

Math 2550-01

9/4/24



Ex:

$$\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & -5 \\ 7 & 10 \end{pmatrix} = \begin{pmatrix} 1+0 & 2-5 \\ 3+7 & -1+10 \end{pmatrix} \\ = \begin{pmatrix} 1 & -3 \\ 10 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ -1 & 6 \end{pmatrix} - \begin{pmatrix} 2 & -2 \\ 3 & 4 \\ -6 & 10 \end{pmatrix} = \begin{pmatrix} 1-2 & 0+2 \\ 2-3 & 1-4 \\ -1+6 & 6-10 \end{pmatrix} \\ = \begin{pmatrix} -1 & 2 \\ -1 & -3 \\ 5 & -4 \end{pmatrix}$$

$$5 \begin{pmatrix} 1 & 2 \\ -1 & 6 \\ 0 & -10 \end{pmatrix} = \begin{pmatrix} 5(1) & 5(2) \\ 5(-1) & 5(6) \\ 5(0) & 5(-10) \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ -5 & 30 \\ 0 & -50 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 \\ -1 & 6 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 0 & 7 \end{pmatrix}$$

3×2 2×2

← undefined
the sizes
don't
match
up

Def: Let A be an $m \times r$ matrix
and B be an $r \times n$ matrix.
We define the product AB
to be the $m \times n$ matrix C
whose entry in row i and
column j is equal to the
dot product of row i from A
and column j from B .

$$C = A B$$

C is $m \times n$
 A is $m \times r$
 B is $r \times n$

The dimensions $m \times r$ and $r \times n$ must be equal for matrix multiplication to be possible.

Ex: Calculate AB , if possible,
 When $A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

$$A B = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

A is 2×2
 B is 2×3

The inner dimensions (2 and 2) are the same, so the multiplication is possible.

Answer is 2×3

(row 1 of A) · (col 1 of B) (row 1 of A) · (col 2 of B) (row 1 of A) · (col 3 of B)

$$= \begin{pmatrix} (1 \ 2) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\ (-1 \ 0) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (-1 \ 0) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} & (-1 \ 0) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{pmatrix}$$

(row 2 of A) · (col 1 of B) (row 2 of A) · (col 2 of B) (row 2 of A) · (col 3 of B)

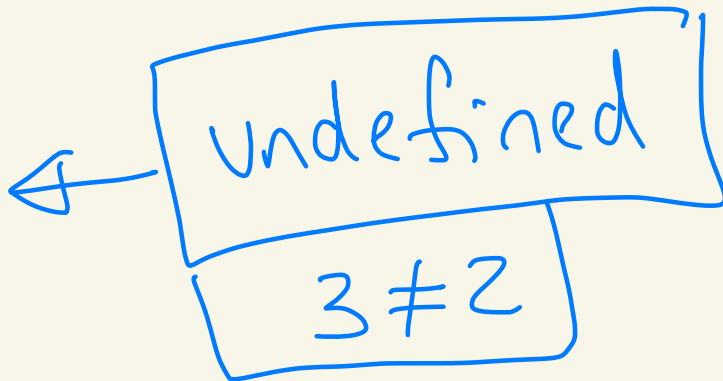
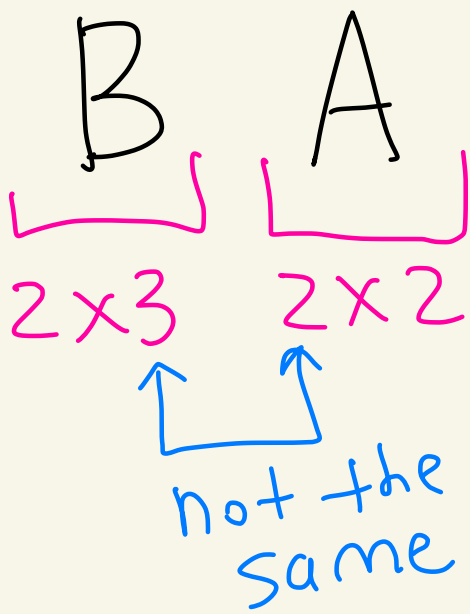
$$= \begin{pmatrix} (1)(1) + (2)(0) & (1)(2) + (2)(1) & (1)(-1) + (2)(0) \\ (-1)(1) + (0)(0) & (-1)(2) + (0)(1) & (-1)(-1) + (0)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

Ex: Using the same matrices

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

can we calculate BA ?



Let's try to multiply anyways.
Why doesn't it work?

$$\begin{matrix} & \text{B} & & \text{A} \\ \left(\begin{array}{ccc|cc} 1 & 2 & -1 & 1 & 2 \\ 0 & 1 & 0 & -1 & 0 \end{array} \right) & & & \end{matrix}$$

$$\begin{matrix} & \text{B} & & \text{A} \\ & 2 \times 3 & & 2 \times 2 \\ & & \text{L} & \end{matrix}$$

(row 1 of B).
(col 1 of A)

$$= \left((1 \ 2 \ -1) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right)$$

you can't calculate that dot product. That's why the mult. of BA is undefined.

Ex: Calculate

$$\begin{array}{c} \overbrace{\left(\begin{array}{cc} 1 & 3 \\ -1 & 7 \end{array} \right)}^M \quad \overbrace{\left(\begin{array}{c} 0 \\ 2 \end{array} \right)}^N \\ \underbrace{\hspace{10em}}_{2 \times 2} \quad \underbrace{\hspace{2em}}_{2 \times 1} \\ \downarrow \\ \underbrace{\hspace{10em}} \end{array}$$

answer is 2×1

(row 1 of M) ·
(col 1 of N)

$$MN = \left(\begin{array}{c} (1 \ 3) \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix} \\ (-1 \ 7) \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix} \end{array} \right) = \begin{pmatrix} (1)(0) + (3)(2) \\ (-1)(0) + (7)(2) \end{pmatrix}$$

(row 2 of M) ·
(col 1 of N)

$$= \begin{pmatrix} 6 \\ 14 \end{pmatrix}$$

Ex: With matrices the equation

$AB = BA$ is not always true.

For example, $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$
$$BA = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

not equal

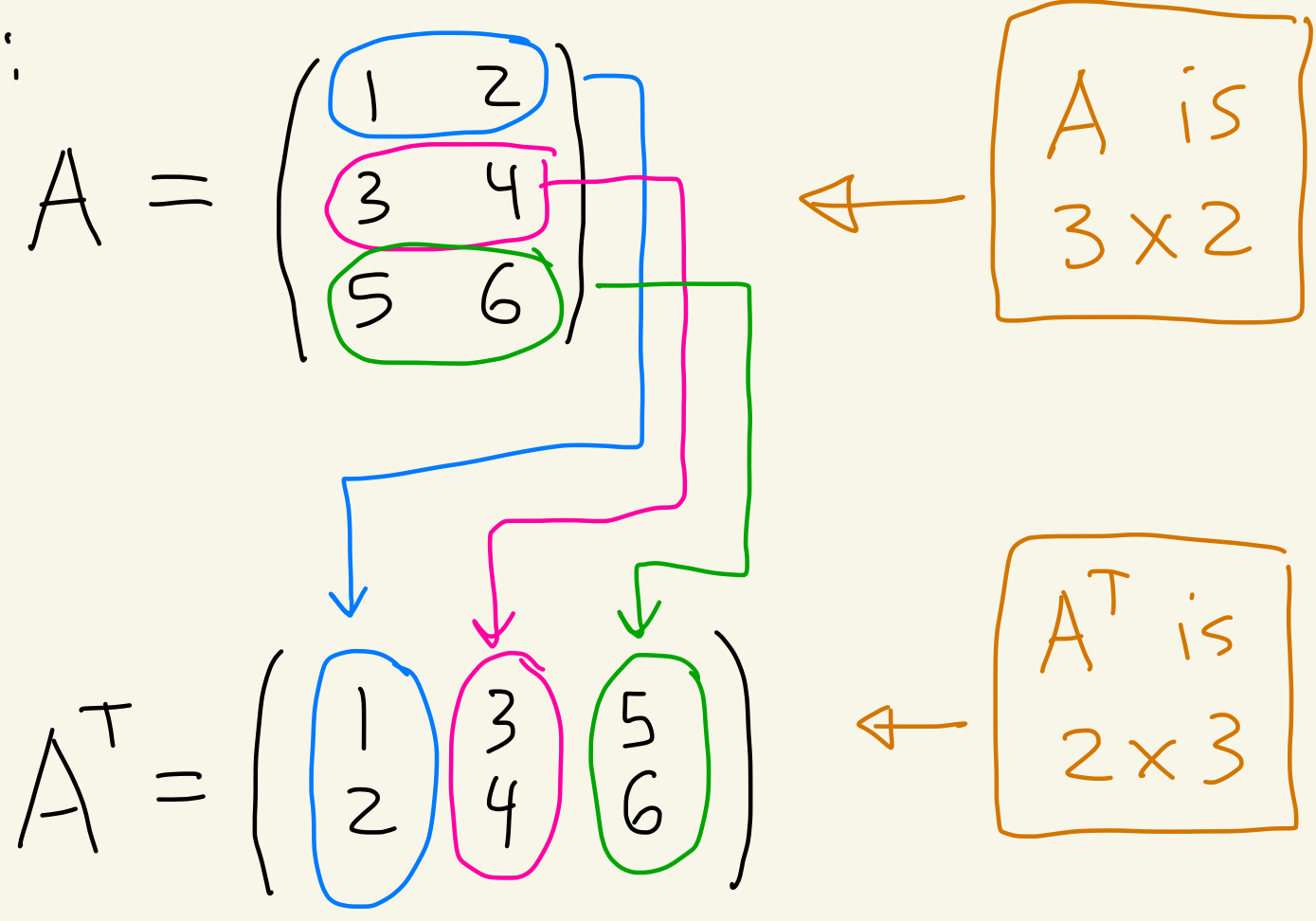
So, $AB \neq BA$ in this case.

Def: Let A be an $m \times n$ matrix. The transpose of A , denoted by A^T , is defined to be the $n \times m$ matrix whose i -th row equals the i -th column of A .

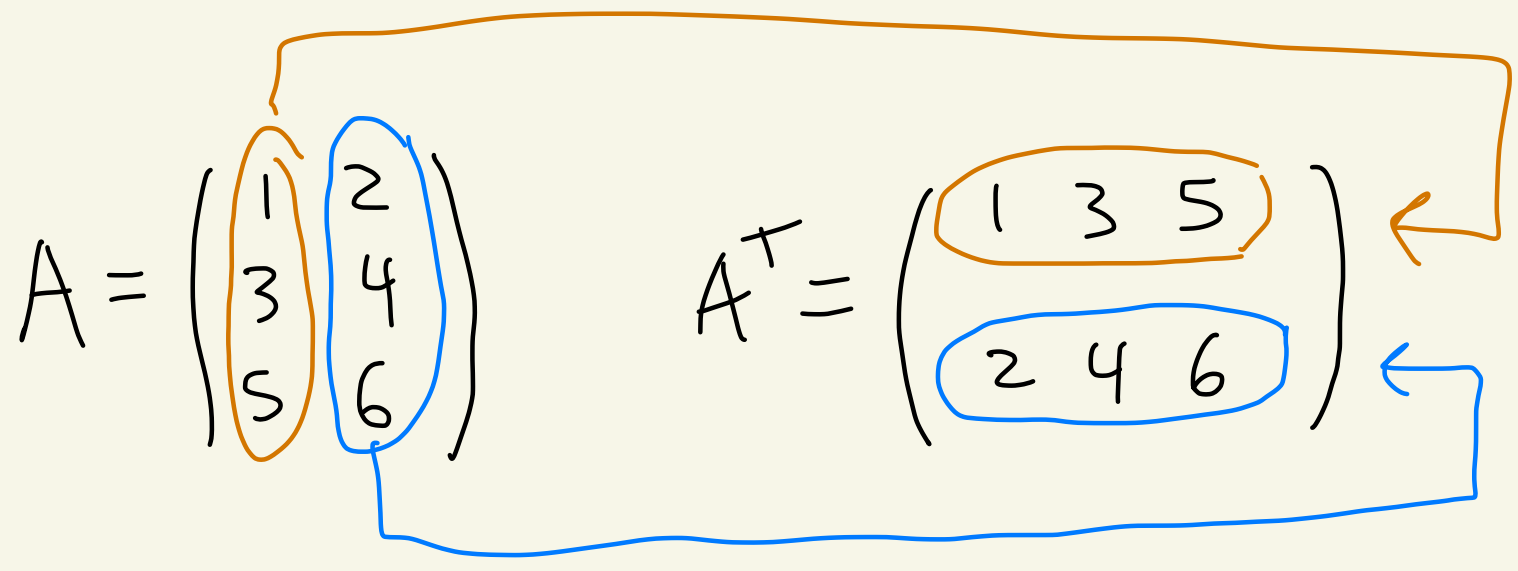
Or you could say that the j -th column of A^T is the j -th row of A .

You're interchanging
the rows and columns

Ex:



OR:



Def: The $m \times n$ zero matrix, denoted by $O_{m \times n}$ or just O , is the $m \times n$ matrix whose entries are all zero

Ex:

$$O_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \quad O_{4 \times 1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$O_{3 \times 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Ex: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$A + O_{2 \times 2} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

$$O_{2 \times 2} + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

$$\text{So, } A + O_{2 \times 2} = A$$

$$O_{2 \times 2} + A = A$$

Def: The $n \times n$ identity matrix, denoted by I_n or just I , is the $n \times n$ matrix whose main diagonal contains all 1's and all other entries are 0's.

Ex:

$$I_1 = (1)$$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and so on...