Math 4680

12/7/22

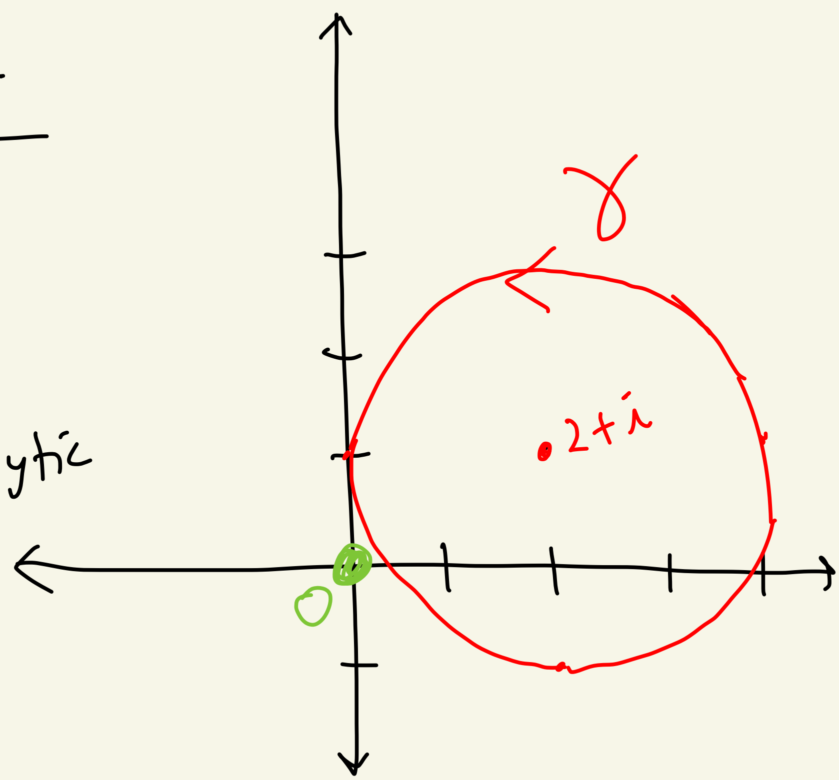
HW 9

① (c) Evaluate $\int_\gamma e^{1/z}\, dz$ where γ is a circle of radius 2 centered at $2+i$.

$f(z) = e^{1/z}$ is analytic everywhere except at 0.

Since $f(z) = e^{1/z}$ is analytic inside and on γ by Cauchy's theorem

$$\int_\gamma e^{1/z}\, dz = 0.$$



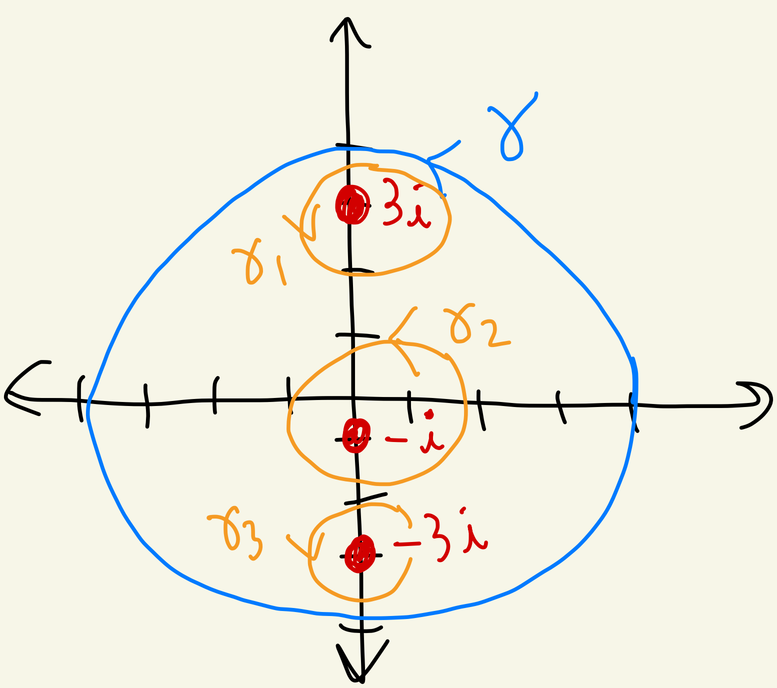
$\boxed{\text{HW 10}}$

① (f) Evaluate $\displaystyle\int_\gamma \frac{z}{(9+z^2)(z+i)^2}\,dz$ where γ is the circle $|z|=4$ oriented counterclockwise.

$\dfrac{z}{(9+z^2)(z+i)^2}$ is not analytic when $\boxed{z \neq \pm 3i, -i}$

$9+z^2=0$ when $z^2=-9$ which is when $\boxed{z=\pm 3i}$

$z+i=0$ when $\boxed{z=-i}$



Our function $\dfrac{z}{(9+z^2)(z+i)^2}$ is analytic on $\gamma, \gamma_1, \gamma_2, \gamma_3$ an in between the curves.

Thus, $\displaystyle\int_\gamma \frac{z}{(9+z^2)(z+i)^2}\,dz$

$$\boxed{z^2+9=(z+3i)(z-3i)}$$

$$= \int_{\gamma_1} \frac{z}{(9+z^2)(z+i)^2}\,dz + \int_{\gamma_2} \frac{z}{(9+z^2)(z+i)^2}\,dz + \int_{\gamma_3} \frac{z}{(9+z^2)(z+i)^2}\,dz$$

$$\int_{\gamma_1} \frac{\frac{z}{(z+3i)(z+i)^2}}{(z-3i)}\,dz = 2\pi i\left[\frac{3i}{(3i+3i)(3i+i)^2}\right] \overset{\text{calculate}}{=} -\frac{\pi i}{16}$$

$$\int_\gamma \frac{f(z)}{z-z_0}\,dz = 2\pi i f(z_0)$$

f is analytic on and in γ

plug $3i$ into top part

$$\int_{\gamma_3} \frac{\frac{z}{(z-3i)(z+i)^2}}{\underbrace{(z+3i)}_{(z-(-3i))}}\,dz = 2\pi i\left[\frac{(-3i)}{(-3i-3i)(-3i+i)^2}\right] = -\frac{\pi i}{4}$$

plug $-3i$ into top part

$$z_0=-i,\ k=1,\ f(z)=\frac{z}{(z-3i)(z+3i)}=\frac{z}{z^2+9}$$

$$f'(z)=\frac{(z^2+9)-2z^2}{(z^2+9)^2}=\frac{-z^2+9}{(z^2+9)^2}$$

$$\int_{\gamma_2} \frac{\frac{z}{(z-3i)(z+3i)}}{\underbrace{(z+i)^2}_{(z-(-i))^2}}\,dz = \frac{2\pi i}{1!}f^{(1)}(-i) = 2\pi i\left[\frac{-(-i)^2+9}{((-i)^2+9)^2}\right] = 2\pi i\left[\frac{10}{64}\right] = \frac{5\pi i}{16}$$

$$\int_\gamma \frac{f(z)}{(z-z_0)^{k+1}}\,dz = \frac{2\pi i}{k!}f^{(k)}(z_0)$$

f is analytic in and on γ

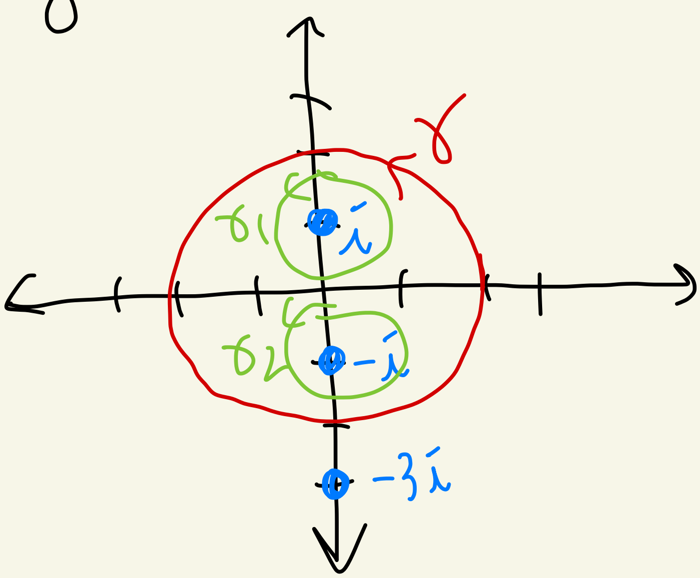
Answer is $\boxed{\frac{-\pi i}{16} - \frac{\pi i}{4} + \frac{5\pi i}{16} = 0}$

What if we modified it

γ circle of radius 2 centered at 0

$$\int_\gamma \frac{z+1}{(z^2+1)(z+3i)}\,dz$$

$$(z^2+1)(z+3i) = (z+i)(z-i)(z+3i)$$



$$\int_\gamma \frac{z+1}{(z^2+1)(z+3i)}\,dz = \int_{\gamma_1} \frac{z+1}{(z^2+1)(z+3i)}\,dz$$

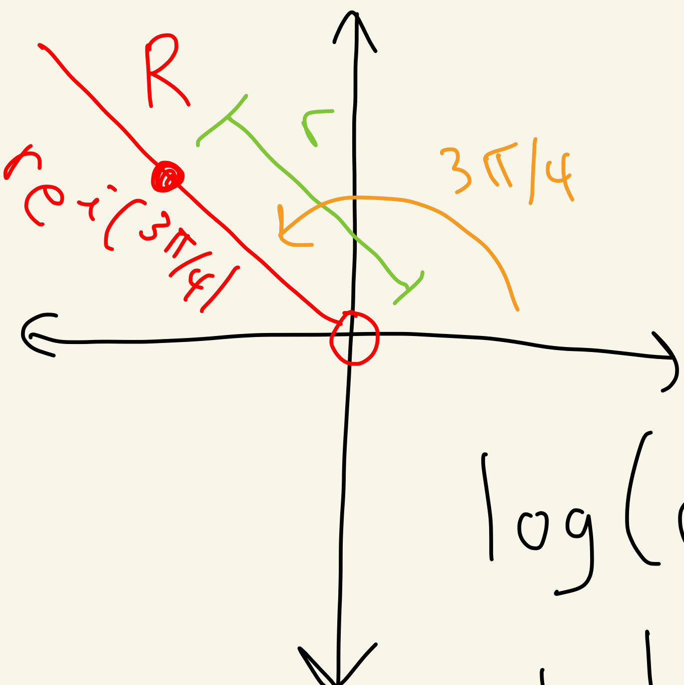
$$+ \int_{\gamma_2} \frac{z+1}{(z^2+1)(z+3i)}\,dz$$

$=$ ... keep going ...

7. $$\boxed{\begin{gathered}\log(z) = \ln|z| + i\arg(z) \\ 0 \le \arg(z) < 2\pi\end{gathered}}$$

$$R = \left\{ re^{i(3\pi/4)} \;\middle|\; r \in \mathbb{Z},\ r > 0 \right\}$$

What does log do to this set?



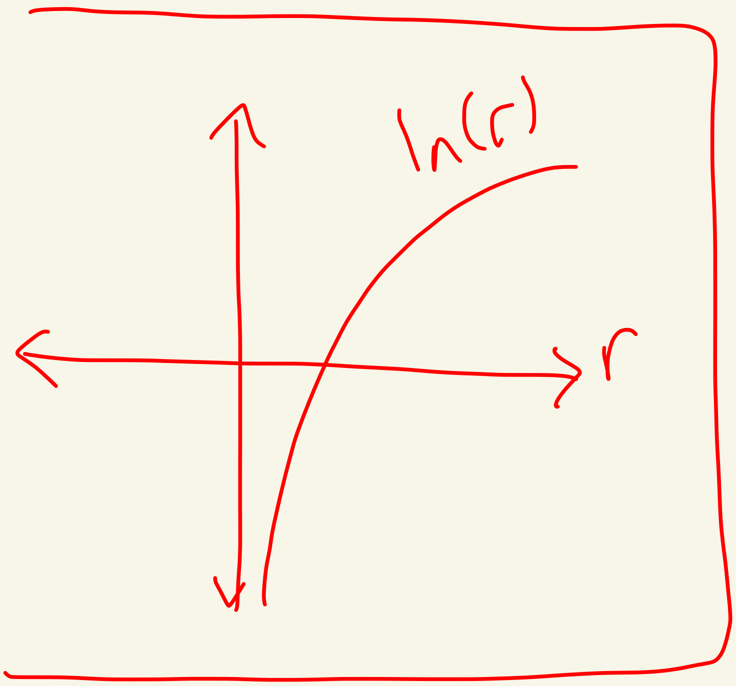
$$\log\left(re^{i(3\pi/4)}\right)$$

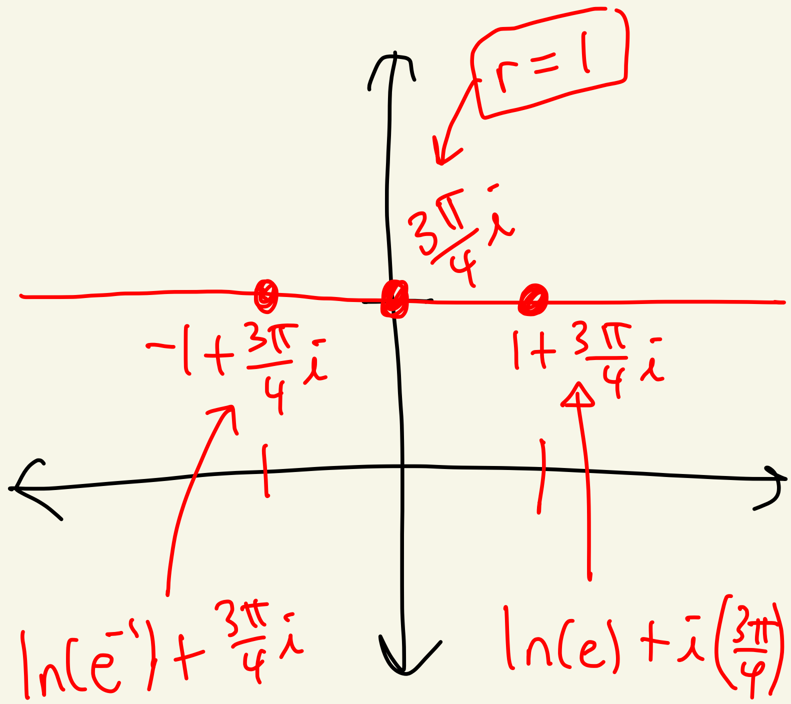
$$= \ln\left|re^{i(3\pi/4)}\right| + i\arg\left(re^{i(3\pi/4)}\right)$$

$$= \ln|r| + i\left(3\pi/4\right)$$

$$= \ln(r) + i\left(3\pi/4\right)$$

Need to graph $\ln(r) + i\left(3\pi/4\right)$ where $r > 0$





We looked at HW 8 #5(a)