Math 4740
2/19/25

(last day of topic 2)

How can we make a probability function when you do two experiments in a row where the outcome of the first experiment does not influence the outcome of the second experiment?

Ex: Suppose you flip a coin and then roll a 4-sided die. Let's make a probability space for this.

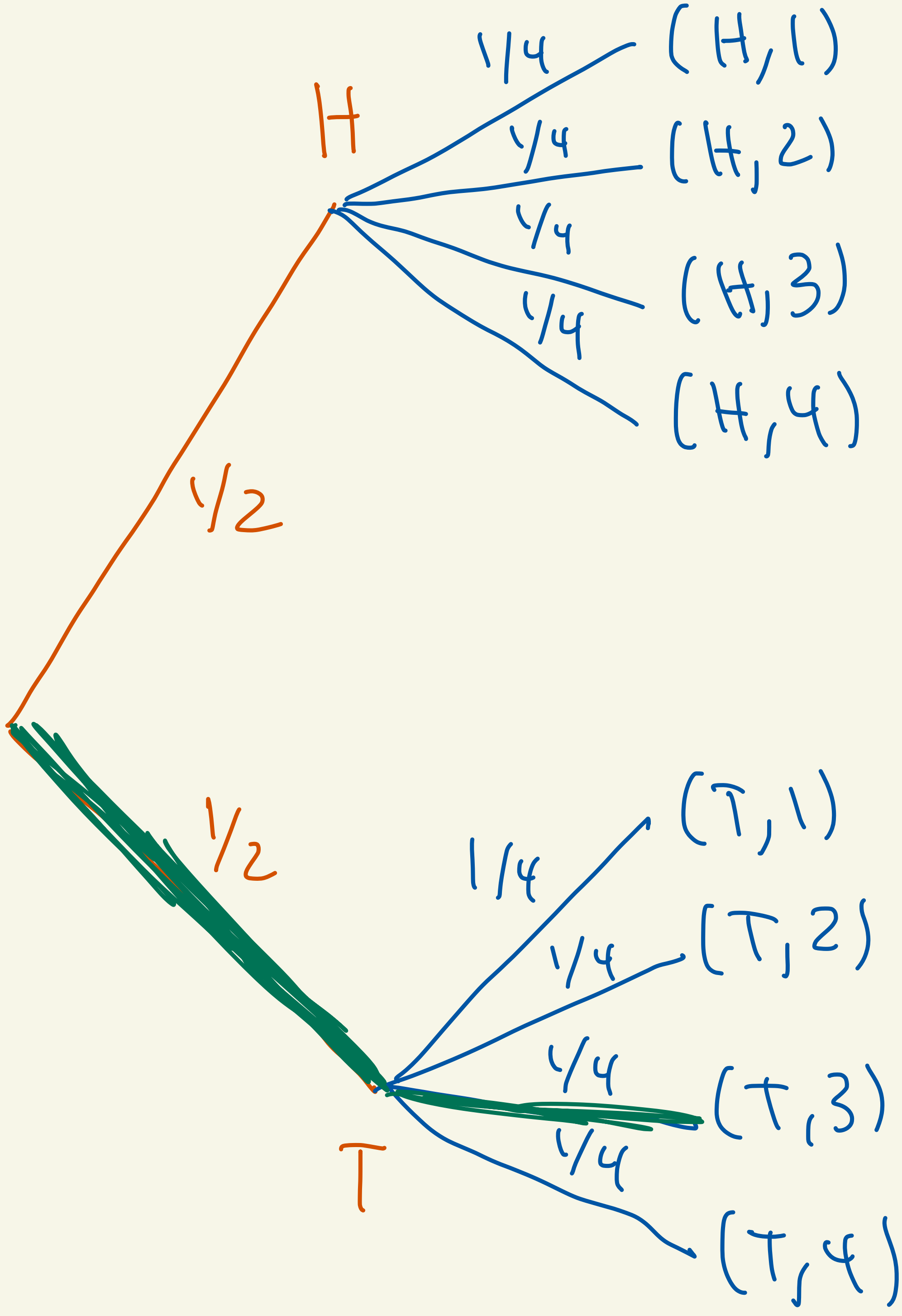
sample space

$$S = \underbrace{\{H, T\}}_{\text{coin sample space}} \times \underbrace{\{1, 2, 3, 4\}}_{\text{die sample space}}$$

$$S = \{(H,1), (H,2), (H,3), (H,4), (T,1), (T,2), (T,3), (T,4)\}$$

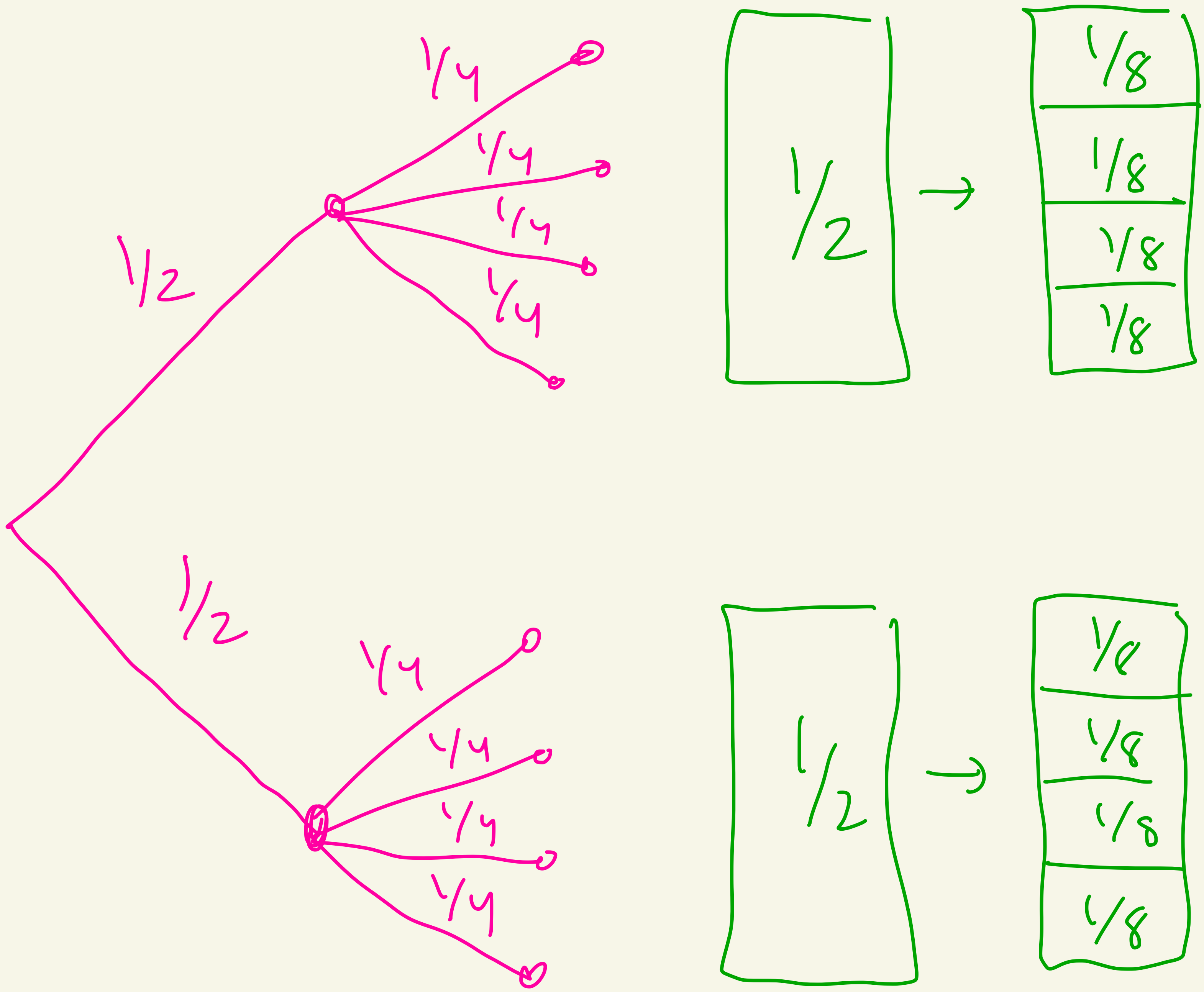
events

Ω is all subsets of S



$$P(\{(T,3)\}) = \underbrace{\frac{1}{2}}_{\text{probability of T in the coin space}} \cdot \overbrace{\frac{1}{4}}^{\text{probability of 3 in die space}} = \frac{1}{8}$$

Why does this make sense?
Why multiply?



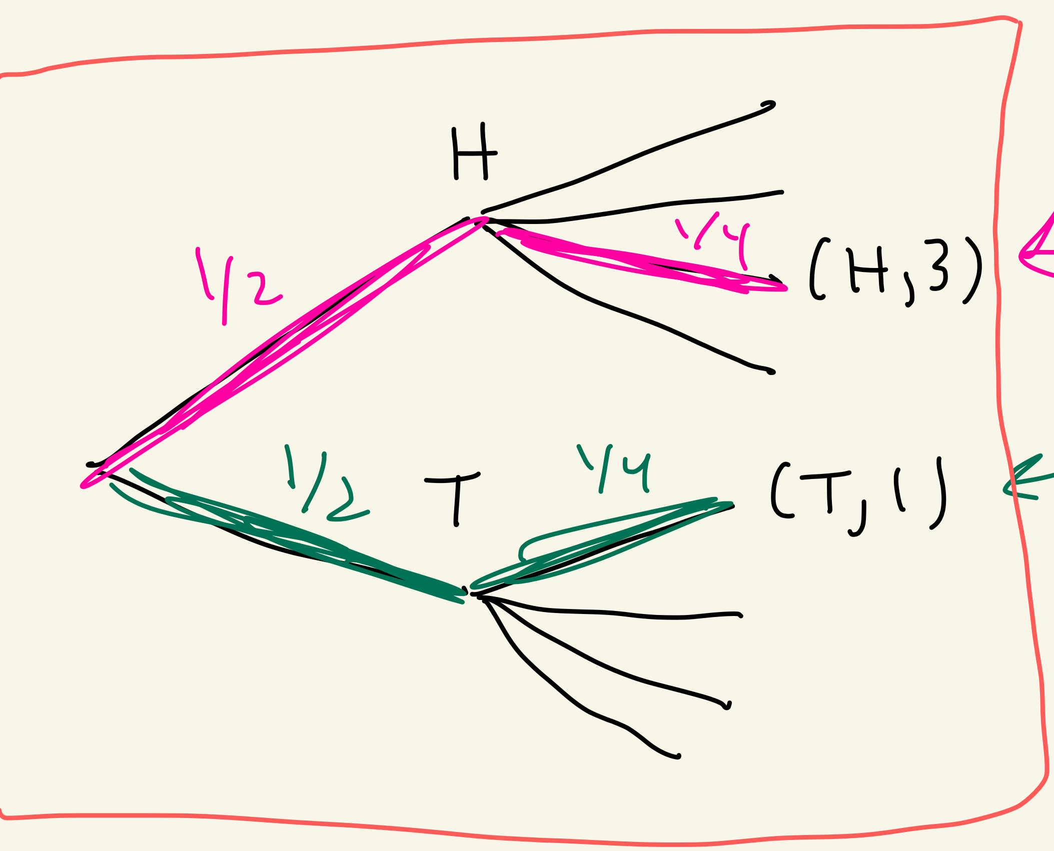
In the above, let

$E = \{(T,1),(H,3)\}$. Then,

$$P(E) = P(\{(T,1)\}) + P(\{(H,3)\})$$

$$= \underbrace{\frac{1}{2}\cdot\frac{1}{4}} + \underbrace{\frac{1}{2}\cdot\frac{1}{4}}$$

$$= \frac{2}{8} = \frac{1}{4}$$



Ex: Suppose you have a 4-sided die but it isn't a normal die. You estimate the following probabilities:

die #	1	2	3	4
probability	$3/8$	$1/8$	$15/32$	$1/32$

Suppose you roll this die and then flip a normal coin. Let's make the probability space.

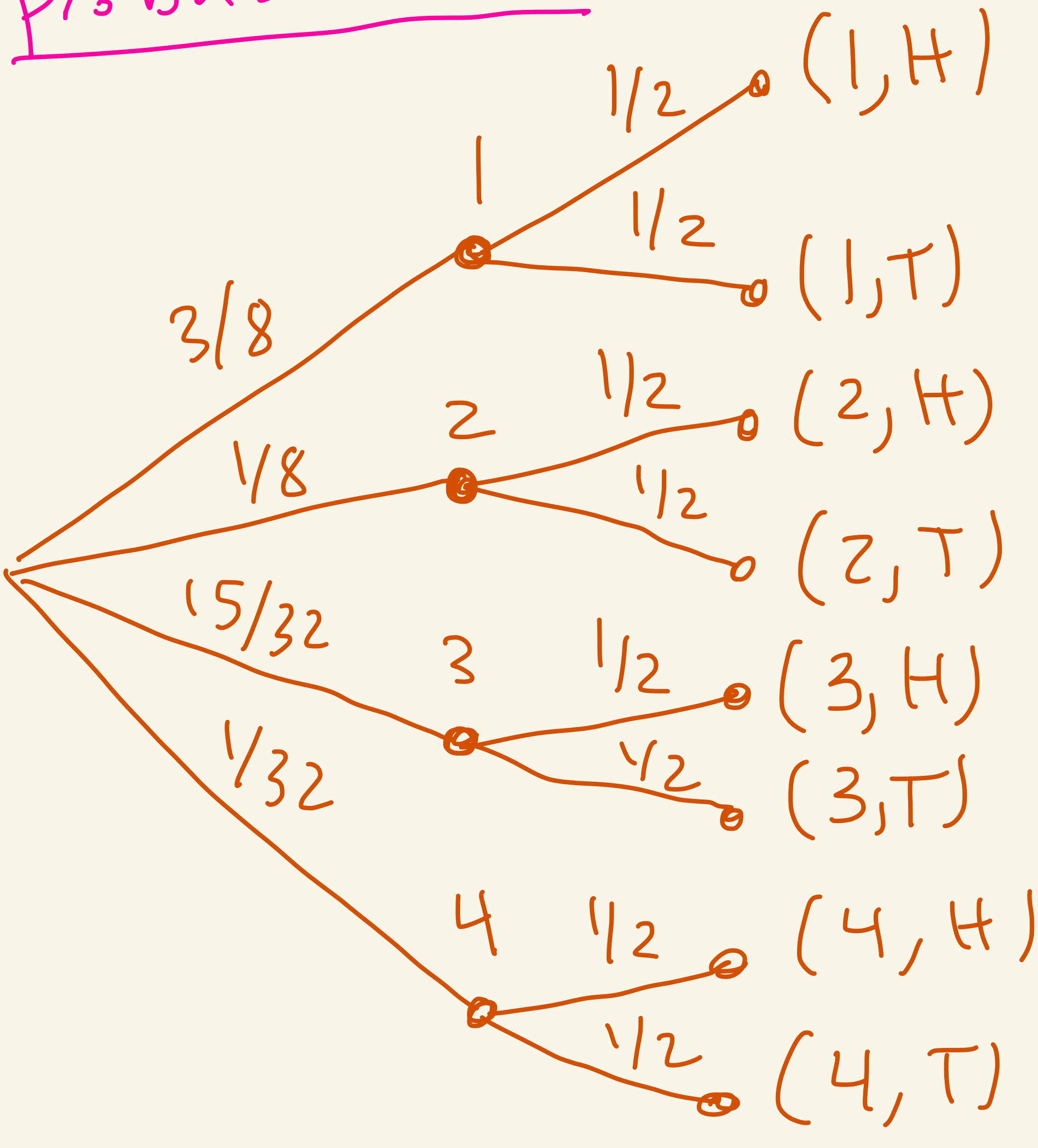
sample space

$S = \{(1,H), (1,T), (2,H), (2,T), (3,H), (3,T), (4,H), (4,T)\}$

events

Ω is set of all subsets of S

probabilities



$$P(\{(1,T)\}) = \frac{3}{8}\cdot\frac{1}{2} = \frac{3}{16} \approx 0{,}1875\ldots$$

$$P(\{(4,H)\}) = \frac{1}{32}\cdot\frac{1}{2} = \frac{1}{64} \approx 0{,}0156\ldots$$

$$P(\{(2,H),(3,T)\})$$
$$= P(\{(2,H)\}) + P(\{(3,T)\})$$
$$= \frac{1}{8}\cdot\frac{1}{2} + \frac{15}{32}\cdot\frac{1}{2} = \frac{19}{64}$$

$\boxed{\text{HW 2}}$ $\boxed{\#9(a)}$

Suppose you toss a coin 20 times. What's the probability that at least two heads occurs?

≥ 2 heads

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sample space

$S = \{$all outcomes of flipping a coin 20 times$\}$

$$|S| = 2^{20}$$
$$= 1{,}048{,}576$$

Let E be the event where at least 2 heads occur.

We want $P(E)$.

Instead calculate $P(\overline{E})$.

$\overline{E}$ is the event where exactly 0 heads or exactly 1 head occur.

We have

$$\overline{E} = \Big\{ \underbrace{(T,T,T,\ldots,T)}_{0 \text{ heads}}, \left.\begin{array}{l} (H,T,T,\ldots,T), \\ (T,H,T,\ldots,T), \\ (T,T,H,\ldots,T), \\ \quad\vdots \\ (T,T,T,\ldots,H) \Big\} \end{array}\right\} 1 \text{ head}$$

$$|\overline{E}| = 1 + 20 = 21$$

$$P(E) = 1 - P(\overline{E}) = 1 - \frac{21}{1{,}048{,}576}$$

$$= \frac{1{,}048{,}555}{1{,}048{,}576}$$

$$\approx 0.9999799 7\ldots$$

$$\approx 99.997997\%$$

HW 2 (11) Suppose 5 numbers are randomly selected from 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ..., 19, 20

What's the probability the smallest number selected is larger than 6.

Examples:

#'s selected	is smallest # greater than 6

$\{4, 18, 13, 7, 10\}$	No
$\{10, 15, 20, 19, 7\}$	YES

elements of S

S is sample space of all possible outcomes (ways to pick 5 #'s from 1-20)

$$|S| = \binom{20}{5} = \frac{20!}{5!\,15!}$$

$$= \frac{20 \cdot 19 \cdot 18 \cdot 17 \cdot 16 \cdot \cancel{15!}}{5!\,\cancel{15!}}$$

$$= \boxed{15{,}504}$$

Let E be the event where all the numbers selected are greater than 6.

$$|E| = \underbrace{\binom{14}{5}}_{\substack{\text{pick 5}\\\text{from}\\ \underbrace{7,8,9,10,11,\ldots,20}_{\text{14 \#'s here}}}} = \frac{14!}{5!\,9!} = 2{,}002$$

Answer

$$P(E) = \frac{|E|}{|S|} = \frac{2{,}002}{15{,}504} \approx 0.129\ldots$$

$$\approx 12.9\%$$