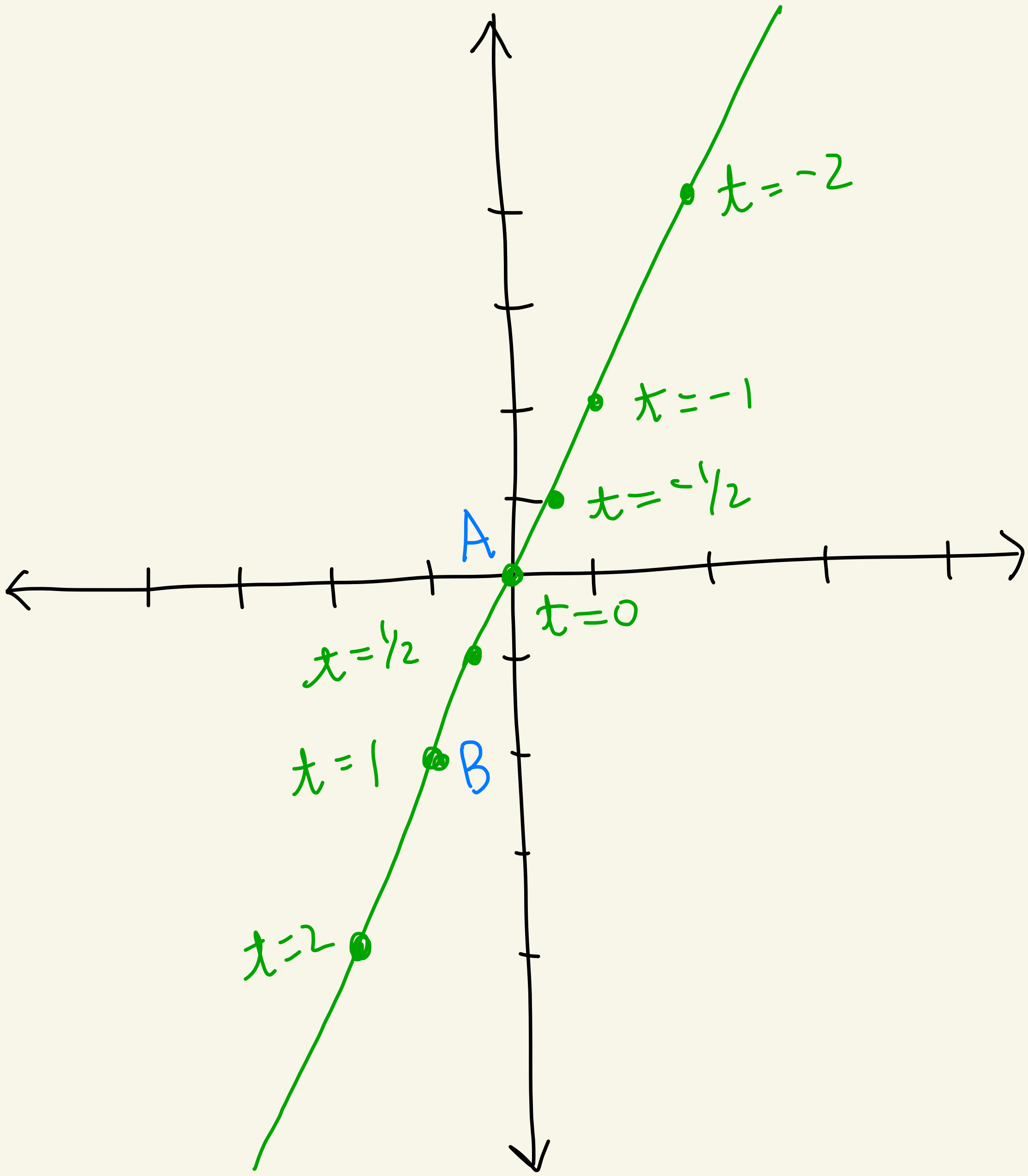
Math 4300
Homework 3
Solutions

$\boxed{① (a)}$ $A=(0,0)$, $B=(-1,-2)$

$$L_{AB} = \{ A + t(B-A) \mid t \in \mathbb{R} \}$$
$$= \{ (0,0) + t(-1,-2) \mid t \in \mathbb{R} \}$$
$$= \{ (-t,-2t) \mid t \in \mathbb{R} \}$$

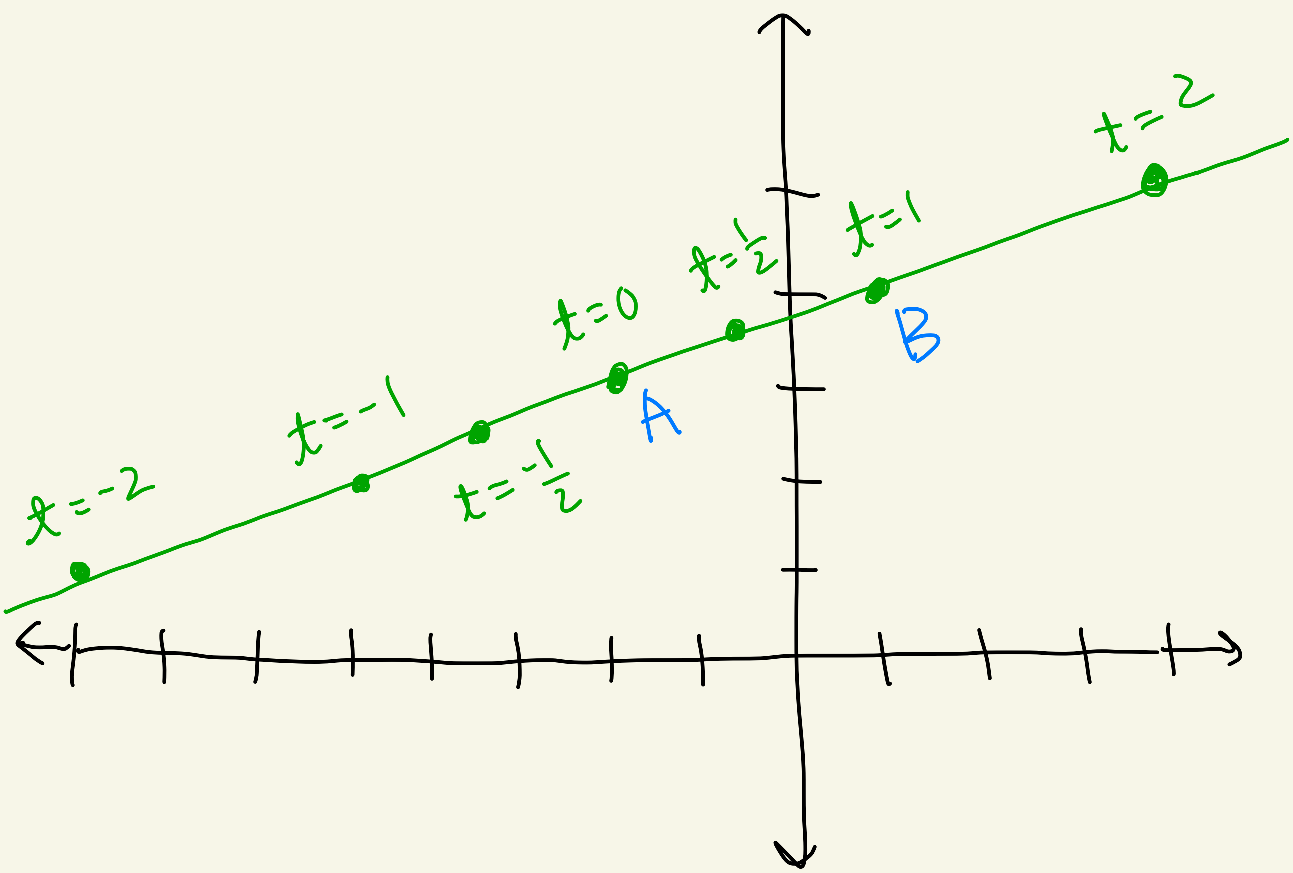


t	$(-t,-2t)$
-2	$(2,4)$
-1	$(1,2)$
$-\frac{1}{2}$	$(\frac{1}{2},1)$
0	$(0,0)$
$\frac{1}{2}$	$(-\frac{1}{2},-1)$
1	$(-1,-2)$
2	$(-2,-4)$

$\boxed{①(b)}$ $A=(-2,3)$, $B=(1,4)$

$$L_{AB}=\{A+t(B-A)\mid t\in\mathbb{R}\}$$
$$=\{(-2,3)+t(3,1)\mid t\in\mathbb{R}\}$$
$$=\{(-2+3t,3+t)\mid t\in\mathbb{R}\}$$

t	$(-2+3t,3+t)$
-2	$(-8,1)$
-1	$(-5,2)$
$-\frac{1}{2}$	$(-\frac{7}{2},5/2)$
0	$(-2,3)$
$1/2$	$(-1/2,7/2)$
1	$(1,4)$
2	$(4,5)$



② For all of problem 2, let

$A=(x_1,y_1)$, $B=(x_2,y_2)$, $C=(x_3,y_3)\in\mathbb{R}^2$

and $r,s\in\mathbb{R}$.

(a)
$$\begin{aligned} A+B &= (x_1,y_1)+(x_2,y_2) \\ &= (x_1+x_2,\, y_1+y_2) \\ &= (x_2+x_1,\, y_2+y_1) \\ &= (x_2,y_2)+(x_1,y_1) \\ &= B+A \end{aligned}$$

In $\mathbb{R}$
$a+b=b+a$

(b)
$$\begin{aligned} (A+B)+C &= \big((x_1,y_1)+(x_2,y_2)\big)+(x_3,y_3) \\ &= (x_1+x_2,\, y_1+y_2)+(x_3,y_3) \\ &= \big((x_1+x_2)+x_3,\, (y_1+y_2)+y_3\big) \\ &= \big(x_1+(x_2+x_3),\, y_1+(y_2+y_3)\big) \\ &= (x_1,y_1)+\big((x_2+x_3,\, y_2+y_3)\big) \\ &= (x_1,y_1)+\big((x_2,y_2)+(x_3,y_3)\big) \\ &= A+(B+C) \end{aligned}$$

In $\mathbb{R}$
$(a+b)+c$
$=a+(b+c)$

(c) $$r(A+B) = r((x_1,y_1)+(x_2,y_2))$$
$$= r(x_1+x_2, y_1+y_2)$$
$$= (r(x_1+x_2), r(y_1+y_2))$$
$$= (rx_1+rx_2, ry_1+ry_2)$$
$$= (rx_1, ry_1) + (rx_2, ry_2)$$
$$= r(x_1,y_1) + r(x_2,y_2)$$
$$= rA + rB$$

(d) $$(r+s)A = (r+s)(x_1,y_1)$$
$$= ((r+s)x_1, (r+s)y_1)$$
$$= (rx_1+sx_1, ry_1+sy_1)$$
$$= (rx_1, ry_1) + (sx_1, sy_1)$$
$$= r(x_1,y_1) + s(x_1,y_1)$$
$$= rA + sA$$

$$
\begin{aligned}
\text{(e)}\quad \langle A, B\rangle &= \langle (x_1, y_1), (x_2, y_2)\rangle \\
&= x_1 x_2 + y_1 y_2 \\
&= x_2 x_1 + y_2 y_1 \\
&= \langle (x_2, y_2), (x_1, y_1)\rangle \\
&= \langle B, A\rangle
\end{aligned}
$$

$$
\begin{aligned}
\text{(f)}\quad \langle rA, B\rangle &= \langle r(x_1, y_1), (x_2, y_2)\rangle \\
&= \langle (rx_1, ry_1), (x_2, y_2)\rangle \\
&= (rx_1)x_2 + (ry_1)y_2 \\
&= r[x_1 x_2 + y_1 y_2] \\
&= r\langle (x_1, y_1), (x_2, y_2)\rangle \\
&= r\langle A, B\rangle
\end{aligned}
$$

$$
\begin{aligned}
\text{(g)}\quad \langle A+B, C\rangle &= \langle (x_1, y_1) + (x_2, y_2), (x_3, y_3)\rangle \\
&= \langle (x_1 + x_2, y_1 + y_2), (x_3, y_3)\rangle \\
&= (x_1 + x_2)x_3 + (y_1 + y_2)y_3 \\
&= x_1 x_3 + x_2 x_3 + y_1 y_3 + y_2 y_3 \\
&= (x_1 x_3 + y_1 y_3) + (x_2 x_3 + y_2 y_3)
\end{aligned}
$$

$$= \langle (x_1, y_1), (x_3, y_3) \rangle + \langle (x_2, y_2), (x_3, y_3) \rangle$$

$$= \langle A, C \rangle + \langle B, C \rangle$$

(h) $\|rA\| = \sqrt{\langle rA, rA \rangle}$

$$= \sqrt{\langle r(x_1, y_1), r(x_1, y_1) \rangle}$$

$$= \sqrt{\langle (rx_1, ry_1), (rx_1, ry_1) \rangle}$$

$$= \sqrt{(rx_1)(rx_1) + (ry_1)(ry_1)}$$

$$= \sqrt{r^2 x_1^2 + r^2 y_1^2}$$

$$= \sqrt{r^2}\sqrt{x_1^2 + y_1^2}$$

$$= |r| \sqrt{\langle (x_1, y_1), (x_1, y_1) \rangle}$$

$$= |r| \sqrt{\langle A, A \rangle}$$

$$= |r| \cdot \|A\|$$

(i) Note that $\|A\| = \sqrt{x_1^2 + y_1^2}$.

And, $\sqrt{x_1^2+y_1^2} > 0$ iff $x_1^2 + y_1^2 > 0$

iff $x_1 \neq 0$ or $y_1 \neq 0$

iff $A = (x_1, y_1) \neq (0,0)$.

$\boxed{/\!/}$

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3 Let $A = (x_1, y_1)$, $B = (x_2, y_2)$

Then,

$$d_E(A,B) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{\langle (x_1 - x_2, y_1 - y_2), (x_1 - x_2, y_1 - y_2) \rangle}$$

$$= \sqrt{\langle A - B, A - B \rangle}$$

$$= \| A - B \|$$