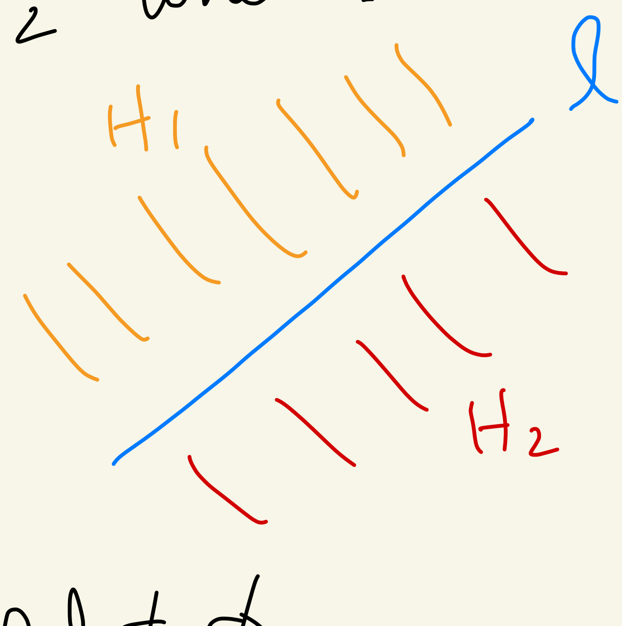
Math 4300

12/4/23

$\boxed{\text{HW 7}}$

7) Let $(\mathcal{P}, \mathcal{L}, d)$ be a metric geometry satisfying the PSA.
Let $P, Q, R \in \mathcal{P}$, $\ell \in \mathcal{L}$.
If P, Q are on opposite side of ℓ
and Q, R are on opposite sides of ℓ,
then P, R are on the same side of ℓ.

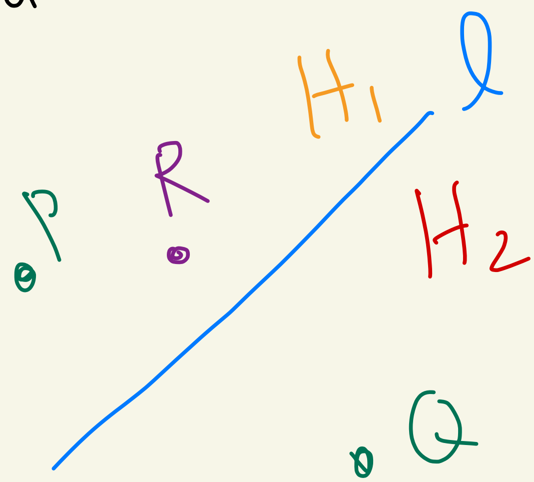
<u>proof:</u> By the PSA there exist two half-planes H_1, H_2 where:



- $\mathcal{P} - \ell = H_1 \cup H_2$
- $H_1 \cap H_2 = \phi$
- H_1, H_2 are convex
- $A \in H_1, B \in H_2 \rightarrow \overline{AB} \cap \ell \neq \phi$

Since P and Q are on opposite sides of l, either $\boxed{P \in H_1, Q \in H_2}$ or $\boxed{P \in H_2, Q \in H_1}$.

<u>Case 1:</u> Suppose $P \in H_1$ and $Q \in H_2$.

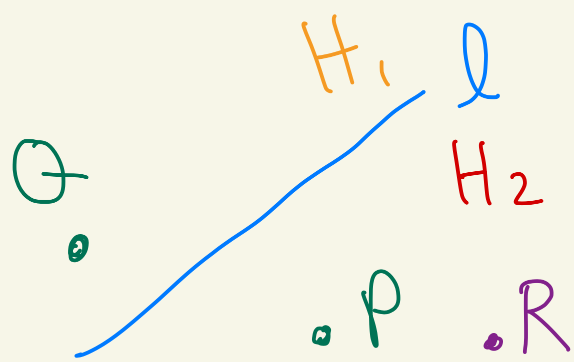


Since Q and R are also on opposite sides of l, this implies that $R \in H_1$.

So, $P \in H_1$ and $R \in H_1$

Thus, P, R are on the same side of l.

<u>Case 2:</u> Suppose $P \in H_2$ and $Q \in H_1$.



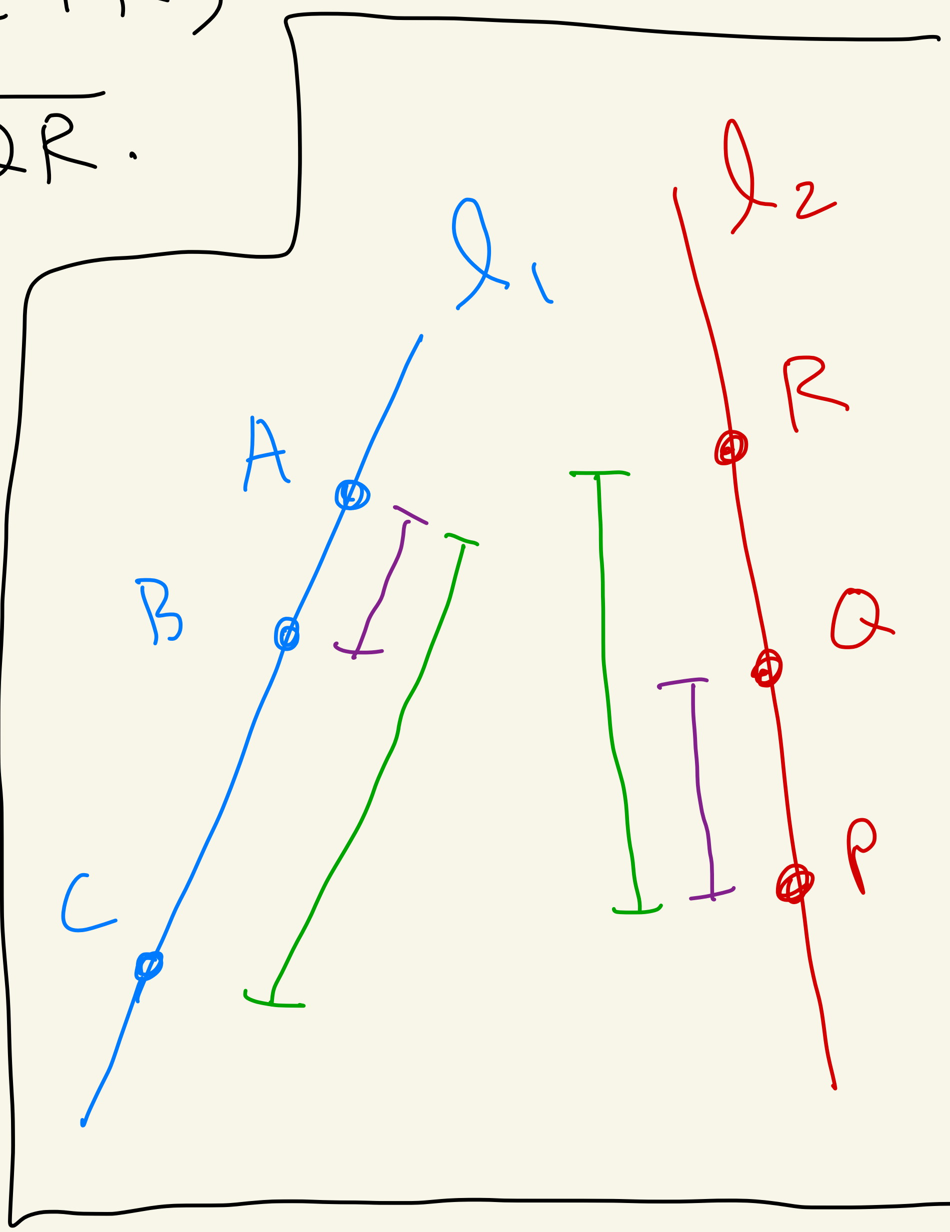
Since Q and R are on opposite sides of l, this implies that $R \in H_2$.

Since $P \in H_2$ and $R \in H_2$, we know P and R are on the same side of ℓ.

$\blacksquare$

HW 5

8 Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry.
Let $A, B, C, P, Q, R \in \mathscr{P}$.
If $A-B-C$, $P-Q-R$, $\overline{AB} \simeq \overline{PQ}$,
and $\overline{AC} \simeq \overline{PR}$,
then $\overline{BC} \simeq \overline{QR}$.



<u>proof:</u>

Since $A-B-C$ there exists a line l_1 with $A, B, C \in l_1$ and

$$\boxed{d(A,B)+d(B,C) = d(A,C).}$$

Since $P-Q-R$, there exists a line ℓ_2 where $P, Q, R \in \ell_2$ and $\boxed{d(P,Q)+d(Q,R)=d(P,R).}$

Since $\overline{AB} \simeq \overline{PQ}$ we know $\boxed{d(A,B)=d(P,Q)}$

Since $\overline{AC} \simeq \overline{PR}$ we know $\boxed{d(A,C)=d(P,R).}$

Thus,

$$\begin{aligned} d(B,C) &= d(A,C) - d(A,B) \\ &= d(P,R) - d(P,Q) \\ &= d(Q,R). \end{aligned}$$

So, $\overline{BC} \simeq \overline{QR}$. $\blacksquare$

$\boxed{\text{Hw 4}}$

(9) Let $(\mathscr{P}, \mathscr{L}, d)$ be a metric geometry. Let $A, B, P, Q \in \mathscr{P}$. If $\boxed{A-Q-B}$ and $A-P-B$ and $P-C-Q$,

then $A-C-B$.

<u>proof:</u> Since $A-Q-B$ and $A-P-B$ and $P-C-Q$ we know:

$A, B, P, Q, C \in l$ where $l = \overleftrightarrow{AB}$.

Let $f: l \to \mathbb{R}$ be a ruler for l.

Since $A-Q-B$ we know either:

$f(A) < f(Q) < f(B)$ ← Case 1

or $f(B) < f(Q) < f(A)$. ← Case 2

Let's prove case 1 where

$$\boxed{f(A) < f(Q) < f(B),} \quad (*)$$

Case 2 is similar.

Since $A - P - B$ we know either

$$f(A) < f(P) < f(B)$$

or $f(B) < f(P) < f(A)$.

Can't happen because $f(A) < f(B)$

We must have

$$\boxed{f(A) < f(P) < f(B)} \quad (**)$$

Since $P - C - Q$ either

$$f(P) < f(C) < f(Q) \quad \text{Case (a)}$$

or $f(Q) < f(C) < f(P)$ Case (b)

<u>Case (a):</u> Suppose $f(P) < f(C) < f(Q)$

Then,

$$f(A) \overset{(**)}{<} f(P) < f(C) < f(Q) \overset{(*)}{<} f(B)$$

So, $f(A) < f(C) < f(B)$.

Thus, $A - C - B$.

<u>Case (b):</u> Suppose $f(Q) < f(C) < f(P)$.

Then,

$$f(A) \overset{(*)}{<} f(Q) < f(C) < f(P) \overset{(**)}{<} f(B)$$

So, $f(A) < f(C) < f(B)$

Thus, $A - C - B$.

Try case 2 for practice. ∎