Math 3450
3/12/24

$\boxed{\text{HW 3}}$ $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

8 (c)

$S = \mathbb{N} \times \mathbb{N} = \{(1,1), (1,2), (2,1), (3,1), \ldots\}$

Define

$(a,b) \sim (c,d)$ to mean $a+d = b+c$

Prove $\sim$ is an equivalence relation.

proof:

(reflexive)

Let $(x,y) \in S$.

Then, $(x,y) \sim (x,y)$ because $x+y = y+x$

(symmetric)

Let $(x,y), (w,z) \in S$.

Suppose $(x,y) \sim (w,z)$.

Then, $x+z=y+w$.

This implies $w+y=z+x$
which gives us $(w,z)\sim(x,y)$.

(transitive)

Let $(x,y),(w,z),(s,t)\in S$.
Assume that $(x,y)\sim(w,z)$
and $(w,z)\sim(s,t)$.

This means that $x+z=y+w$
and $w+t=z+s$.

Adding gives $x+z+w+t=y+w+z+s$

Subtracting $w+z$ from both
sides gives $x+t=y+s$.

Thus, $(x,y)\sim(s,t)$.

$\blacksquare$

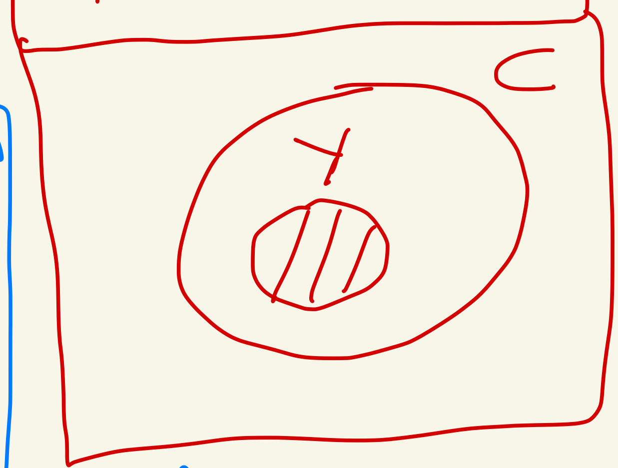
$\boxed{\text{Hw 2}}$

14 (a) Let A and B be sets.
Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

proof:

$\boxed{\subseteq}$: Let's show $\mathcal{P}(A \cap B) \subseteq \mathcal{P}(A) \cap \mathcal{P}(B)$.

Def: $Y \in \mathcal{P}(C)$ means $Y \subseteq C$



Let $X \in \mathcal{P}(A \cap B)$.

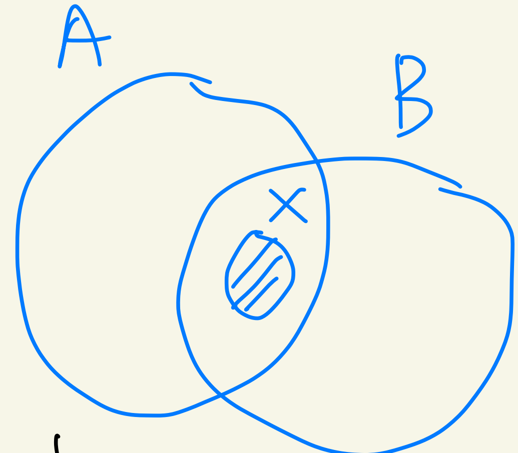
Then, $X \subseteq A \cap B$.

Thus, $X \subseteq A$ and $X \subseteq B$.

So, $X \in \mathcal{P}(A)$

and $X \in \mathcal{P}(B)$.

Thus, $X \in \mathcal{P}(A) \cap \mathcal{P}(B)$.



$\boxed{\supseteq}$: Let's now show that
$\mathcal{P}(A) \cap \mathcal{P}(B) \subseteq \mathcal{P}(A \cap B)$.

Let $Y \in \mathcal{P}(A) \cap \mathcal{P}(B)$.

Then, $Y \in \mathcal{P}(A)$ and $Y \in \mathcal{P}(B)$.

So, $Y \subseteq A$ and $Y \subseteq B$.

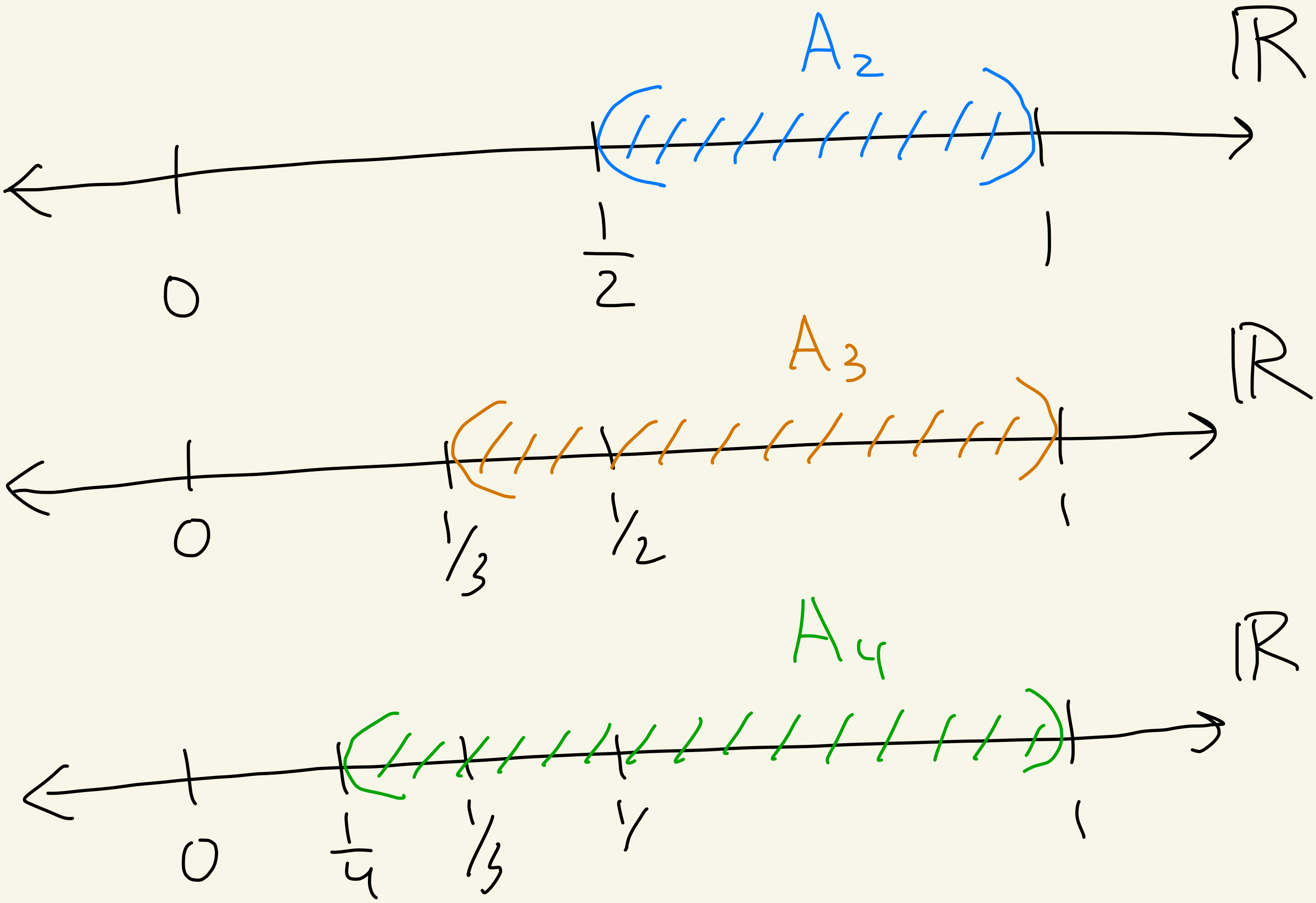
Then, $Y \subseteq A \cap B$.

Thus, $Y \in \mathcal{P}(A \cap B)$. ∎

$\boxed{\text{Hw 2}}$

9(b) $A_n = \left(\frac{1}{n}, 1\right)$ ← interval in $\mathbb{R}$

Find $\bigcup_{n=2}^{\infty} A_n$ and $\bigcap_{n=2}^{\infty} A_n$



$$\bigcap_{n=2}^{\infty} A_n = \left(\frac{1}{2}, 1\right) \qquad \bigcup_{n=2}^{\infty} A_n = (0, 1)$$

Practice Test

(4) $A_n = \{-2n, 0, 2n\}$

$A_1 = \{-2, 0, 2\}$

$A_2 = \{-4, 0, 4\}$

$A_3 = \{-6, 0, 6\}$

$A_4 = \{-8, 0, 8\}$

$A_5 = \{-10, 0, 10\}$

$A_6 = \{-12, 0, 12\}$

$$\bigcap_{n=1}^{\infty} A_n = \{0\}$$

even integers

$$\bigcup_{n=1}^{\infty} A_n = \{\ldots, -8, -6, -4, -2, 0, 2, 4, 6, 8, \ldots\}$$

$$= \{2k \mid k \in \mathbb{Z}\}$$

$$\bigcup_{n=4}^{\infty} A_n = \{\ldots, -12, -10, -8, 0, 8, 10, 12, \ldots\}$$

$$= \{2k \mid |k| \geq 4,\ k \in \mathbb{Z}\}$$

$$= \{2k \mid k \in \mathbb{Z},\ k \leq -4 \text{ or } k \geq 4\}$$

Look at HW 2 - 8, 9 / Practice test #4

$\boxed{\text{Ex:}}$

List 2 elements from S.

$$S = \{5x + y^2 - z \mid x, y, z \in \mathbb{R},\ 0 \le x \le 1,\ 1 < y \le 5\}$$

$\boxed{\begin{array}{l} \underline{x = 1,\ y = 2,\ z = 3:} \\ 5(1) + (2)^2 - 3 = 6 \\ 6 \in S \end{array}}$

$\boxed{\begin{array}{l} \underline{x = 0,\ y = 4,\ z = -1:} \\ 5(0) + 4^2 - (-1) = 17 \\ 17 \in S \end{array}}$

Hammock

Ch. 8 #25

25 Prove
$(A\times B)\cup(C\times D)\subseteq(A\cup C)\times(B\cup D)$

proof:

Let $w\in(A\times B)\cup(C\times D)$.

Then $w\in A\times B$ or $w\in C\times D$.

case 1: Suppose $w\in A\times B$.

Then, $w=(a,b)$ where $a\in A$, $b\in B$.

So, $w=(a,b)$ where $a\in A\cup C$
and $b\in B\cup D$.

Thus, $w\in(A\cup C)\times(B\cup D)$.

Case 2: Suppose $w \in C \times D$.

Then, $w = (c, d)$ where $c \in C$, $d \in D$.

So, $w = (c, d)$ where $c \in A \cup C$
and $d \in B \cup D$.

Hence, $w \in (A \cup C) \times (B \cup D)$.

$\blacksquare$

$\boxed{\text{Hw 3}}$

3. $S = \mathbb{Z}$

Define $x \sim y$ to mean $3x - 5y$ is even

Prove $\sim$ is an equivalence relation on $\mathbb{Z}$.

proof:

(reflexive)

Let $a \in \mathbb{Z}$.

Then, $3a - 5a = -2a = 2(-a)$ is even, so $a \sim a$.

(symmetric)

Let $a, b \in \mathbb{Z}$.

Assume that $a \sim b$.

$x \sim y$
$3x - 5y$ even

Then, $3a - 5b$ is even.

So, $3a - 5b = 2k$ where $k \in \mathbb{Z}$.

Thus,

$$-8a + 8b + \underbrace{(3a-5b)}_{2k} = -8a + 8b + 2k$$

This gives

$$3b - 5a = 2\underbrace{(-4a + 4b + k)}_{\text{this is still an integer}}$$

So, $3b - 5a$ is even and $b \sim a$.

(transitive)

$x \sim y$
$3x - 5y$ even

Let $a, b, c \in \mathbb{Z}$.

Suppose $a \sim b$ and $b \sim c$.

Then, $3a - 5b$ is even
and $3b - 5c$ is even.

So, $3a - 5b = 2k$ and
$3b - 5c = 2l$ where $k, l \in \mathbb{Z}$

Adding gives
$$3a - 2b - 5c = 2k + 2l$$

So,
$$3a - 5c = 2k + 2l + 2b$$

Thus, $3a - 5c = 2\underbrace{(k + l + b)}_{\text{this is an integer}}$

So, $3a-5c$ is even and $a \sim c$. $\blacksquare$

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Ch. 8

29 Suppose $A \neq \emptyset$.
Prove $A \times B \subseteq A \times C$ iff $B \subseteq C$.

<u>proof:</u> Suppose $A \neq \emptyset$.

($\Leftarrow$) Suppose $B \subseteq C$.
Let $x \in A \times B$.
Then, $x = (a, b)$ where $a \in A$ and $b \in B$.

Since $B \subseteq C$ we know $b \in C$.
So, $x = (a, b) \in A \times C$.

Thus, $A \times B \subseteq A \times C$.

($\Leftarrow$) Suppose $A \times B \subseteq A \times C$.

We want to show that $B \subseteq C$.

Let $b \in B$.

Since $A \neq \emptyset$ there exists some $a \in A$.

Then, $(a, b) \in A \times B$.

Since $A \times B \subseteq A \times C$, we get $(a, b) \in A \times C$.

So, $b \in C$.

Thus, $B \subseteq C$.

$\blacksquare$

Hammock
Ch. 8

31) Suppose $B \neq \emptyset$ and $A \times B \subseteq B \times C$. Prove $A \subseteq C$.

<u>proof:</u> Assume $B \neq \emptyset$ and $A \times B \subseteq B \times C$.

Let $a \in A$.

Since $B \neq \emptyset$ there exists some $b \in B$.

assumption

Then, $(a,b) \in A \times B \subseteq B \times C$.

So, $a \in B$ and $b \in C$.

Then, $(a, a) \in A \times B \subseteq B \times C$

$a \in A$
$a \in B$

assumption

Thus, $a \in C$.

So, $A \subseteq C$. $\blacksquare$

Hammock

1.8 #3

For each $n \in \mathbb{N}$, define $\underbrace{A_n = \{0, 1, 2, \ldots, n\}}$

$$A_n = \{x \mid x \in \mathbb{Z} \text{ and } 0 \leq x \leq n\}$$

$$A_1 = \{0, 1\}$$

$$A_2 = \{0, 1, 2\}$$

$$A_3 = \{0, 1, 2, 3\}$$

$$A_4 = \{0, 1, 2, 3, 4\}$$

$$\bigcap_{n=1}^{\infty} A_n = \{0, 1\}$$

$$\bigcup_{n=1}^{\infty} A_n = \{0, 1, 2, 3, \ldots\} = \mathbb{N} \cup \{0\} = \{x \mid x \in \mathbb{Z} \text{ and } x \geq 0\}$$

$(0,\infty)$