1

$$514 = 4 \cdot 120 + 34$$
$$120 = 3 \cdot 34 + 18$$
$$34 = 1 \cdot 18 + 16$$
$$18 = 1 \cdot 16 + 2$$
$$16 = 8 \cdot 2 + 0$$

$\boxed{\gcd(514, 120) = 2}$

(2)

$$14 = 1\cdot 110 - 4\cdot 24$$
$$10 = 1\cdot 24 - 1\cdot 14$$
$$4 = 1\cdot 14 - 1\cdot 10$$
$$2 = 1\cdot 10 - 2\cdot 4$$

$$2 = 1\cdot\boxed{10} - 2\cdot\boxed{4}$$
$$= 1\cdot\left[1\cdot\boxed{24} - 1\cdot\boxed{14}\right] - 2\cdot\left[1\cdot\boxed{14} - 1\cdot\boxed{10}\right]$$
$$= 1\cdot\boxed{24} - 3\cdot\boxed{14} + 2\cdot\boxed{10}$$
$$= 1\cdot\boxed{24} - 3\cdot\left[1\cdot\boxed{110} - 4\cdot\boxed{24}\right] + 2\cdot\left[1\cdot\boxed{24} - 1\cdot\boxed{14}\right]$$
$$= -3\cdot\boxed{110} + 15\cdot\boxed{24} - 2\cdot\boxed{14}$$
$$= -3\cdot\boxed{110} + 15\cdot\boxed{24} - 2\cdot\left[1\cdot\boxed{110} - 4\cdot\boxed{24}\right]$$
$$= -5\cdot 110 + 23\cdot 24$$

So, $110(-5) + 24(23) = 2$

particular solution: $x = -5,\ y = 23$

all solutions

$$x = -5 - t\left(\tfrac{24}{2}\right) = -5 - 12t$$
$$y = 23 + t\left(\tfrac{110}{2}\right) = 23 + 55t, \quad t \in \mathbb{Z}$$

(3)

(a) $\gcd(12,8)=4$

$4 \nmid 14$

There are no integer solutions to

$12x+8y=14$

(b)

Since $a \mid (b-c)$ we have $b-c=ak$ where $k \in \mathbb{Z}$.

Since $a \mid d$ we have $d=al$ where $l \in \mathbb{Z}$.

Thus,

$$\begin{aligned} 2bd-2cd &= 2d(b-c) \\ &= 2(al)(ak) \\ &= a^2(2lk) \end{aligned}$$

Since $2lk \in \mathbb{Z}$ this implies that

$a^2 \mid (2bd-2cd)$.

4

A HW 2 #9

B HW 1 #7(a)

5 c

Since $e = \gcd(a,b)$ we know that $e \mid a$ and $e \mid b$.

Thus, $a = ek$ and $e = bl$ where $k, l \in \mathbb{Z}$.

So, $a+b = e(k+l)$ and $a-b = e(k-l)$

So, $e \mid (a+b)$ and $e \mid (a-b)$.

So, e is a common divisor of $a+b$ and $a-b$.

But $d = \gcd(a+b, a-b)$.

That is, d is the greatest common divisor of $a+b$ and $a-b$.

Thus, $e \leq d$ $\blacksquare$

5 D

Since $d = \gcd(m,n)$ we know $d | m$ and $d | n$.
Thus, $m = dk$ and $n = dl$ where $k, l \in \mathbb{Z}$.
Since $m | (a-b)$ and $n | (a-c)$ we know
that $a-b = ms$ and $a-c = nt$
where $s, t \in \mathbb{Z}$.

Thus,

$$\begin{aligned} b-c &= (a-ms)-(a-nt) \\ &= -ms+nt \\ &= -dks+dlt \\ &= d[-ks+lt]. \end{aligned}$$

So, $d | (b-c)$. $\blacksquare$