

# 4460 Test 2 Solutions

pg 1

① (a)  $4 - (-1001) = 1005 = 3 \cdot 335$

Since  $3 \mid (4 - (-1001))$  we

know that  $4 \equiv -1001 \pmod{3}$

(TRUE)

$$\begin{array}{r} 335 \\ 3 \overline{)1005} \\ \underline{-9} \phantom{00} \\ 10 \phantom{0} \\ \underline{-9} \phantom{0} \\ 15 \\ \underline{-15} \\ 0 \end{array}$$

(b)  ~~$68 - (-3) = 71$~~

$68 - (-3) = 71$

$12 \nmid 71$

Thus,  $\overline{-3} \neq \overline{68}$

So the statement  ~~$\overline{-3} = \overline{68}$~~  is false

$$\begin{array}{r} 5 \\ 12 \overline{)71} \\ \underline{-60} \\ 11 \end{array}$$

remainder

② (a) 
$$\begin{aligned} \overline{3} \cdot \overline{5} + \overline{2}^3 + \overline{4} \cdot \overline{4} \cdot \overline{-3} &= \overline{15} + \overline{8} + \overline{16} \cdot \overline{-3} \\ &= \overline{3} + \overline{2} + \overline{4} \cdot \overline{3} \\ &= \overline{5} + \overline{12} \\ &= \overline{5} + \overline{0} = \overline{5} \end{aligned}$$

② (b) 
$$\overline{-14}^4 = \overline{-2}^4 = \overline{16} = \overline{4}$$

$$\begin{array}{c} \uparrow \\ \overline{-14} = \overline{-2} \\ \text{in } \mathbb{Z}_6 \end{array}$$

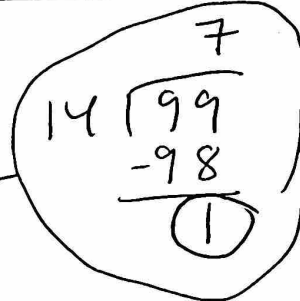
$$\begin{array}{c} \uparrow \\ \overline{16} = \overline{4} \text{ in } \mathbb{Z}_6 \end{array}$$

③(a)

$$\mathbb{Z}_{20}^* = \{ \overline{1}, \overline{3}, \overline{7}, \overline{9}, \overline{11}, \overline{13}, \overline{17}, \overline{19} \}$$

---

③(b)  $\overline{9} \cdot \overline{11} = \overline{99} = \overline{1}$



So,  $\overline{9}^{-1} = \overline{11}$

---

Ⓐ HW 4 - Problem #10

Ⓑ HW 3 - Problem #6

---

ⓐ Suppose  $\sqrt{\frac{3}{5}}$  is rational.

Then  $\sqrt{\frac{3}{5}} = \frac{a}{b}$  where  $a, b \in \mathbb{Z}$ ,  $b \neq 0$ , and  $\gcd(a, b) = 1$ .

Thus,  $\frac{3}{5} = \frac{a^2}{b^2}$ .

So,  $3b^2 = 5a^2$ .

Then  $3 \mid 5a^2$ .

Since 3 is prime  $3 \mid 5$  or  $3 \mid a^2$ .

Since  $3 \nmid 5$  we know  $3 \mid a^2$ .

Since 3 is prime and  $3 \mid a \cdot a$  we know  $3 \mid a$ .

Thus  $a = 3k$  where  $k \in \mathbb{Z}$ .

So,  $3b^2 = 5(3k)^2$ .

Thus,  $3b^2 = 5 \cdot 3^2 \cdot k^2$ .

So,  $b^2 = 3 \cdot 5 \cdot k^2$ .

Thus,  $3 \mid b^2$ .

Since 3 is prime and  $3 \mid b \cdot b$ , we know  $3 \mid b$ .

But then  $3 \mid a$  and  $3 \mid b$  which contradicts  $\gcd(a, b) = 1$ .

Therefore,  $\sqrt{\frac{3}{5}}$  is irrational. ◻

(D) Suppose by way of contradiction that there exist integers  $x, y$  with  $15x^2 - 10x + 7y^2 = 11$ .

Then in  $\mathbb{Z}_5$  we would have

$$\overline{15} \overline{x}^2 + \overline{(-10)} \overline{x} + \overline{7} \overline{y}^2 = \overline{11}$$

$$\begin{aligned} \overline{15} &= 0 \\ \overline{-10} &= 0 \\ &\text{in } \mathbb{Z}_5 \end{aligned}$$

So,  $\overline{7} \overline{y}^2 = \overline{11}$

Thus,  $\overline{2} \overline{y}^2 = \overline{1}$ .

$$\begin{aligned} \overline{7} &= \overline{2} \\ \overline{11} &= \overline{1} \end{aligned} \text{ in } \mathbb{Z}_5$$

So,  $\overline{3} \cdot \overline{2} \overline{y}^2 = \overline{3}$ .

$$\overline{3} \cdot \overline{2} = \overline{6} = \overline{1} \text{ in } \mathbb{Z}_5$$

Thus,  $\overline{y}^2 = \overline{3}$ .

However in  $\mathbb{Z}_5$  we have

$$\overline{0}^2 = \overline{0}, \overline{1}^2 = \overline{1}, \overline{2}^2 = \overline{4}, \overline{3}^2 = \overline{9} = \overline{4}, \overline{4}^2 = \overline{16} = \overline{1}$$

Thus, there is no  $\overline{y} \in \mathbb{Z}_5$  with  $\overline{y}^2 = \overline{3}$ .

Contradiction.

Thus,  $15x^2 - 10x + 7y^2 = 11$  has no integer solutions.