$Math 2550 - 01$ Math 2550-01 10/23/24

Last time we showed that L G S T TIMI<br><del>a</del> = < 1, 1 >,  $\zeta = \langle -1, 1 \rangle$ are linearly ast time we showed that<br>  $\vec{\lambda} = \langle 1, 1 \rangle$ ,  $\vec{b} = \langle -1, 1 \rangle$ <br>  $\vec{c}$  are linearly<br>
independent.<br>
Since  $\beta = [\vec{a}, \vec{b}]$ <br>
onsists of 2 linearly independ<br>
cectures in IR<sup>2</sup> we know that<br>
rectures in IR<sup>2</sup> we know that  $u$ re linearig<br>
independent.<br>
Since  $\beta = \begin{bmatrix} a \\ b \end{bmatrix}$ 5] consists of 2 linearly independent rectors in IR2 we know that vecinis in institution of system<br>B is a basis/coordinate system for R? So any vector V rector v 3 is a basis/cordinal form  $\overline{R}$ . So any vector V<br>for  $\mathbb{R}^2$ . So any vector V<br>in  $\mathbb{R}^2$  will be able to be for  $\mathbb{R}^2$ . So any vector<br>in  $\mathbb{R}^2$  will be able,<br>written  $\vec{v} = c_1 \vec{a} + c_2 \vec{b}$  $W$ here  $C_{1}$  $c_{\nu}$ are unique numbers  $associated with \frac{1}{v}caled$ called its coordinates.

Let 
$$
\vec{v} = \langle 3,1 \rangle
$$
.  
\nLet  $\vec{v}$  find  $\vec{v}$  is coordinates.  
\nNeed to solve  
\n $\langle 3,1 \rangle = c_1 \langle 1,1 \rangle + c_2 \langle -1,1 \rangle$   
\n $\langle 3,1 \rangle = c_1 \langle 1,1 \rangle + c_2 \langle -1,1 \rangle$   
\nThis gives  
\n $\langle 3,1 \rangle = \langle c_1, c_1 \rangle + \langle -c_2, c_2 \rangle$   
\nSo  
\n $\langle 3,1 \rangle = \langle c_1, c_2 \rangle + c_2 \langle -1, c_3 \rangle$   
\nNeed the solve  
\n $\langle c_1 - c_2 = 3 \rangle$   
\n $\langle c_1 + c_2 = 1 \rangle$ 

Need 
$$
\frac{\text{10} \cdot \text{10}}{1 - 1} = \frac{1}{2}
$$
  
2 = 1 2

 $(1) + (2)$  gives  $2c_1 = 4$ .  $S_{0}$   $C_{1} = 2$ <br>2 +  $C_{2} = 1$ , Plug into  $(2)$  to get  $2+c_2=1$ .  $S_{0}/C_{2}=-1.$  $D+Q$  gives  $2c_1=4$ . So  $C_1=2$ <br>
Plug into 2 to get  $2+c_2=1$ .<br>
So,  $C_2=-1$ .<br>
Thus,<br>  $\langle 3,1 \rangle = 2\langle 1,1 \rangle - 1\cdot\langle -1,1 \rangle$ <br>  $\sqrt{2}$   $\sqrt{2}$   $\sqrt{2}$   $\sqrt{2}$   $\sqrt{2}$ 

So  $\frac{1}{2}$  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}$  $\left[\begin{array}{c}p\\y\end{array}\right]_{\beta}=\left\langle 2\right\rangle$ -  $\begin{matrix} \vert \ \vert \ \end{matrix}$  $\frac{2}{\sqrt[3]{s}}$  B-coordinates  $\beta = [\vec{a}, \vec{b}]$ 

Let's draw <sup>a</sup> picture.



Suppose you know that  $Suppose Y04$ <br> $\begin{matrix} \overrightarrow{u} \\ \overrightarrow{w} \end{matrix} = \begin{matrix} 4 \\ 4 \end{matrix}$  $\begin{array}{ccc}\n\text{Know that} & \rightarrow \\ -5\text{?} & \text{What is } \text{w?}\n\end{array}$  $\frac{Suppose}{LNDB}$  $\beta$ -coordinates  $\frac{1}{\left|\lambda\right|}$  $\beta$  =(00%)<br>B =  $\begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix}$ you know that<br>
= <4,-5> What is W?<br>
dinates<br>
), a = <1,1>, b = <-1,1><br>
= <9,-1><br>
= <9,-1><br>
+a = <4,4> ,  $= \langle 1, 1 \rangle, 1, 1 \rangle$ We get  $\frac{1}{2}$  $(56 - 4(1, 1) - 5(-1, 1))$ 1) =  $\begin{align} - & | \searrow \rangle \\ & \leq \langle \heartsuit \rangle \end{align}$ - 17 **1**  $4a = 4,47$  $B = [a, b]$ ,  $\vec{a} = \langle 1, 1 \rangle, \vec{b} =$ <br>  $B = [a, b]$ ,  $\vec{a} = \langle 1, 1 \rangle, \vec{b} =$ <br>  $\vec{w} = \sqrt{\vec{a} - 5\vec{b}} = \frac{4}{5}\langle 9 \rangle$ <br>  $\vec{w} = \frac{4}{5}\langle 4, 4 \rangle$ <br>  $\vec{a} = \langle 4, 4 \rangle$  $\in$  $\frac{1}{\omega^{3}}$ <br>  $\frac{1}{\omega^{5}}$  $-1$ ) = 42 -56  $\begin{bmatrix} \downarrow \downarrow \downarrow \downarrow \end{bmatrix}$ ↓-  $\ddagger$  $\frac{1}{\pm}$  $\vec{a}$ <br>  $\vec{a$  $\overline{ }$  $56 = 451$ <br>  $= 69$ <br>  $= 69$ <br>  $= 65$ 

Ex:	In $\mathbb{R}^3$ , let $\overline{\lambda} = \langle 1, 0, 0 \rangle$
$\overline{1} = \langle 0, 1, 0 \rangle$ , $\overline{k} = \langle 0, 0, 1 \rangle$	
$\overline{2}$	In the How from the two from the two the second in the second independent.
$\langle 0, \beta = [\overline{\lambda}, \overline{\lambda}, \overline{k}]$ is a basis or coordinate system for $\mathbb{R}^3$ .	
$\overline{2}$	In the two independent.
$\text{basis}$ for $\mathbb{R}^3$ .	
$\overline{L}f$ for example, $\overline{v} = \langle 1, 2, 3 \rangle$ .	

Then,

 $\vec{y} = \langle 1, 2, 3 \rangle$  $\langle 1,0,0 \rangle + \langle 0,2,0 \rangle + \langle 0,0,3 \rangle$  $= |C_1, C_2, C_3| + 2 < 0, |C_1, C_4| > + 3 < 0, |C_1| >$  $= 1 \cdot i + 2 \cdot j + 3 \cdot k$  $SO$  $\langle j, \overline{c}, \overline{s} \rangle$  $\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$  $\boxed{ }$ 



Recall that if 
$$
\vec{u}
$$
 and  $\vec{v}$   
\nare in  $\mathbb{R}^2$  or  $\mathbb{R}^3$  we have  
\n $\vec{u} \cdot \vec{v} = ||\vec{u}|| \cdot ||\vec{v}|| \cdot cos(\theta)$   
\nwhere  $\theta$  is the angle between  
\n $\vec{u}$  and  $\vec{v}$   
\n $\int_{\mathbf{v}} \theta = 90^\circ$   
\nprecisely when  
\n $\vec{u} \cdot \vec{v} = ||\vec{u}|| \cdot ||\vec{v}|| \cos(90^\circ) = 0$   
\n $\frac{p_{e}f}{\vec{u} \cdot \vec{v}} = ||\vec{u}|| \cdot ||\vec{v}|| \cos(90^\circ) = 0$   
\n $\frac{p_{e}f}{\vec{u} \cdot \vec{v}} = \frac{p_{\dot{u}}}{\vec{u} \cdot \vec{v}} \text{ in } \mathbb{R}^n$ , we say  
\nthat  $\vec{u} \cdot \vec{u} = 0$ .  
\nif  $\vec{a} \cdot \vec{b} = 0$ .

$$
\frac{1}{a}
$$
 and  $\frac{1}{b}$  in  $\mathbb{R}^{n}$ , we say  
that  $\frac{1}{a}$  and  $\frac{1}{b}$  are orthogonal  
if  $\frac{1}{a} \cdot \frac{1}{b} = 0$ .

$$
\frac{Ex\ddot{i} + \pi}{2\vec{i} - 1} = \langle 1, 0 \rangle \cdot \langle 0, 1 \rangle = (1)(0) + (0)(1) = 0
$$
\n
$$
\frac{1}{3} \frac{1}{4} \frac{S_{0j} \ddot{j} + \pi}{2}
$$
\n
$$
\frac{1}{3} \frac{1}{4} \frac{S_{0j} \ddot{j} + \pi}{2}
$$
\n
$$
\frac{E_x \ddot{i} + \pi}{2} \frac{1}{a} \frac{1}{b} \frac{1}{b} = \langle 1, 1 \rangle \cdot \langle 1, 1 \rangle
$$
\n
$$
\frac{1}{b} \frac{1}{b} \frac{1}{b} \frac{1}{a} \frac{1}{a} \frac{1}{b} \frac{1}{a} \frac{1}{b} = \langle 1, 1 \rangle \cdot \langle 1, 1 \rangle
$$
\n
$$
= (1)(-1) + (1)(1)
$$
\n
$$
= 0
$$



