Math 2550-01
9/4/24

Ex:

$$\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & -5 \\ 7 & 10 \end{pmatrix} = \begin{pmatrix} 1+0 & 2-5 \\ 3+7 & -1+10 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -3 \\ 10 & 9 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ -1 & 6 \end{pmatrix} - \begin{pmatrix} 2 & -2 \\ 3 & 4 \\ -6 & 10 \end{pmatrix} = \begin{pmatrix} 1-2 & 0+2 \\ 2-3 & 1-4 \\ -1+6 & 6-10 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 2 \\ -1 & -3 \\ 5 & -4 \end{pmatrix}$$

$$5\begin{pmatrix} 1 & 2 \\ -1 & 6 \\ 0 & -10 \end{pmatrix} = \begin{pmatrix} 5(1) & 5(2) \\ 5(-1) & 5(6) \\ 5(0) & 5(-10) \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ -5 & 30 \\ 0 & -50 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 1 & 2 \\ -1 & 6 \\ 0 & 1 \end{pmatrix}}_{3\times 2} + \underbrace{\begin{pmatrix} 2 & -1 \\ 0 & 7 \end{pmatrix}}_{2\times 2} \leftarrow \boxed{\text{undefined the sizes don't match up}}$$

Def: Let A be an $m\times r$ matrix and B be an $r\times n$ matrix. We define the **product** AB to be the $m\times n$ matrix C whose entry in row i and column j is equal to the dot product of row i from A and column j from B.

$$\underbrace{C}_{m\times n} = \underbrace{A}_{m\times r}\,\underbrace{B}_{r\times n}$$

have to equal

Ex: Calculate AB, if possible, when $A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

$$\underbrace{A}_{2\times 2}\,\underbrace{B}_{2\times 3} = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

same ✓

answer is 2×3

$$= \begin{pmatrix} \overbrace{(1\ \ 2)\cdot\begin{pmatrix}1\\0\end{pmatrix}}^{\text{(row 1 of A)}\cdot\text{(col 1 of B)}} & \overbrace{(1\ \ 2)\cdot\begin{pmatrix}2\\1\end{pmatrix}}^{\text{(row 1 of A)}\cdot\text{(col 2 of B)}} & \overbrace{(1\ \ 2)\cdot\begin{pmatrix}-1\\0\end{pmatrix}}^{\text{(row 1 of A)}\cdot\text{(col 3 of B)}} \\ \underbrace{(-1\ \ 0)\cdot\begin{pmatrix}1\\0\end{pmatrix}}_{\text{(row 2 of A)}\cdot\text{(col 1 of B)}} & \underbrace{(-1\ \ 0)\begin{pmatrix}2\\1\end{pmatrix}}_{\text{(row 2 of A)}\cdot\text{(col 2 of B)}} & \underbrace{(-1\ \ 0)\cdot\begin{pmatrix}-1\\0\end{pmatrix}}_{\text{(row 2 of A)}\cdot\text{(col 3 of B)}} \end{pmatrix}$$

$$= \begin{pmatrix} (1)(1)+(2)(0) & (1)(2)+(2)(1) & (1)(-1)+(2)(0) \\ (-1)(1)+(0)(0) & (-1)(2)+(0)(1) & (-1)(-1)+(0)(0) \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}$$

Ex: Using the same matrices

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

can we calculate BA?

$$\underbrace{B}_{2\times 3}\ \underbrace{A}_{2\times 2}$$

not the same

$\boxed{\text{undefined}}$ $\boxed{3 \neq 2}$

Let's try to multiply anyways.

Why doesn't it work?

$$\overbrace{\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}}^{B} \overbrace{\begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}}^{A}$$

$$\begin{array}{cc} B & A \\ 2\times 3 & 2\times 2 \end{array}$$

$$= \begin{pmatrix} \overbrace{\begin{pmatrix} 1 & 2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}}^{(\text{row } 1 \text{ of } B)\cdot(\text{col } 1 \text{ of } A)} & \\ & \end{pmatrix}$$

You can't calculate that dot product. That's why the mult. of BA is undefined.

Ex: Calculate

$$\underbrace{\overbrace{\begin{pmatrix} 1 & 3 \\ -1 & 7 \end{pmatrix}}^{M}}_{2\times 2}\underbrace{\overbrace{\begin{pmatrix} 0 \\ 2 \end{pmatrix}}^{N}}_{2\times 1}$$

answer is 2x1

$$MN = \begin{pmatrix} \overbrace{\begin{pmatrix} 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix}}^{(\text{row 1 of } M)\cdot(\text{col 1 of } N)} \\ \\ \underbrace{\begin{pmatrix} -1 & 7 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \end{pmatrix}}_{(\text{row 2 of } M)\cdot(\text{col 1 of } N)} \end{pmatrix} = \begin{pmatrix} (1)(0)+(3)(2) \\ \\ (-1)(0)+(7)(2) \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 14 \end{pmatrix}$$

__Ex:__ With matrices the equation $AB = BA$ is __not__ always true.

For example, $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

not equal

$$BA = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

So, $AB \neq BA$ in this case.

__Def:__ Let A be an $m \times n$ matrix. The __transpose__ of A, denoted by A^T, is defined to be the $n \times m$ matrix whose i-th row equals the i-th column of A.

Or you could say that the j-th column of A^T is the j-th row of A.

You're interchanging the rows and columns

Ex:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix}$$

A is 3x2

$$A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

A^T is 2x3

OR:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \qquad A^T = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

Def: The $m \times n$ <u>zero matrix</u>, denoted by $0_{m \times n}$ or just 0, is the $m \times n$ matrix whose entries are all zero

---

Ex:

$$0_{2\times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \qquad 0_{4\times 1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$0_{3\times 4} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Ex: $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

$$A + 0_{2\times 2} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

$$0_{2\times2} + A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

So, $A + 0_{2\times2} = A$

$0_{2\times2} + A = A$

<u>Def:</u> The $n \times n$ <u>identity matrix</u>, denoted by I_n or just I, is the $n \times n$ matrix whose main diagonal contains all 1's and all other entries are 0's.

<u>Ex:</u>

$$I_1 = (1)$$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and so on...