Math 4740
2/10/25

I made a study guide with practice tests for Test 1.
It's on the website.

Combinations:

Consider a set of n objects. The number of subsets of size k where $0 \le k \le n$ is

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

proof in notes online

read: "n choose k"

This is the same as the # of ways to choose k objects from n objects where the order doesn't matter.

Ex: $S = \{1, 2, 3, 4\}$

How many subsets of size $k=2$ are there?

Subsets of size $k=2$

$\{1,2\}$ $\{2,3\}$

$\{1,3\}$ $\{2,4\}$

$\{1,4\}$ $\{3,4\}$

there are 6 of them

And the formula gives

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{24}{2\cdot 2} = 6$$

We get the right answer

Why does this work?

ways to pick where order matters

12
13
14
21
23
24
31
32
34
41
42
43

$\underset{\text{possibilities}}{4}$ $\underset{\text{possibilities}}{3}$ $= \underset{\text{possibilities}}{12}$

$$4 \cdot 3 = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1}$$

$$= \frac{4!}{2!} = \frac{n!}{(n-k)!}$$

Divide out double counting so order doesn't matter.

Divide by $2! = k!$

So we get: $\frac{4!}{2!\,2!} = \binom{4}{2}$

Ex: Suppose a dealer has the following cards:

2♣ Q♦ J♥ A♠

How many ways can the dealer deal you two cards from these four? Order doesn't matter.

Ex: Q♦ A♠ ← same as A♠ Q♦

Answer:

6 ways

2♣	Q♦	Q♦	J♥
2♣	J♥	Q♦	A♠
2♣	A♠	J♥	A♠

Note: $\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!\,2!}$

$$= \frac{\overset{2}{\cancel{4}} \cdot 3 \cdot \cancel{(2!)}}{\cancel{2!}\,\cancel{2!}} = 6$$

<u>Ex:</u> A dealer has a standard 52-card deck. They deal you 5 cards. How many possible hands are there that you can get? Order doesn't matter.

Ex hand: K♠ Q♠ J♠ 10♠ A♠

(called a royal flush)

possible # hands

$$= \binom{52}{5} = \frac{52!}{5!(52-5)!}$$

$$= \frac{52!}{5!\,47!} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot \cancel{(47!)}}{5!\,\cancel{47!}}$$

$$= \frac{\overset{26}{\cancel{52}} \cdot \overset{17}{\cancel{51}} \cdot \overset{10}{\cancel{50}} \cdot 49 \cdot \overset{12}{\cancel{48}}}{\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1} = 26 \cdot 17 \cdot 10 \cdot 49 \cdot 12$$

$$= \boxed{2{,}598{,}960}$$

CA SuperLotto Plus

A ticket consists of

- 5 "lucky" numbers chosen from 1 – 47
- 1 "mega" number chosen from 1 – 27

- No repeat #s amongst the lucky #s. But the mega # can repeat a lucky #.
- Order doesn't matter for the lucky #s. They are always written in sequential order on a ticket.

Example tickets:

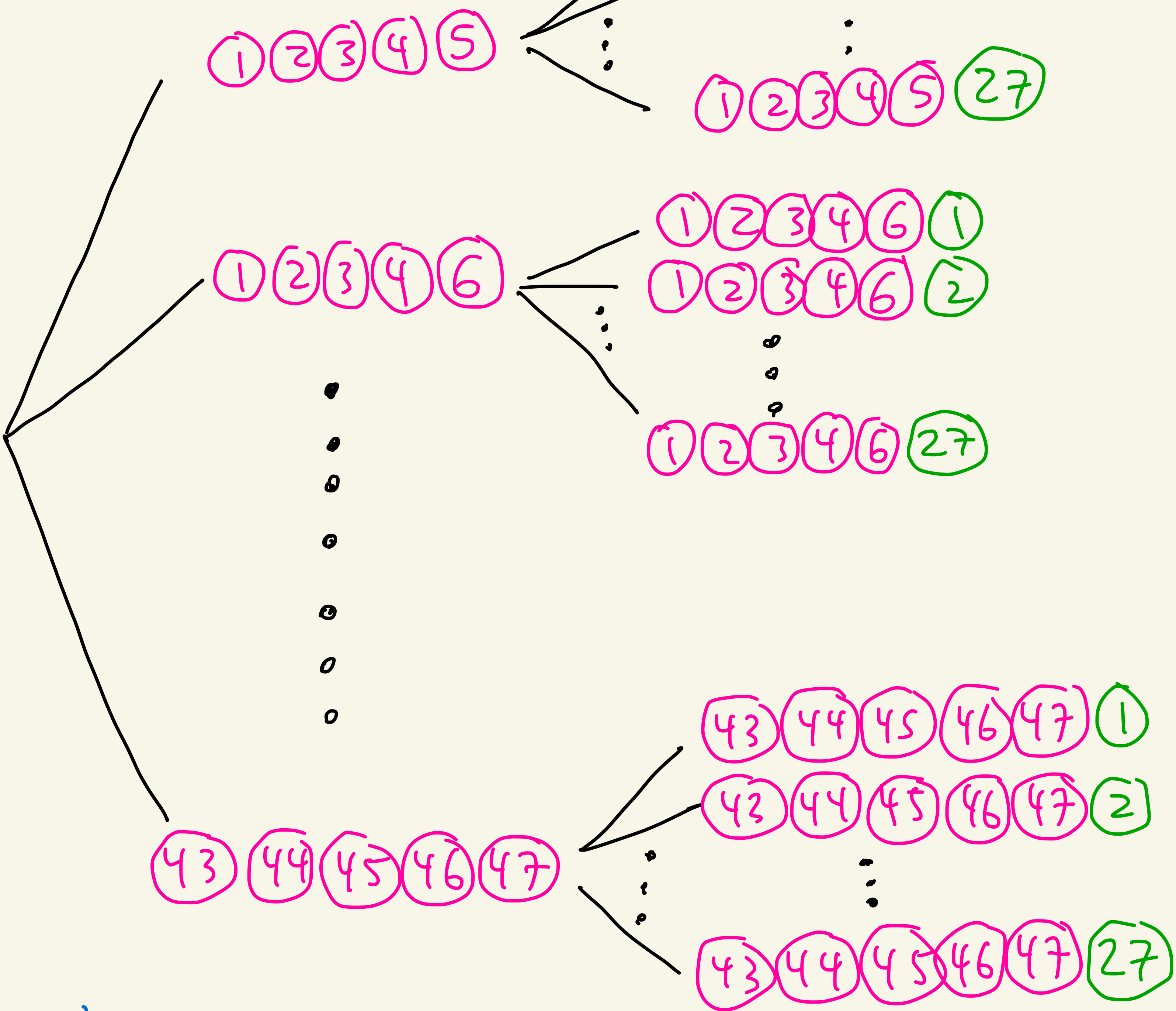
$$\underbrace{1 \quad 7 \quad 15 \quad 21 \quad 47}_{\text{lucky \#s}} \qquad \underbrace{11}_{\text{mega \#}}$$

$$\underbrace{2 \quad 3 \quad 19 \quad 22 \quad 25}_{\text{lucky \#s}} \qquad \underbrace{25}_{\text{mega \#}}$$

How many possible tickets are there?

$$\underbrace{\binom{47}{5}}_{\text{choose 5 of the 47 lucky \#s}} \cdot \overbrace{\binom{27}{1}}^{\text{choose 1 mega \# from the 27}}$$

Tree picture



$\binom{47}{5}$ branches

each has $\binom{27}{1}$ subbranches

The total of tickets is

$$\binom{47}{5}\cdot\binom{27}{1} = \frac{47!}{5!(47-5)!}\cdot 27$$

$$= \frac{47!}{5!\,42!}\cdot 27$$

$$= \frac{47\cdot 46\cdot 45\cdot 44\cdot 43\cdot \cancel{42!}}{5!\,\cancel{42!}}\cdot 27$$

$$= \frac{47\cdot 46\cdot 45\cdot 44\cdot 43\cdot 27}{120}$$

$$= \boxed{41{,}416{,}353 \text{ possible tickets}}$$

$$\binom{n}{1} = \frac{n!}{1!(n-1)!}$$

$$= \frac{n\left[\cancel{(n-1)!}\right]}{\cancel{(n-1)!}}$$

$$= n$$

Ex:

$$5! = 5\,[4!]$$