Math 4740
2/12/25

Q: What's the probability that if you buy 1 ticket you win the jackpot, that is you get all 5 lucky #'s correct and and the mega # correct?

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Let S be the sample space of all possible outcomes that the lottery machines can create

S is set of all possible tickets.

So, $|S| = 41,416,353$.

Let E be the event with just the ticket you bought.

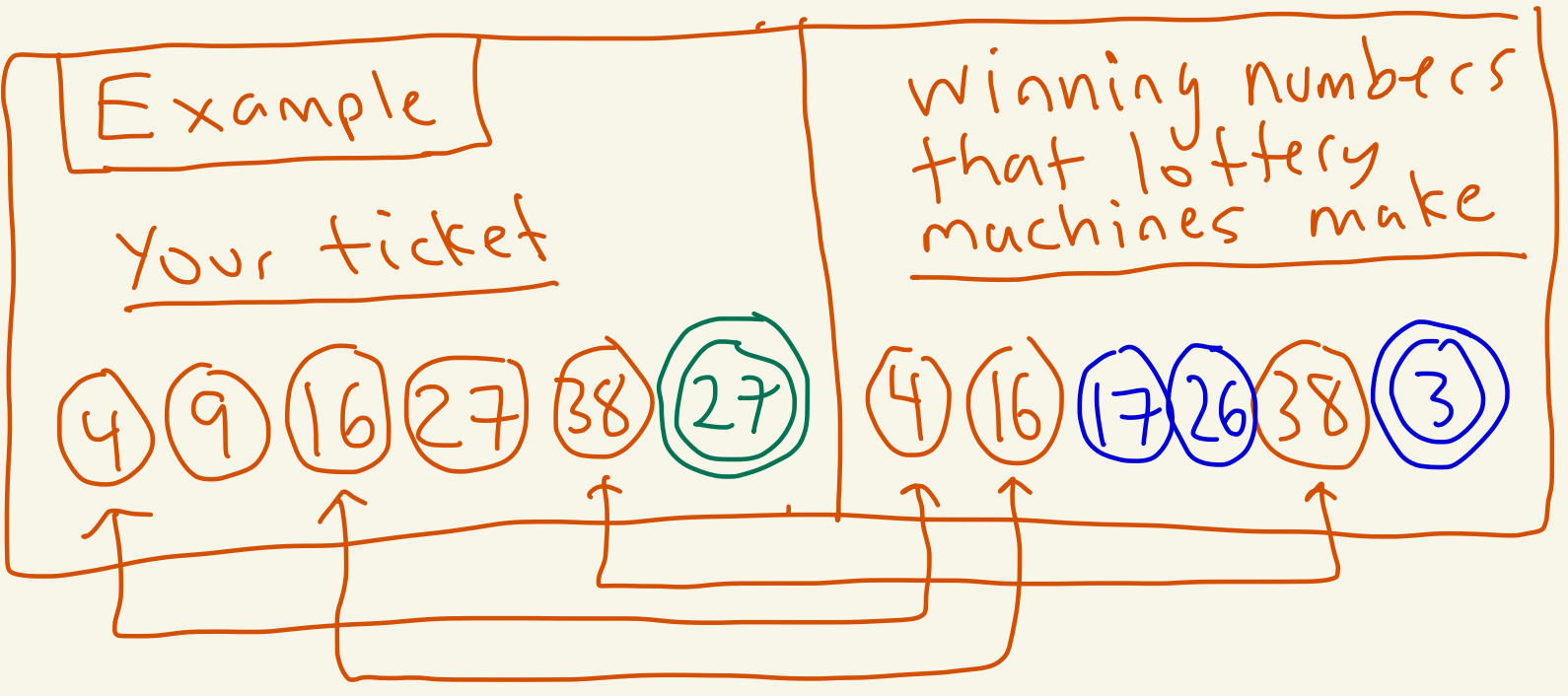
Ex: E = { (1) (5) (11) (22) (44) (7) } ← your ticket

$$P(E) = \frac{|E|}{|S|} = \frac{1}{41,416,353}$$

$\approx 0.000000002414\ldots$

$\approx 0.000002414\ \%$

**Ex:** What is the probability that you get exactly 3 of the 5 lucky numbers correct and not the mega number correct? You bought one ticket



$S$ = set of all possible tickets

$|S| = 41,416,353$

$E$ is the set of all tickets that match exactly 3 of our lucky #s and don't match our mega #.

In our example above

$E = \{$ (4)(16)(17)(26)(38)(3), (4)(9)(16)(18)(41)(1), ... $\}$

$$|E| = \underbrace{\binom{5}{3}}_{\text{3 of our 5 lucky \#s appear}} \cdot \underbrace{\binom{47-5}{2}}_{\text{2 are not our lucky \#s}} \cdot \underbrace{\binom{26}{1}}_{\text{not our mega \#}}$$

$$= \frac{5!}{3!(5-3)!} \frac{42!}{2!(42-2)!} \frac{26!}{1!(26-1)!}$$

$$= \frac{5!}{3!2!} \frac{42!}{2!40!} \frac{26!}{25!}$$

$$= \frac{120}{6 \cdot 2} \frac{42 \cdot 41 \cdot (40!)}{2 \cdot (40!)} \frac{26(25!)}{25!}$$

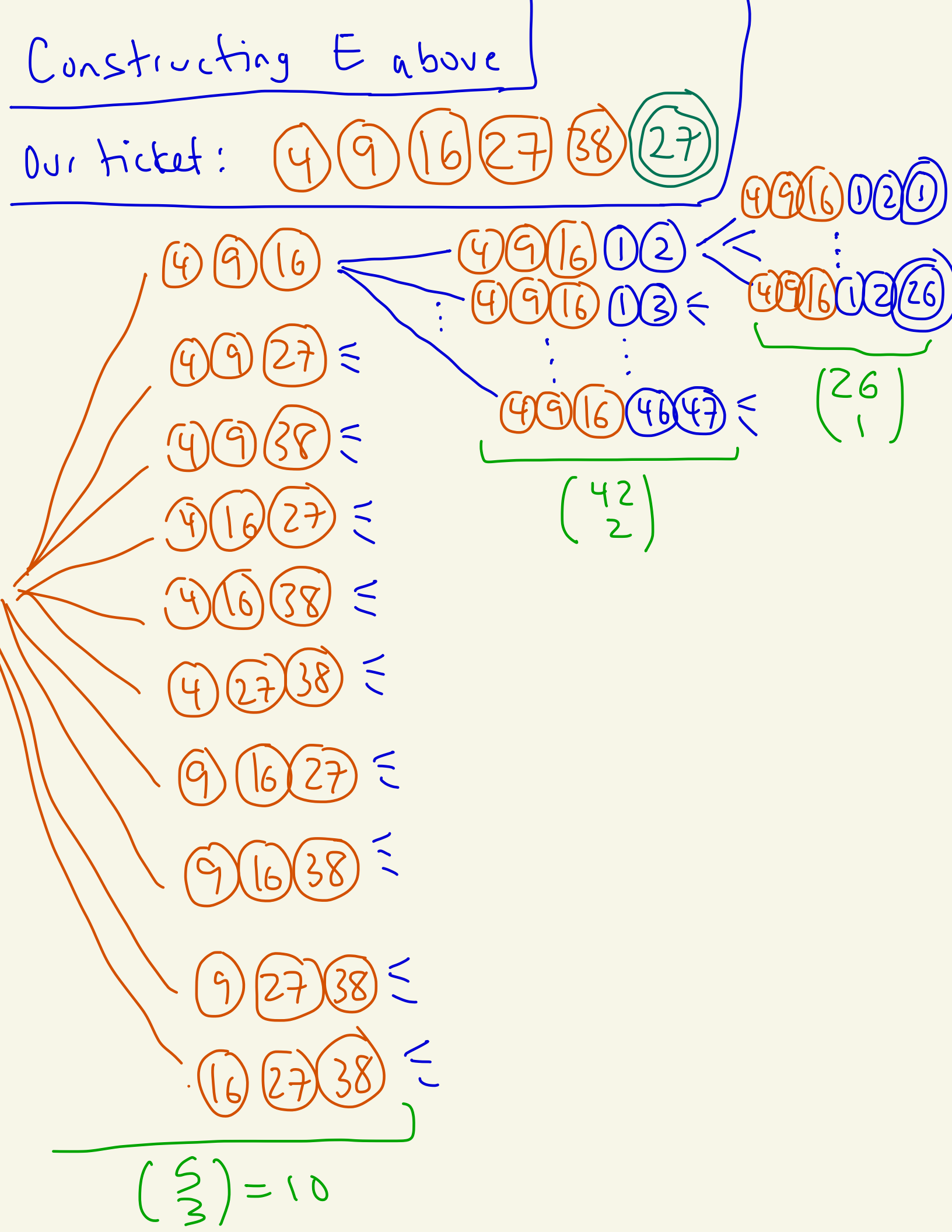
$$= 223,860$$

$$P(E) = \frac{|E|}{|S|} = \frac{223,860}{41,416,353}$$

$$\approx 0.005405111\ldots$$

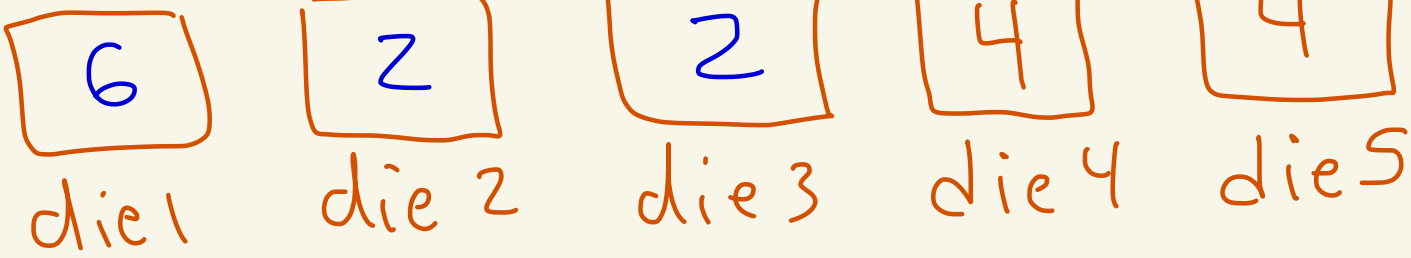
$$\approx 0.540511\%$$

The lottery website says the probability is about $\frac{1}{185} \approx 0.540541\%$


Constructing E above
Our ticket: 4 9 16 27 38 27
4 9 16
4 9 16 1 2
4 9 16 1 3
4 9 16 1 2 1
4 9 16 1 2 26
4 9 16 46 47
(26 1)
(42 2)
4 9 27
4 9 38
4 16 27
4 16 38
4 27 38
9 16 27
9 16 38
9 27 38
16 27 38
(5 3) = 10

Ex: Suppose five 6-sided dice are rolled. What's the probability that exactly two of the dice have 4's showing?

Example



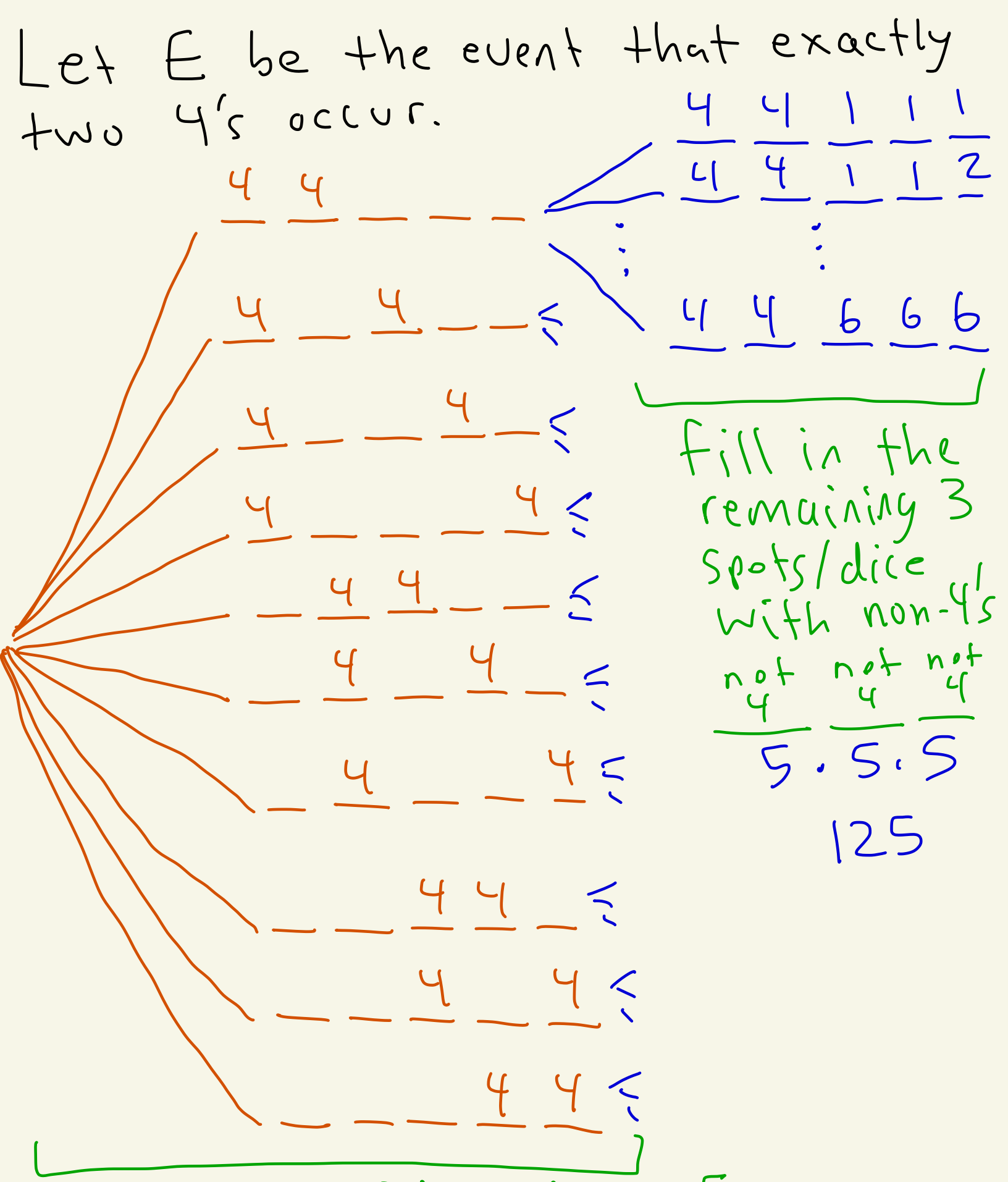

Let S be the sample space of all possible out comes.

$\underline{1\text{-}6}$ $\underline{1\text{-}6}$ $\underline{1\text{-}6}$ $\underline{1\text{-}6}$ $\underline{1\text{-}6}$

die 1 die 2 die 3 die 4 die 5

$$|S| = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5 = \boxed{7,776}$$

Let E be the event that exactly two 4's occur.




fill in the remaining 3 spots/dice with non-4's

not 4 · not 4 · not 4

5 · 5 · 5

125

$\binom{5}{2} = 10$ pick 2 of the 5 spots/dice where the 4's go

So, $|E| = 10 \cdot 125 = 1,250$

$$P(E) = \frac{|E|}{|S|} = \frac{1250}{7776} \approx 0.16075$$

$$\approx 16.075\%$$

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## Another way to count E:

**Step 1:** Pick where the 4's go.

$\binom{5}{2} = 10$ ways

_ _ 4 4 _

pick 2 of the 5 spots/dice

**Step 2:** Fill in the other 3 dice

$5 \cdot 5 \cdot 5 = 125$ ways

1 6 4 4 1

↑ not 4 ↑ not 4 ↑ not 4

Answer: $|E| = 10 \cdot 125 = 1250$