Math 2550-01
10/7/24

HW 1 - Part 1

$\vec{a} = \langle -1, 2, 4, 6 \rangle$

$\vec{b} = \langle 3, 1, 0, -2 \rangle$

$2\vec{a} - 3\vec{b}$

$= \langle -2, 4, 8, 12 \rangle + \langle -9, -3, 0, 6 \rangle$

$= \boxed{\langle -11, 1, 8, 18 \rangle}$

$\vec{a} \cdot \vec{b} = \langle -1, 2, 4, 6 \rangle \cdot \langle 3, 1, 0, -2 \rangle$

$= (-1)(3) + (2)(1) + (4)(0) + (6)(-2)$

$= -3 + 2 + 0 - 12 = \boxed{-13}$

$$\|\vec{b}\| = \sqrt{3^2 + 1^2 + 0^2 + (-2)^2}$$
$$= \sqrt{14}$$

Q: Find three vectors in S.

$$S = \{c_1\langle 1,0\rangle + c_2\langle 2,-1\rangle + c_3\langle 5,1\rangle \mid c_1, c_2, c_3 \in \mathbb{R}\}$$

$c_1 = 1,\ c_2 = 0,\ c_3 = -1$:

$$1\cdot\langle 1,0\rangle + 0\cdot\langle 2,-1\rangle - \langle 5,1\rangle = \boxed{\langle -4,-1\rangle}$$

$c_1 = 0,\ c_2 = 0,\ c_3 = 0$:

$$0\cdot\langle 1,0\rangle + 0\cdot\langle 2,-1\rangle + 0\cdot\langle 5,1\rangle = \boxed{\langle 0,0\rangle}$$

$c_1 = 1,\ c_2 = 0,\ c_3 = 0:$

$$1 \cdot \langle 1, 0 \rangle + 0 \cdot \langle 2, -1 \rangle + 0 \cdot \langle 5, 1 \rangle = \boxed{\langle 1, 0 \rangle}$$

HW 2 - Part 1

$$A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ -3 & 1 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix}$$

$$2A - B = \begin{pmatrix} 2 & -2 & 0 \\ 0 & 4 & 2 \\ -6 & 2 & -4 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & -1 \\ -2 & 0 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & -3 & 0 \\ -1 & 4 & 1 \\ -8 & 2 & -6 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 0 & -3 \\ -1 & 2 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$

$$AB = \underbrace{\begin{pmatrix} 1 & -1 & 0 \\ 0 & 2 & 1 \\ -3 & 1 & -2 \end{pmatrix}}_{3\times 3} \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 0 & 2 \end{pmatrix}}_{3\times 3}$$

✓

answer is 3x3

$$= \begin{pmatrix} 0-1+0 & 1+0+0 & 0-1+0 \\ 0+2+2 & 0+0+0 & 0+2+2 \\ 0+1-4 & -3+0+0 & 0+1-4 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 1 & -1 \\ 4 & 0 & 4 \\ -3 & -3 & -3 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 4 \end{pmatrix}}_{3 \times 2} \underbrace{\begin{pmatrix} 1 & 3 \\ 1 & 3 \\ 1 & 3 \end{pmatrix}}_{3 \times 2} \leftarrow \text{undefined}$$

not equal

Hw 1 - Part 2

① (d) Let $\vec{u}, \vec{v} \in \mathbb{R}^2$ and $\alpha \in \mathbb{R}$.

Prove that

$$\alpha(\vec{u} \cdot \vec{v}) = (\alpha \vec{u}) \cdot \vec{v}$$

Let $\vec{u} = \langle a, b \rangle$, $\vec{v} = \langle c, d \rangle$.

Then,

$$\alpha(\vec{u} \cdot \vec{v}) = \alpha(\langle a, b \rangle \cdot \langle c, d \rangle)$$

$$= \alpha(ac + bd)$$

$$= \alpha ac + \alpha bd$$

And,

$$(\alpha\vec{u})\cdot\vec{v} = \left(\alpha\langle a,b\rangle\right)\cdot\langle c,d\rangle$$

$$= \langle \alpha a, \alpha b\rangle \cdot \langle c,d\rangle$$

$$= \alpha ac + \alpha bd$$

EQUAL

So, $\alpha(\vec{u}\cdot\vec{v}) = (\alpha\vec{u})\cdot\vec{v}$. $\blacksquare$

HW 3

1 (c) Solve

$$\begin{aligned} x - y + 2z - w &= -1 \\ 2x + y - 2z - 2w &= -2 \\ -x + 2y - 4z + w &= 1 \\ 3x \qquad\qquad - 3w &= -3 \end{aligned}$$

have 1 ✓

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{array}\right)$$

make these 0

$$\xrightarrow[\substack{R_1 + R_3 \to R_3 \\ -3R_1 + R_4 \to R_4}]{-2R_1 + R_2 \to R_2} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right)$$

make this 1

$$\xrightarrow{R_2 \leftrightarrow R_3} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{array}\right)$$

make these 0

$$\xrightarrow[-3R_2+R_4 \to R_4]{-3R_2+R_3 \to R_3} \left(\begin{array}{cccc|c} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

in row echelon form

Back to the land of equations...

$$\begin{aligned} x - y + 2z - w &= -1 \\ y - 2z &= 0 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

leading variables: x, y

free variables: z, w

Solve for leading variables and give the free variables a new name.

$$\begin{aligned} x &= -1 + y - 2z + w \quad (1) \\ y &= 2z \quad (2) \\ z &= t \quad (3) \\ w &= u \quad (4) \end{aligned}$$

Back substitute:

(4) $w = u$

(3) $z = t$

(2) $y = 2z = 2t$

(1) $x = -1 + y - 2z + w$

$= -1 + 2t - 2t + u$

$= -1 + u$

$\boxed{\begin{array}{ll} \underline{\text{Answer:}} & \\ x = -1+u & \\ y = 2t & \text{where } t, u \text{ can be any real numbers} \\ z = t & \\ w = u & \end{array}}$