Math 4300
11/6/23

$\boxed{\text{Hw 4}}$

$\boxed{\#4}$ $A, B \in \mathscr{P}$ and $A \neq B$.

Let $C \in \overleftrightarrow{AB}$.

Then either:

$C-A-B$, $C=A$, $A-C-B$, $C=B$, or $A-B-C$

<u>proof:</u> Let $f : \overleftrightarrow{AB} \to \mathbb{R}$ be a ruler for $\overleftrightarrow{AB}$.

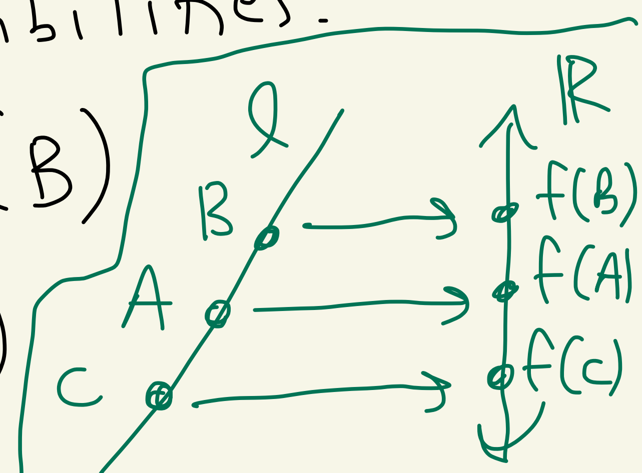
We know $f(A) \neq f(B)$ since $f$ is 1-1 and $A \neq B$.

Here are all the possibilities:

(i) $f(C) < f(A) < f(B)$

(ii) $f(C) < f(B) < f(A)$

(iii) $f(C) = f(A)$




pic for (i)

(iv) $f(C) = f(B)$

(v) $f(A) < f(C) < f(B)$

(vi) $f(B) < f(C) < f(A)$

(vii) $f(A) < f(B) < f(C)$

(viii) $f(B) < f(A) < f(C)$

This gives:

(i) We get $C-A-B$.

(ii) We get $C-B-A$.
This implies $A-B-C$.

If $X-Y-Z$, then $Z-Y-X$.

(iii) Since $f$ is 1-1, we get $A=C$.

(iv) Since $f$ is 1-1 we get $C=B$.

(v) This gives $A-C-B$.

(vi) This gives B-C-A.
This implies A-C-B.

(vii) This gives A-B-C.

(viii) This gives B-A-C.
which yields C-A-B.

$\blacksquare$

HW 4

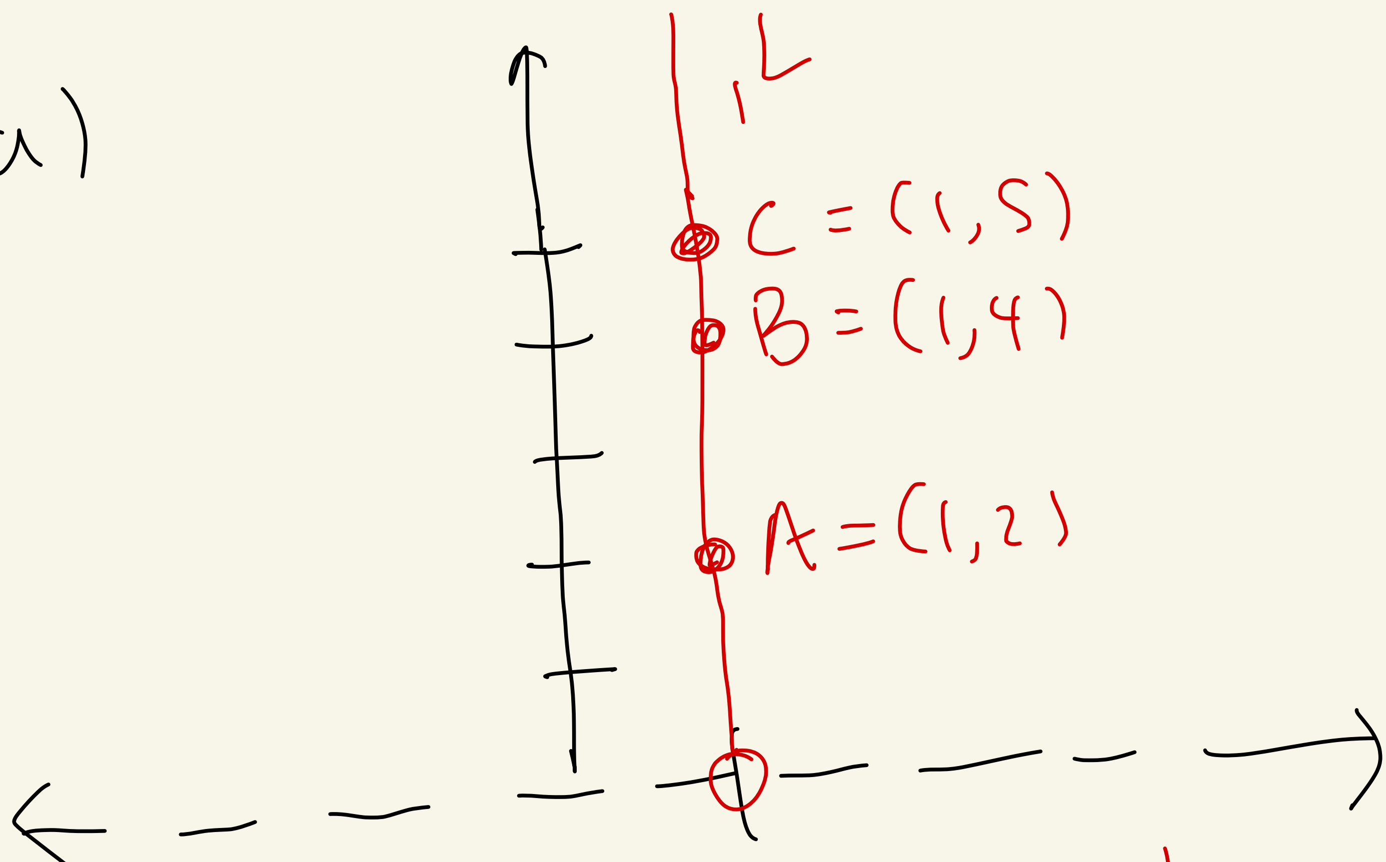
(3) In the hyperbolic plane let $A=(1,2)$, $B=(1,4)$, $C=(1,5)$.

(a) Determine if collinear.

(b) Determine if $A-B-C$, $A-C-B$, or $B-A-C$.

---

(a)




They are collinear and on ${}_1L$.

(b) $\boxed{\text{Ruler way - easier}}$

Standard ruler $f: {}_1L \to \mathbb{R}$

is $f(1, y) = \ln(y)$

We get

$f(A) = \ln(2) \approx 0.693$
$f(B) = \ln(4) \approx 1.386$
$f(C) = \ln(5) \approx 1.609$

Since $f(A) < f(B) < f(C)$,

we know $A - B - C$.

main Topic 4 theorem

$\boxed{\text{Distance way}}$

$$d_H(A, B) = \left|\ln\left(\frac{2}{4}\right)\right| = \left|\ln\left(\frac{1}{2}\right)\right| = -\ln\left(\frac{1}{2}\right)$$

$$d_H(B, C) = \left|\ln\left(\frac{4}{5}\right)\right| = -\ln\left(\frac{4}{5}\right)$$

$$d_H(A,C) = \left|\ln\left(\frac{2}{5}\right)\right| = -\ln\left(\frac{2}{5}\right)$$

Then,

$$d_H(A,B) + d_H(B,C) = -\ln\left(\frac{1}{2}\right) - \ln\left(\frac{4}{5}\right)$$

$$-\ln(A) = \ln(A^{-1})$$

$$= \ln(2) + \ln\left(\frac{5}{4}\right)$$

$$= \ln\left(2 \cdot \frac{5}{4}\right)$$

$$= \ln\left(\frac{5}{2}\right)$$

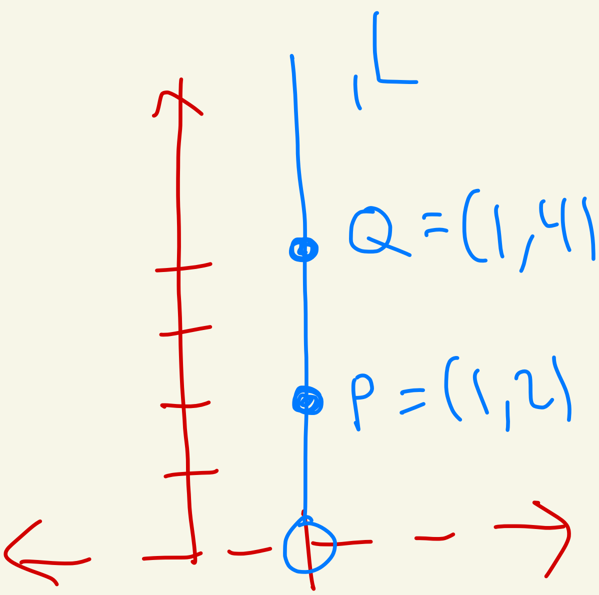
$$= -\ln\left(\frac{2}{5}\right) = d_H(A,C)$$

So, $A - B - C$.

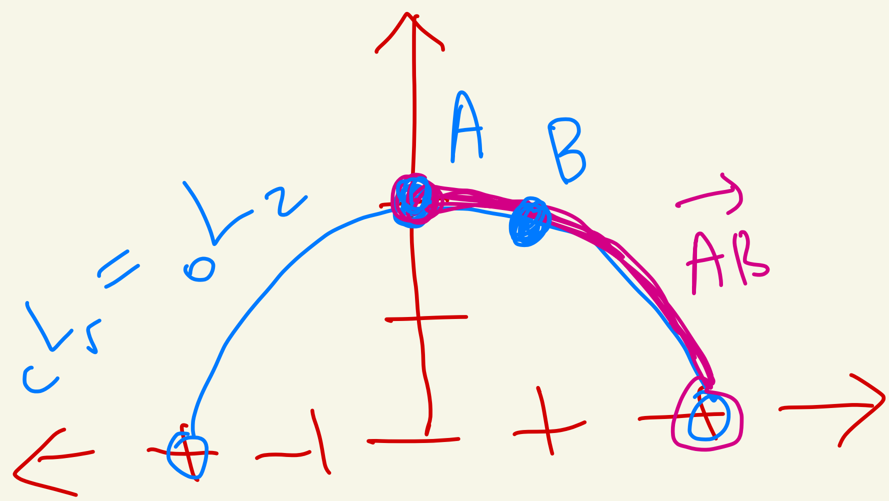
$\boxed{\text{HW 5}}$ Hyperbolic plane

#5 $P = (1,2)$, $Q = (1,4)$
$A = (0,2)$, $B = (1,\sqrt{3})$

Find $C \in \overrightarrow{AB}$ where $\overline{AC} \cong \overline{PQ}$.




you can calculate:
$\overleftrightarrow{AB} = {}_0L_2$




Measure $\overline{PQ}$:

$$d_H(P,Q) = \left|\ln\left(\frac{4}{2}\right)\right| = |\ln(2)| = \ln(2)$$

Want: Find $C \in \overrightarrow{AB}$ where $d_H(A,C) = \ln(2)$

Let $C=(x,y)$.

Want to solve:

$$\ln(2) = d_H(A,C)$$

$$= d_H((0,2),(x,y))$$

$$= \left| \ln\left( \frac{\frac{0-0+2}{2}}{\frac{x-0+2}{y}} \right) \right|$$

$$= \left| \ln\left( \frac{1}{\frac{x+2}{y}} \right) \right| = \left| \ln\left( \frac{y}{x+2} \right) \right|$$

So we need $\ln(2) = \pm \ln\left(\frac{y}{x+2}\right)$.

So either

$$\ln(2) = \ln\left(\frac{y}{x+2}\right) \quad \text{or} \quad \ln(2) = \underbrace{-\ln\left(\frac{y}{x+2}\right)}_{\ln\left(\frac{x+2}{y}\right)}$$

So either

$$z = \frac{y}{x+2} \quad \text{or} \quad 2 = \frac{x+2}{y}.$$

So either

$$\boxed{y = 2x+4} \; 1 \quad \text{or} \quad \boxed{y = \tfrac{1}{2}x+1} \; 2$$

Now plug these into $\underbrace{{}_0L_2}_{x^2+y^2=4}$ to get $C$

We get these two possibilities:

$$\boxed{\begin{aligned} &x^2+(2x+4)^2 = 4 \quad 1 \\ &x^2+\left(\tfrac{1}{2}x+1\right)^2 = 4 \quad 2 \end{aligned}}$$

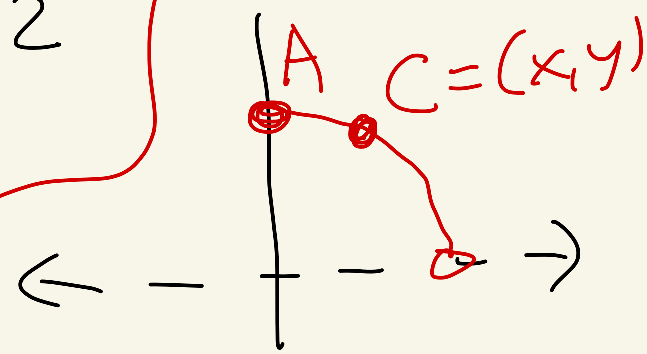
These become:

$$\boxed{\begin{aligned} 5x^2+16x+12&=0 \quad (1)\\ 5x^2+4x-12&=0 \quad (2)\end{aligned}}$$

(1) becomes: $(5x+6)(x+2)=0$

So, $x=-\frac{6}{5}$ or $x=-2$

We need C on the right side of A, so neither of these x's work.




(need $x>0$)

(2) becomes: $(5x-6)(x+2)=0$

So, $\boxed{x=\frac{6}{5}}$ or $x=-2$.

Only $x=\frac{6}{5}$ is positive.

Now plug $C=\left(\frac{6}{5},y\right)$ into $\underbrace{{}_0L_2}_{x^2+y^2=4}$

to get:

$$\left(\frac{6}{5}\right)^2+y^2=4$$

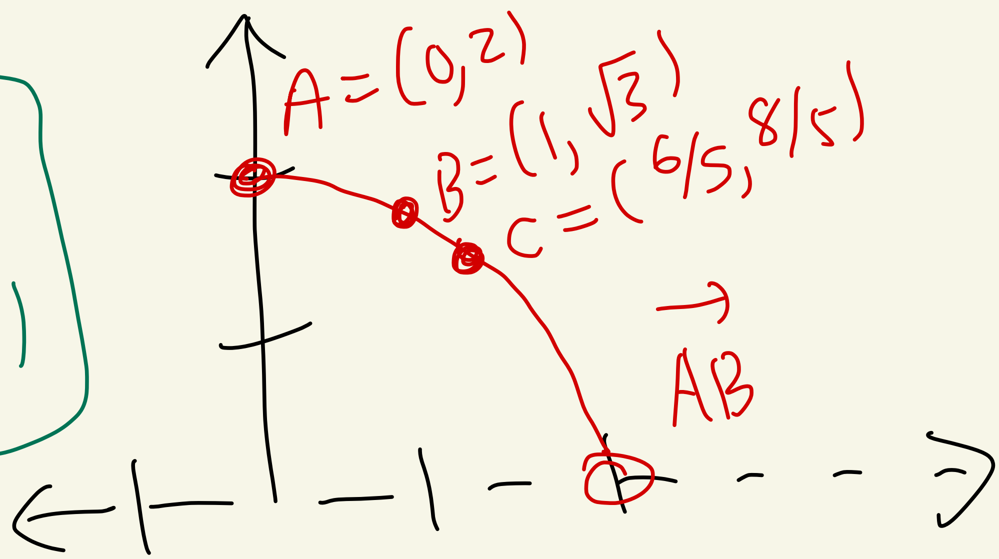
This gives $y^2=4-\frac{36}{25}=\frac{100-36}{25}=\frac{64}{25}$

So, $y=\pm\sqrt{\frac{64}{25}}=\pm\frac{8}{5}$

Need $y>0$ so we get $y=\frac{8}{5}$.

Thus, $C=\left(\frac{6}{5},\frac{8}{5}\right)$.

We should have $d_H(A,C)=\ln(2)$



Check:

$$d_H(A, C) = \left| \ln \left( \frac{\frac{0-0+2}{2}}{\frac{6/5-0+2}{8/5}} \right) \right|$$

$$= \left| \ln \left( \frac{1}{\frac{16/5}{8/5}} \right) \right| = \left| \ln\left(\frac{8}{16}\right) \right|$$

$$= \left| \ln\left(\frac{1}{2}\right) \right| = -\ln\left(\frac{1}{2}\right) = \ln(2)$$