \beta = [\overrightarrow{\lambda}, \overrightarrow{j}]$ where \vec{J} = <1,0>, \vec{J} = <0,1) in R. We have 2 linearly independent vectors in R² So, B is a basis or courdinate system f_{0} $\left[\frac{R^{2}}{1-1-1-1}-\frac{1}{1}-\frac{2}{1}3\right]=2\frac{7}{1}+3\frac{7}{1}$ $\frac{1}{1-1-\frac{1}{1-\frac$ $-1 - 1 - 1$ + -1 $\frac{1}{2}$ $(-3,1)=-3\vec{\lambda}+\vec{3}$ $\overline{}$ $-2\overline{r}$ -3λ -25 -351

 $\beta = [\begin{array}{cc} \rightarrow & \rightarrow \\ \downarrow & \end{array}]$ is called the Standard basis or standard standura possible por IR2.

 $\overrightarrow{y} = [\overrightarrow{z}, \overrightarrow{j}]$
 $\overrightarrow{y} = (z, 3)$
 $\overrightarrow{y} = 2\overrightarrow{i} + 3\overrightarrow{j}$
 $\overrightarrow{y} = 2\overrightarrow{i} + 3\overrightarrow{j}$
 $[\overrightarrow{y}]_p = (z, 3)$
 $[\overrightarrow{y}]_p = (-3, 1)$
 $\beta = [\overrightarrow{i}, \overrightarrow{j}]$ is called the

standard basis or standard

coordinate system for IR,

coordi $[7]_{p} = \langle 2,3 \rangle$ $\lfloor w \rfloor_{p} = \langle -3,1 \rangle$
 $p = [\overrightarrow{i}, \overrightarrow{j}]$ is called the

<u>Standard basis</u> or standard

<u>Courdinate system</u> for IR,
 $\frac{Ex}{b}$ $\lfloor e \rfloor_{p} = [\overrightarrow{a}, \overrightarrow{b}]$ where
 $\frac{1}{b}$ $\frac{1}{b}$ $\frac{1}{a}$ $\frac{1}{b}$ $\frac{1}{a$ $\frac{1}{\alpha}$ (, 1), $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$ Let's show a, b are linearly independent

and hence make β a basis and hence make p
or coordinate system. Want to solve C_{1}^{\dagger} $A + C_{2}^{\dagger}$ $B = 0$ We get $e, \begin{array}{c} 0 \leq t \\ 0 < 1, \end{array}$ \backslash $1) = 0,$ \circ $>$ This gives and hence make p a ba

coordinate system.

ant to solve
 c_1 at c_2 b = 0

get
 c_1c_1 at c_2 c_1 c_2 c_1 c_2
 c_1c_2 c_1+c_2
 $c_2c_1+c_2$ = c_1
 c_2 = 0

get
 $c_1+c_2=0$

get
 $c_1+c_2=0$

get
 c gives
 $\begin{array}{ccc} 0 & 0 & 0 \end{array}$ $\begin{array}{ccc} 0 & 0 & 0 \end{array}$ and hence v

or coordinate

Nant to solute
 c_1 at c_2 b

Ne get
 c_1 (1) to 2

Nis gives
 $\leq c_1$ (2) \leq 1
 \leq 1 -1 0) \leq R_1 + 1

1 -1 0) $\leq R_1$ + 1

1 -1 0) $\leq R_1$ + 1

1 -1 0) $\leq R_1$ + 1

We get

We get We get
 $(1 -1) 0$ = R₁+R₂+R₂ (1-1)

(02) $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

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\frac{V_{2} P_{2} P_{2}}{C_{1} - C_{2} = 0} \quad (1 - 1) 0
$$
\n
$$
\frac{S_{0}}{C_{1} - C_{2} = 0} \quad (2)
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\nThus,
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Q_{1} = C_{2} = 0
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Q_{1} = C_{2} = 0
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\nThus,
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