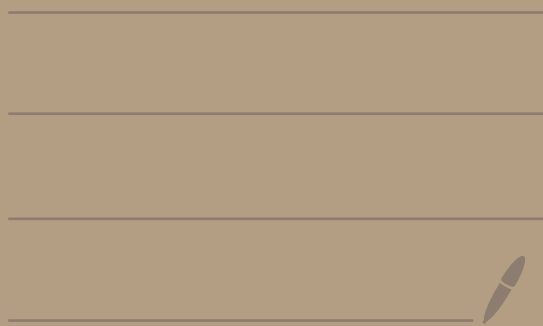


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# Assumptions

We will assume the set of integers exists.

$$\mathbb{Z} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$$

We will assume all the basic algebraic properties of  $\mathbb{Z}$  like:

(closure) If  $a, b \in \mathbb{Z}$ , then  
 $a + b \in \mathbb{Z}$  and  $a \cdot b \in \mathbb{Z}$

other kinds of properties

$$a \cdot (b + c) = ab + ac$$

$$a \cdot 1 = a$$

$$a + 0 = a$$

If  $a < b$  and  $c > 0$ ,

then  $ca < cb$

E  
X  
A  
M  
P  
L  
E  
S

# Topic 1 - Division and primes

Def: Let  $x$  and  $y$  be integers with  $x \neq 0$ .

We say that  $x$  divides  $y$  if there exists an integer  $k$  with  $y = xk$ .

If  $x$  divides  $y$  then we call  $x$  a divisor of  $y$  and

write  $x \mid y$ .

read: " $x$  divides  $y$ "

If  $x$  doesn't divide  $y$  then we write  $x \nmid y$ .

Ex:

Divisors of 12:

1, 2, 3, 4, 6, 12

-1, -2, -3, -4, -6, -12

$$12 \mid 12 \quad \text{because} \quad \underbrace{12}_y = \underbrace{(12)}_x \underbrace{(1)}_k$$

$$-4 \mid 12 \quad \text{because} \quad \underbrace{12}_y = \underbrace{(-4)}_x \underbrace{(-3)}_k$$

$100 \nmid 12$  because if  $12 = 100k$   
you'd need  $k = \frac{12}{100} \notin \mathbb{Z}$ .

Def: Let  $p \in \mathbb{Z}$  with  $p > 1$ .

We say that  $p$  is prime if the only positive divisors of  $p$  are 1 and  $p$ .

If  $p$  is not prime, then we call  $p$  a composite number.

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Ex: Let's circle the primes.

1, (2), (3), 4, (5), 6, (7), 8, 9, 10, (11),  
12, (13), 14, 15, 16, (17), 18, (19), 20,  
21, 22, (23), 24, 25, 26, 27,  
28, (29), 30, ...

Theorem: Let  $x$  and  $y$   
be positive integers.

If  $x|y$ , then  $1 \leq x \leq y$ .

proof:

Suppose  $x$  and  $y$  are positive  
integers with  $x|y$ .

Since  $x$  and  $y$  are positive  
integer we get  $1 \leq x$   
and  $1 \leq y$ .

Since  $x|y$  there exists  
an integer  $k$  with  
 $xk = y$ .

Then  $k = \frac{y}{x}$  is positive.

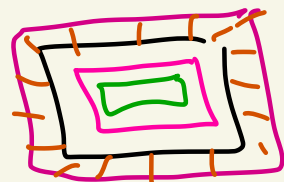
(Using dividing two positive #s is positive)

So,  $1 \leq k$ .

Multiply by  $x$  and since  $x$  is positive we get that  $x \leq \underbrace{xk}_y$ .

So,  $x \leq y$ .

Thus,  $1 \leq x \leq y$ .



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Theorem: Let  $p$  and  $q$  be prime numbers. If  $p|q$ , then  $p=q$ .

Proof:

Suppose  $p$  and  $q$  are primes  
with  $p \mid q$ .

Since  $p$  is prime we  
have  $p > 1$ .

Since  $q$  is prime, the only  
positive divisors are 1 and  $q$ .

Thus since  $p$  is a positive  
divisor of  $q$  we know  
either  $p = 1$  or  $p = q$ .

Since  $p > 1$  we must  
have  $p = q$ .

