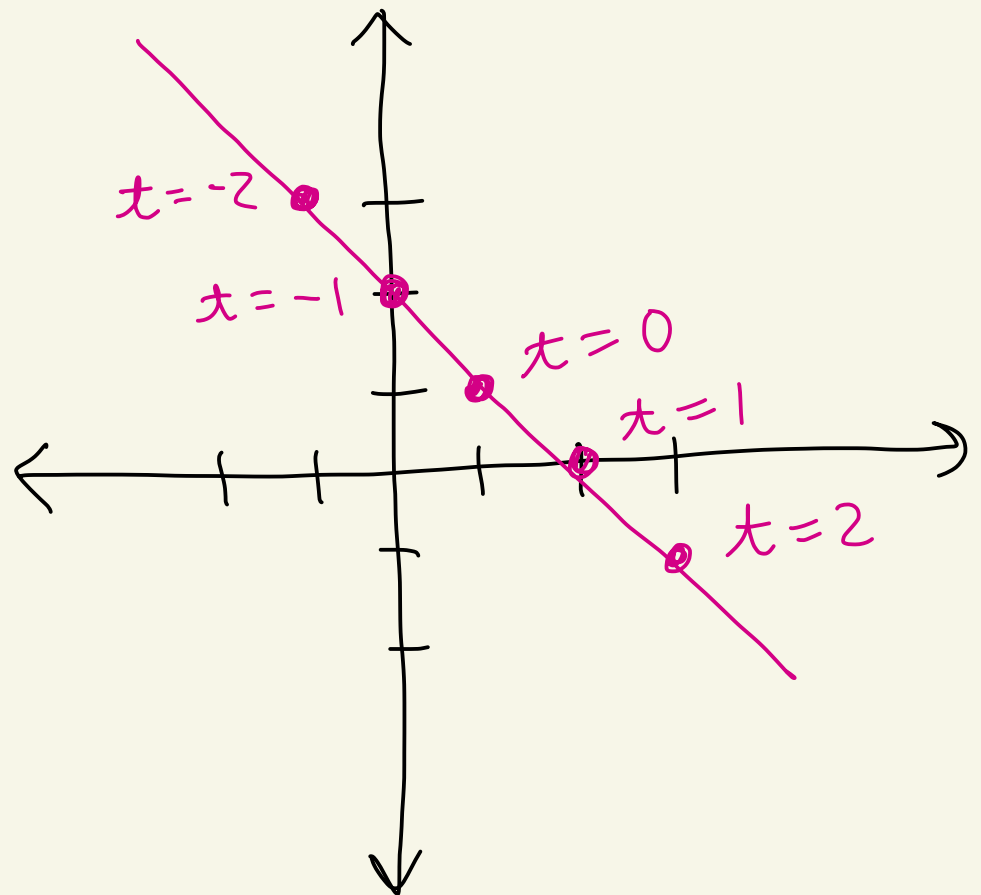


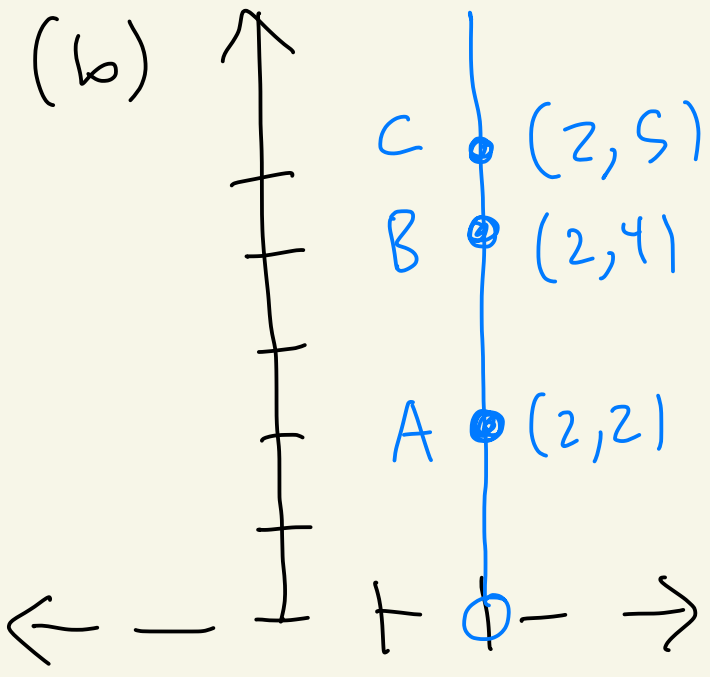
$$\begin{aligned}
 \textcircled{1} L_{AB} &= \{ A + t(B-A) \mid t \in \mathbb{R} \} \\
 &= \{ (1, 1) + t(1, -1) \mid t \in \mathbb{R} \} \\
 &= \{ (1+t, 1-t) \mid t \in \mathbb{R} \}
 \end{aligned}$$

t	$(1+t, 1-t)$
-2	$(-1, 3)$
-1	$(0, 2)$
0	$(1, 1)$
1	$(2, 0)$
2	$(3, -1)$



2

(a) $A, B, C \in {}_2L$ so they are collinear.



Method 1:

$$\begin{aligned}
 d_H(A, B) + d_H(B, C) &= \left| \ln\left(\frac{4}{2}\right) \right| + \left| \ln\left(\frac{5}{4}\right) \right| \\
 &= \left| \ln(2) \right| + \left| \ln\left(\frac{5}{4}\right) \right| \\
 &= \ln(2) + \ln\left(\frac{5}{4}\right) \\
 &= \ln\left(2 \cdot \frac{5}{4}\right) = \ln\left(\frac{5}{2}\right) = d_H(A, C)
 \end{aligned}$$

Since A, B, C are collinear and
 $d_H(A, B) + d_H(B, C) = d_H(A, C)$
we know that $A-B-C$.

Method 2: $f: \mathbb{Z}^+ \rightarrow \mathbb{R}$ given
by $f(z, y) = \ln(y)$ is the
standard ruler.

We have that

$$f(A) = \ln(2)$$

$$f(B) = \ln(4)$$

$$f(C) = \ln(5)$$

So, $f(A) < f(B) < f(C)$.

Thus, since A, B, C are collinear
and $f(A) < f(B) < f(C)$
we know that $A-B-C$.

③

$$\begin{aligned} \vec{AB} &= \{ A + t(B-A) \mid t \geq 0 \} \\ &= \{ (0,0) + t(1,3) \mid t \geq 0 \} \\ &= \{ (t, 3t) \mid t \geq 0 \} \end{aligned}$$

We have that

$$PQ = d_E(P, Q) = \sqrt{(2-1)^2 + (2-1)^2} = \sqrt{2}$$

We must find $C \in \vec{AB}$ where $AC = \sqrt{2}$.

Let $C = (t, 3t)$ where $t \geq 0$.

$$\begin{aligned} \text{Then, } AC &= d_E(A, C) = \sqrt{(t-0)^2 + (3t-0)^2} \\ &= \sqrt{t^2 + 9t^2} \\ &= \sqrt{10t^2} = \sqrt{10} t \end{aligned}$$

↑
since $t \geq 0$

We need $\sqrt{10} t = \sqrt{2}$.

$$\text{So, } t = \frac{\sqrt{2}}{\sqrt{10}} = \sqrt{\frac{2}{10}} = \frac{1}{\sqrt{5}}$$

$$\text{Thus, } C = (t, 3t) = \left(\frac{1}{\sqrt{5}}, \frac{3}{\sqrt{5}}\right)$$

④ (a)

$$(0-c)^2 + 1^2 = r^2 \quad \leftarrow \text{from A}$$

$$(2-c)^2 + 1^2 = r^2 \quad \leftarrow \text{from B}$$

$$\text{So, } c^2 + 1 = r^2 = 4 - 4c + c^2 + 1$$

$$\text{Thus, } 0 = 4 - 4c$$

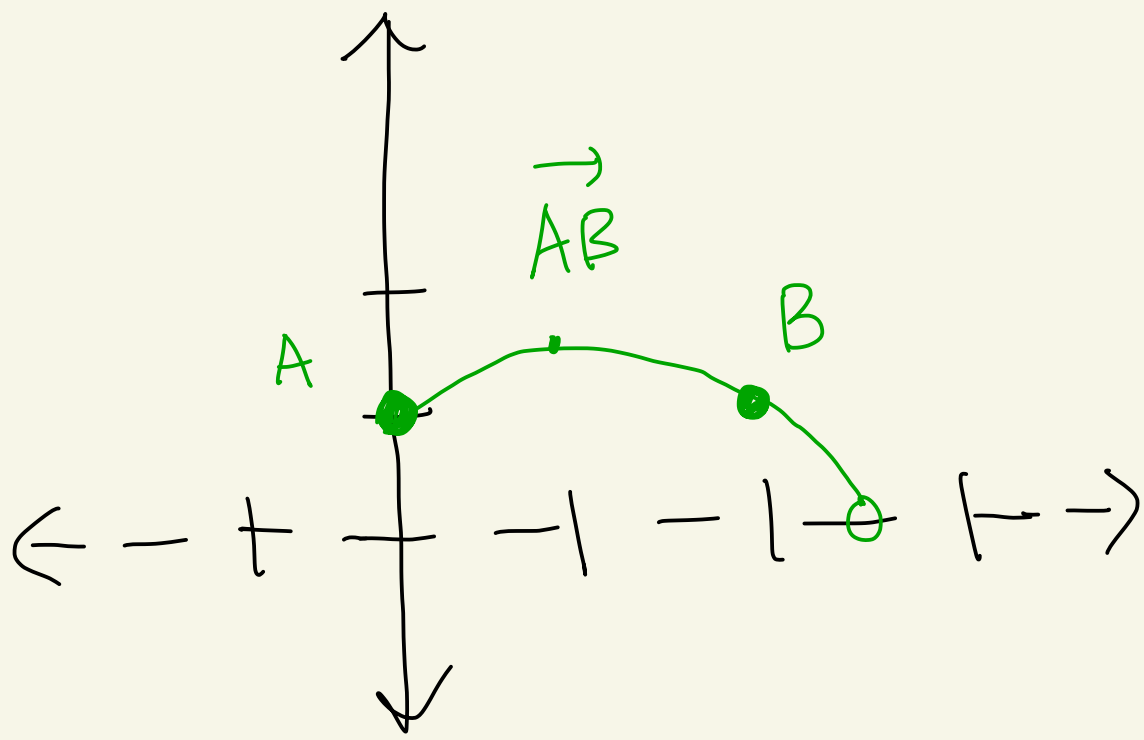
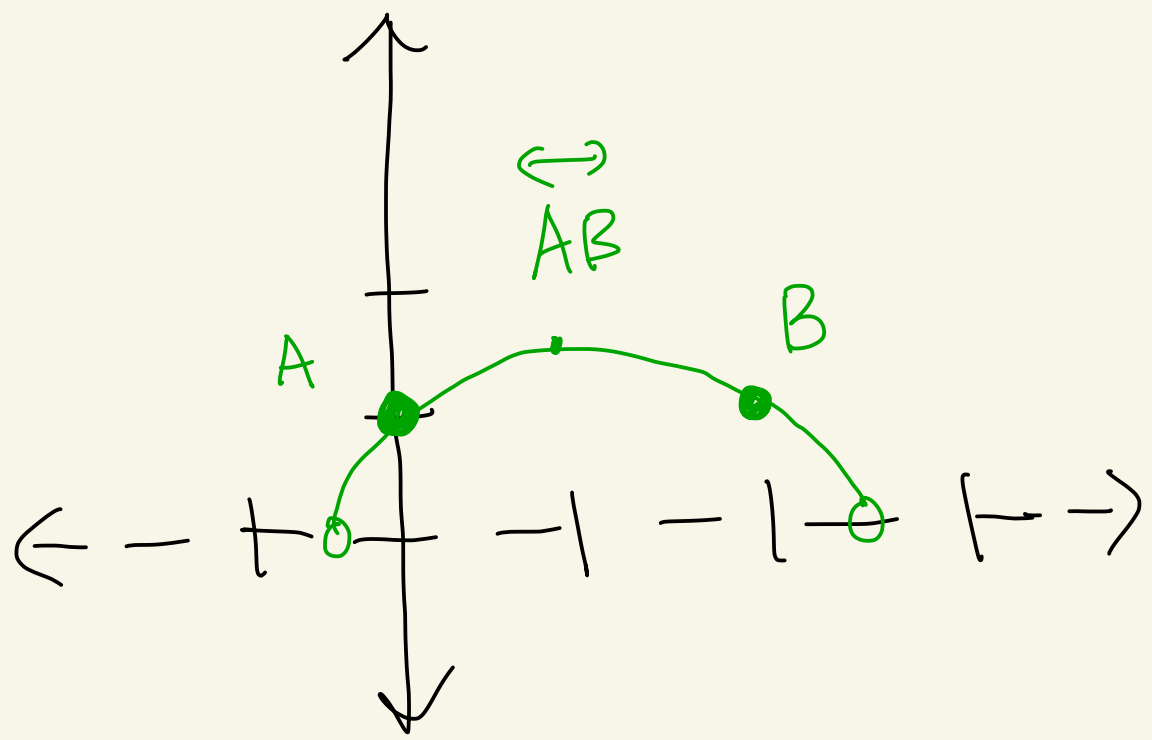
$$\text{So, } c = 1.$$

$$\text{Thus, } r^2 = 1 + c^2 = 2$$

$$\text{So, } r = \sqrt{2}.$$

$$\text{Thus, } \overleftrightarrow{AB} = \sqrt{2}$$

pictures for (a) and (b) :



⑤ See HW 3 problem 2.

⑥ See HW 4 problem 6