Math 4740

10/21/24

Odds Let E be an event.

Define

$$\text{"odds against } E\text{"} = \frac{P(\bar{E})}{P(E)} = \frac{1-P(E)}{P(E)}$$

This is what casinos use for odds calculations.

Ex: Suppose you roll a 4-sided die. Let E be the event that 4 is rolled.

$$(\text{odds against } E) = \frac{P(\bar{E})}{P(E)} = \frac{3/4}{1/4}$$

$$= \frac{3}{1}$$ } written as 3:1

How to convert to probability

Odds against E

$c : d$

$\rightarrow$

$$P(E) = \frac{d}{c+d}$$

Ex: If the odds against E is $3:1$, then

$$P(E) = \frac{1}{3+1} = \frac{1}{4}$$

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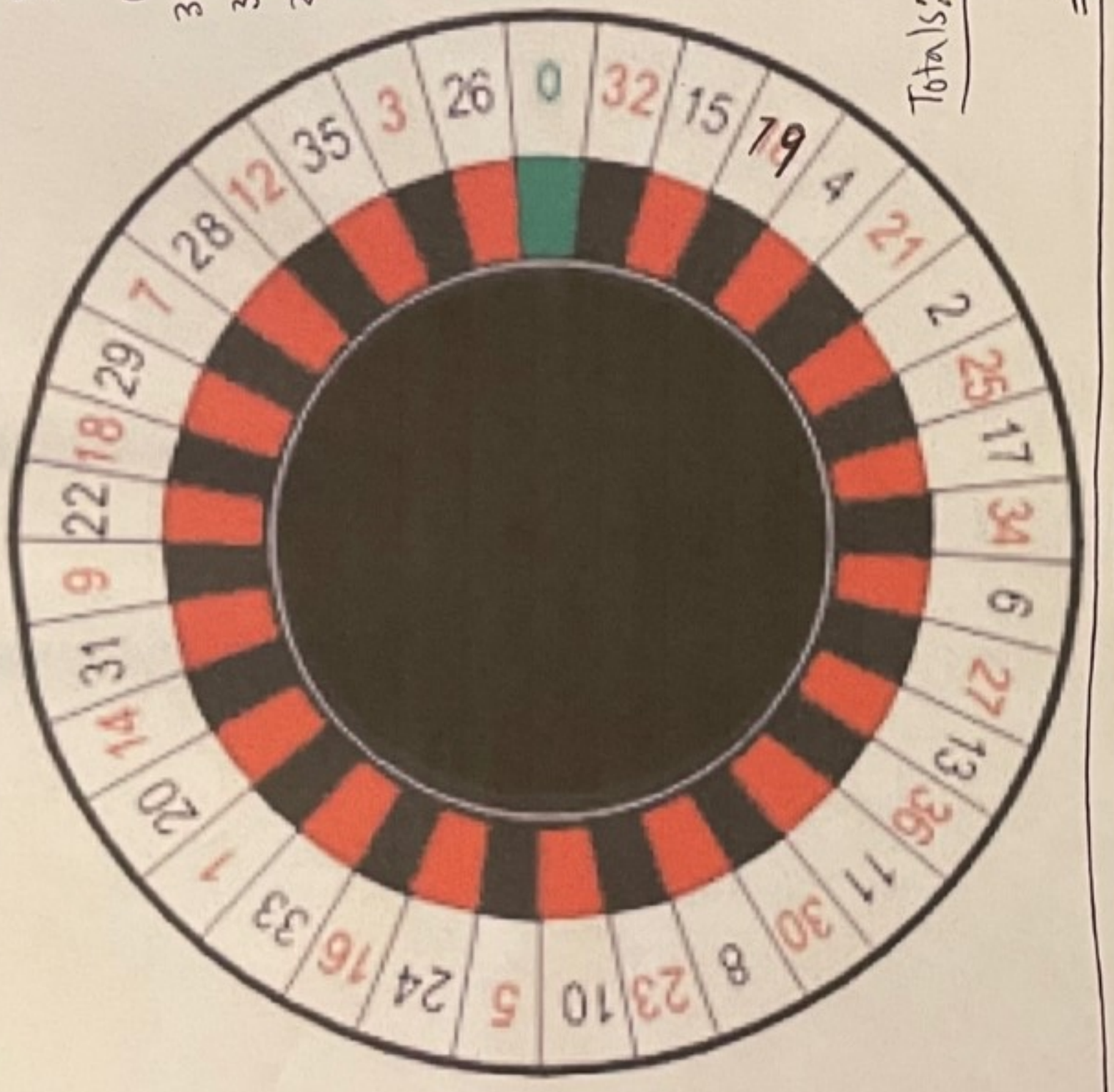
green
0

red
32, 19, 21, 25,
34, 27, 36, 30,
23, 5, 16, 1,
14, 9, 18, 7
12, 3

black
15, 4, 2, 17
6, 13, 11, 8
10, 24, 33, 20
31, 22, 29, 28
35, 26

Totals: 1 green
18 red
18 black
=37 total



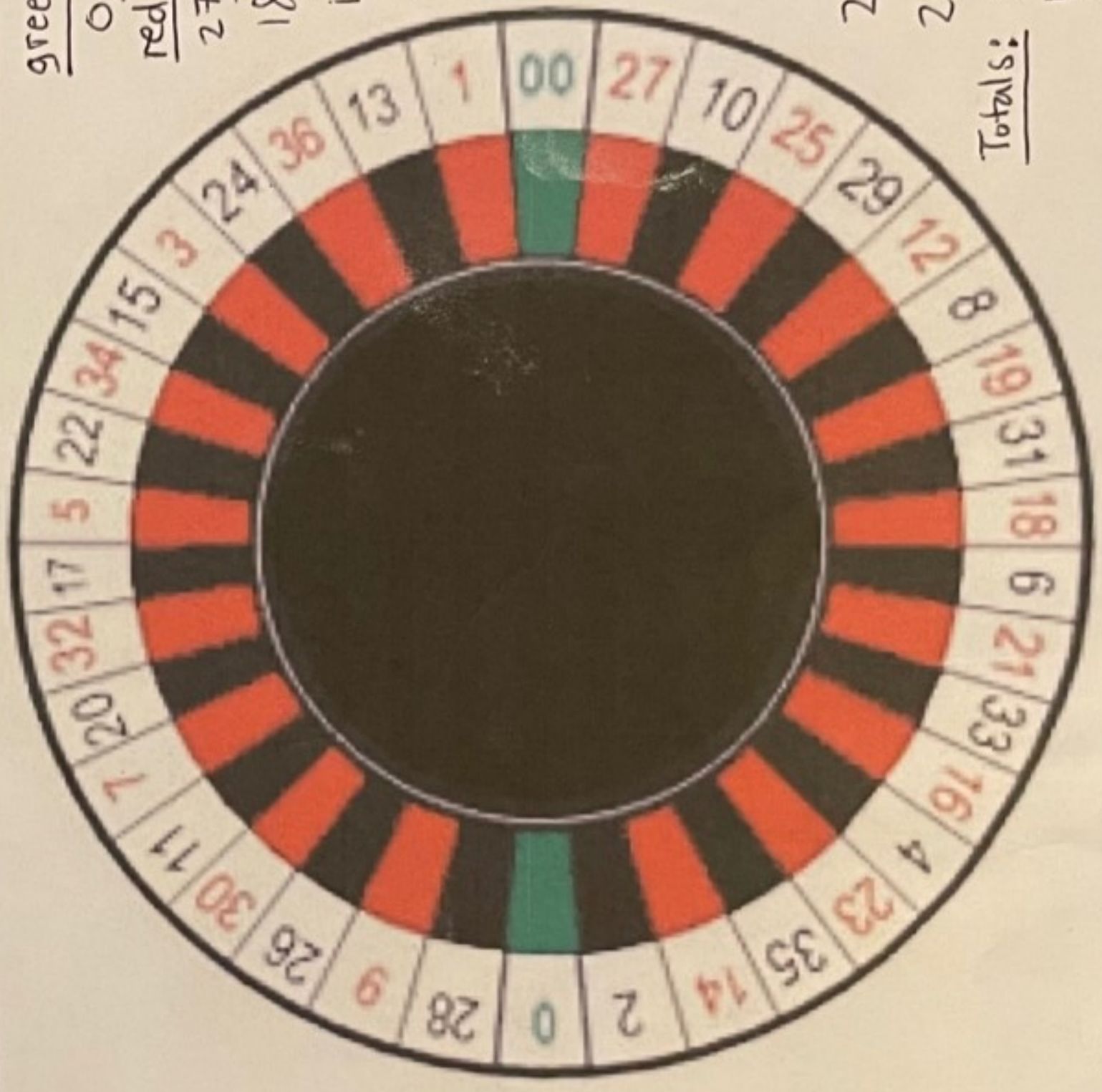
EUROPEAN

green
0, 00

red
27, 25, 12, 19
18, 21, 16, 23
14, 9, 30, 7
32, 5, 34, 3
36, 1

black
10, 29, 8, 31
6, 33, 4, 35
2, 28, 26, 11
20, 17, 22, 15
24, 13

Totals: 2 green
18 red
18 black
= 38 total



AMERICAN

/ Handout

American version

Casino payouts

Type of Bets And [illegible]

Inside bets

Bet Name	Ex.	Numbers to bet on	Payout	Chances	True odds
Straight up	A	30	35:1	38:1	37:1
Split Bet	B	11 or 14	17:1	18:1	36:2
Street Bet	C	19, 20, 21	11:1	12:1	35:3
Corner	D	25, 26, 28, 29	8:1	9:1	34:4
Five Numbers	E	0, 00, 1, 2, 3	6:1	6:1	
Line Bet	F	4, 5, 6, 7, 8, 9	5:1	6:1	32:6

Outside Bets

Bet Name	Ex	Numbers to bet on	Payout	Chances	True odds
Column	G	Set of column numbers	2:1	38:12	26:12
Dozen	H	25 through 36	2:1	38:12	26:12
Red or Black	I	Red numbers	1:1	38:18	20:18
Even or Odd	J	Odd numbers	1:1	38:18	20:18
Low or High	K	19 through 36	1:1	38:18	20:18

Sample space for American wheel

$S = \{0, 00, 1, 2, 3, 4, 5, 6, 7,$
$8, 9, 10, 11, 12, 13, 14,$
$15, 16, 17, 18, 19, 20,$
$21, 22, 23, 24, 25, 26,$
$27, 28, 29, 30, 31, 32,$
$33, 34, 35, 36\}$

Each number is equally likely with probability $1/38$

Straight up bet (35:1 payout)

Suppose we bet \$1 on (10)
What's the expected value of this bet?
Let X be the amount won or lost.

$$X(w) = \begin{cases} -1 & \text{if } w \neq 10 \\ 35 & \text{if } w = 10 \end{cases}$$

$X(w)$: outcome from wheel

Then,

$$E[X] = (-\$1)\underbrace{\left(\frac{37}{38}\right)}_{P(X=-1)} + (\$35)\underbrace{\left(\frac{1}{38}\right)}_{P(X=35)}$$

$$= -\$\frac{2}{38} \approx \boxed{-\$0.0526}$$

$$\approx \boxed{-5.26¢}$$

So on average this bet loses $\underbrace{\$0.0526}_{5.26¢}$ per $1 bet

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In the above example, the casino pays 35:1 on a straight up bet. What are the real odds (that is, the odds against) for this bet?

Here $E = \{10\}$.

we picked

$$(\text{odds against } E) = \frac{P(\bar{E})}{P(E)}$$

$$= \frac{37/38}{1/38} = \frac{37}{1}$$

That is, 37:1.

What if the casino actually paid 37:1 on this bet?

Let Y be the amount won or lost with this payout.

$$Y(w) = \begin{cases} -1 & \text{if } w \neq 10 \\ 37 & \text{if } w = 10 \end{cases}$$

Then,

$$E[Y] = (-\$1)\left(\frac{37}{38}\right) + (\$37)\left(\frac{1}{38}\right)$$

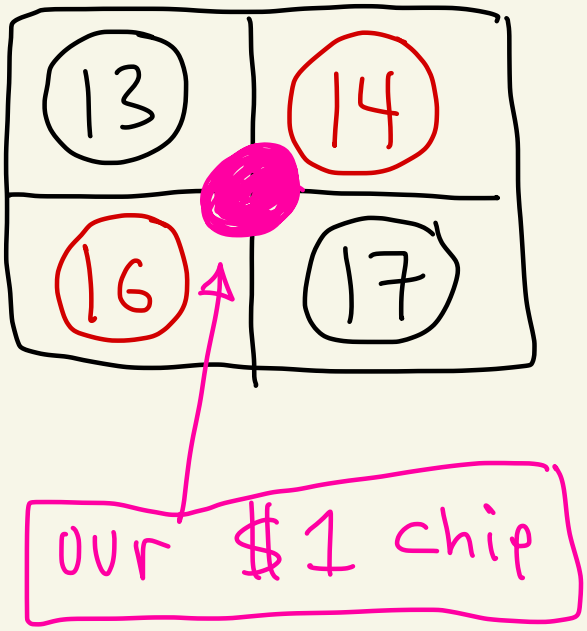
$P(Y=-1)$ $P(Y=37)$

$$= \$0$$

With this payout you break even in the long run. On average you win/lose $0 with many bets.

Corner bet (8:1 payout)

Suppose we put $1 on the corner bet $E = \{13, 14, 16, 17\}$



Let X be the amount won or lost. Then,

$$X(w) = \begin{cases} -1 & \text{if } w \notin E \\ 8 & \text{if } w \in E \end{cases}$$

The expected value of this bet is:

$w \in E$ means w is in E.
$w \notin E$ means w is not in E

$$E[X] = (-\$1)\left(\frac{34}{38}\right) + (\$8)\left(\frac{4}{38}\right)$$

$P(X=-1)$ $P(X=8)$

$$= -\$\frac{2}{38} \approx \boxed{-\$0.0526}$$

$$\approx \boxed{-5.26¢}$$

On average with many bets you lose around 5.26¢ per $1 bet.