Math 4740
8/26/24

Ex: Suppose you flip a coin three times in a row and record each time if we get H = heads or T = tails.

Let's make a sample space to model this experiment.

1st flip = heads
2nd flip = heads
3rd flip = heads

$$S = \{ (H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,T,T), (T,T,H), (T,H,H), (T,H,T) \}$$

T = 1st flip
2nd flip = H
3rd flip = T

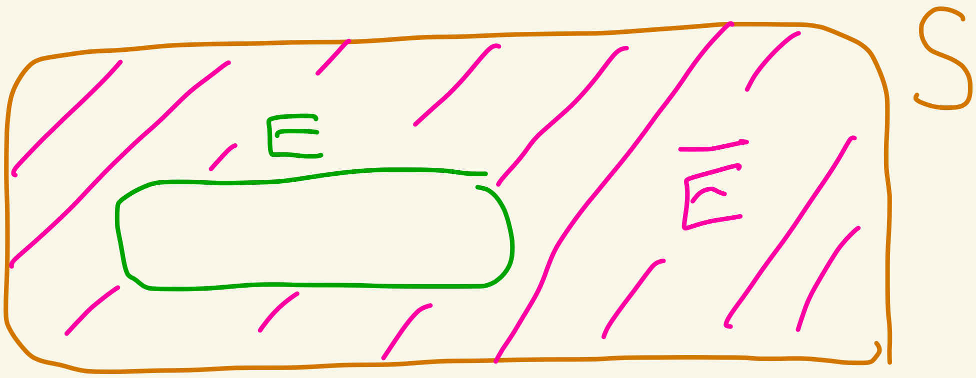
The event that exactly one head occured is

$$E = \{(H,T,T),(T,H,T),(T,T,H)\}$$

<u>Def:</u> Let S be a set and $E \subseteq S$. The <u>complement</u> of E in S is

$$\overline{E} = \{x \mid x \in S \text{ and } x \notin E\}$$

<u>read:</u> $\overline{E}$ consists of all x where x is in S and $x \notin E$

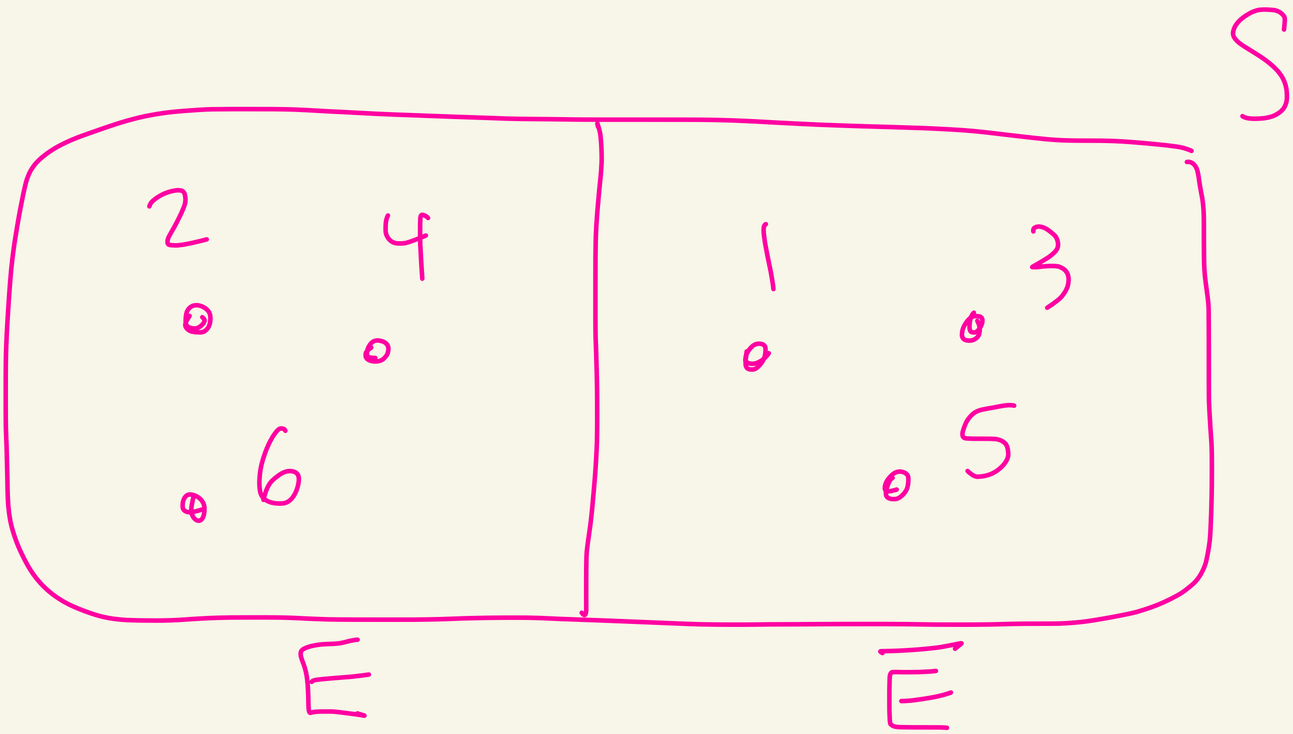


other notations for $\overline{E}$ are
E^c
$S - E$

Ex: $S = \{1, 2, 3, 4, 5, 6\}$

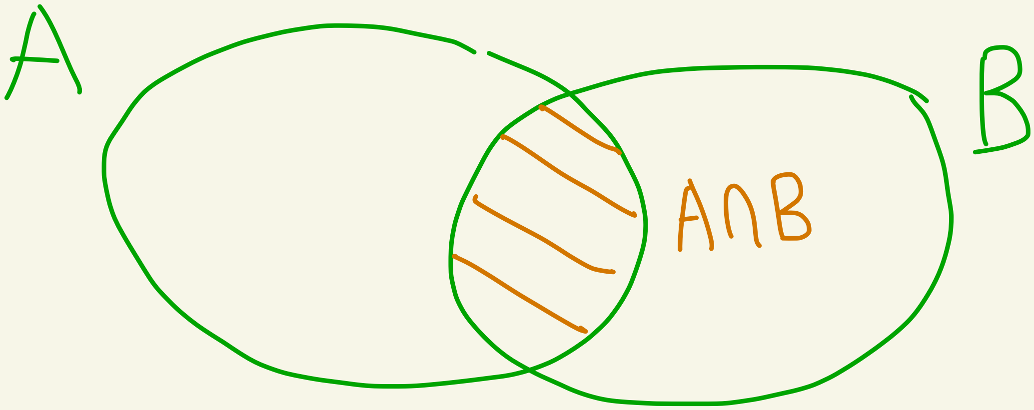
$E = \{2, 4, 6\}$

$\overline{E} = \{1, 3, 5\}$



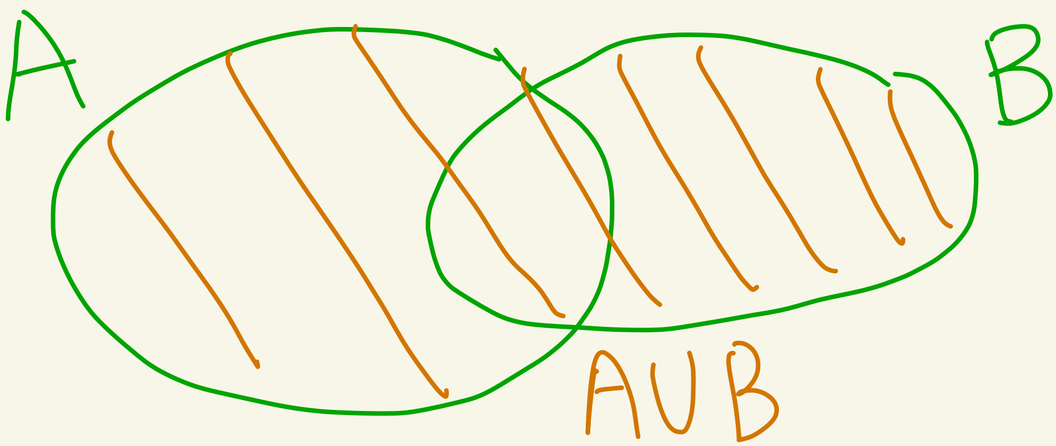
Def: Let A and B be sets. The __intersection__ of A and B is

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$



The union of A and B is

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



The empty set, denoted by $\emptyset$, is the set with no elements.

Ex: Let

$S = \{(H,H,H), (H,H,T), (H,T,H), (H,T,T),$
$(T,H,H), (T,H,T), (T,T,H), (T,T,T)\}$

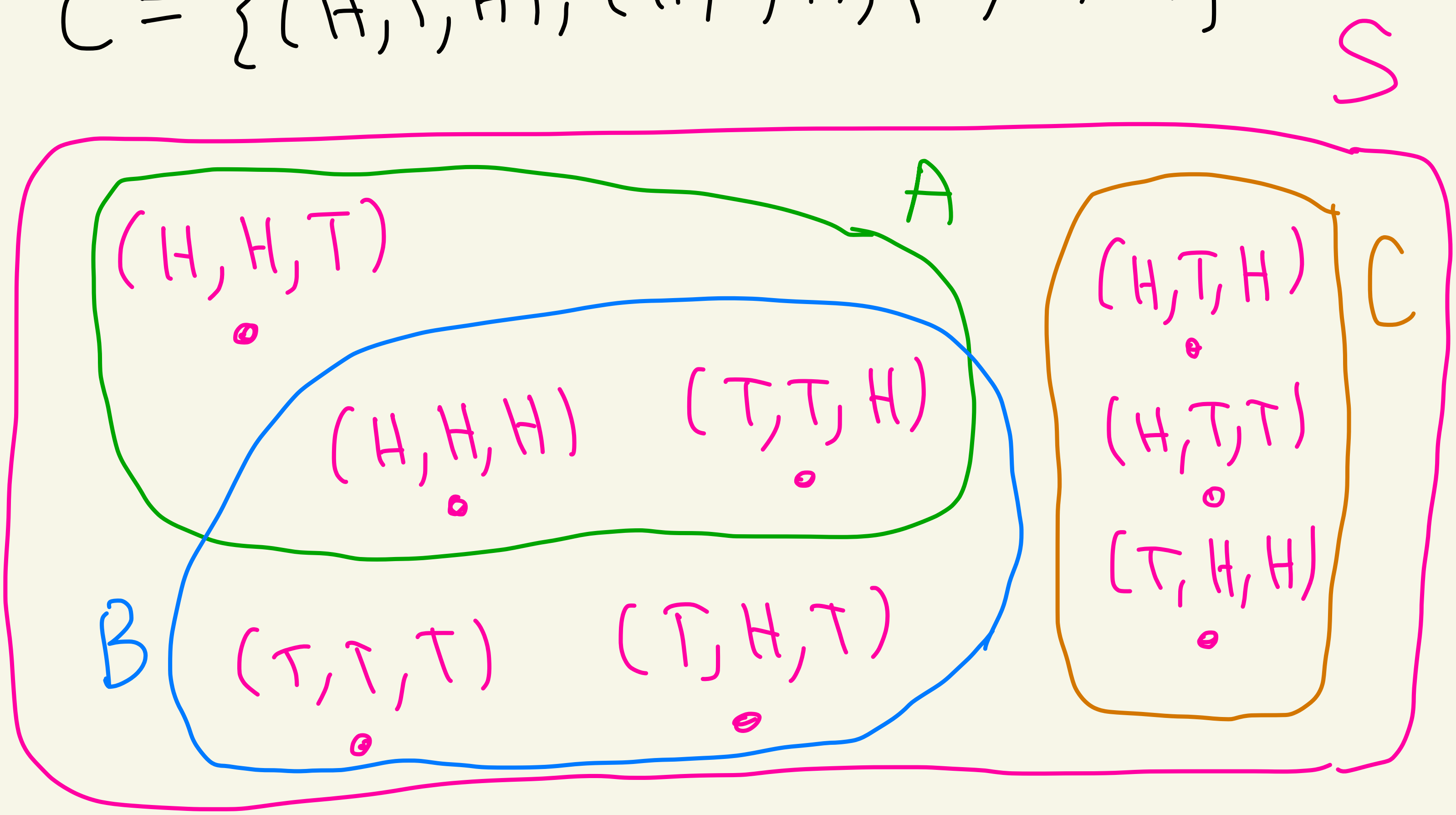
[Flipping a coin three times in a row]

Let

$A = \{(H,H,T), (H,H,H), (T,T,H)\}$

$B = \{(T,T,T), (T,T,H), (H,H,H), (T,H,T)\}$

$C = \{(H,T,H), (H,T,T), (T,H,H)\}$



Then,

$$A \cup B = \{(H,H,T),(H,H,H),(T,T,H),\\ (T,T,T),(T,H,T)\}$$

$$A \cap B = \{(H,H,H),(T,T,H)\}$$

$$A \cap C = \phi$$

$$B \cap C = \phi$$

$$\overline{A} = \{(T,T,T),(T,H,T),(H,T,H),\\ (H,T,T),(T,H,H)\}$$

Def: We say that two sets X and Y are disjoint if $X \cap Y = \phi$.

Ex: $A = \{2, 4, 6\}$
$B = \{1, 3, 5\}$

$$A \cap B = \phi$$

So A and B are disjoint

Def: Let $A_1, A_2, \ldots, A_n$ be sets.

Define the intersection to be

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

$$= \{x \mid \underbrace{x \in A_1 \text{ and } x \in A_2 \text{ and} \cdots \text{ and } x \in A_n}_{x \in A_i \text{ for all } i}\}$$

ie the x's that are in all the A_i

Define the union to be

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n$$

$$= \{x \mid x \in A_1 \text{ or } x \in A_2 \text{ or} \cdots \text{ or } x \in A_n\}$$

$$= \{x \mid x \text{ is in at least one of the } A_i\}$$

Ex: Let

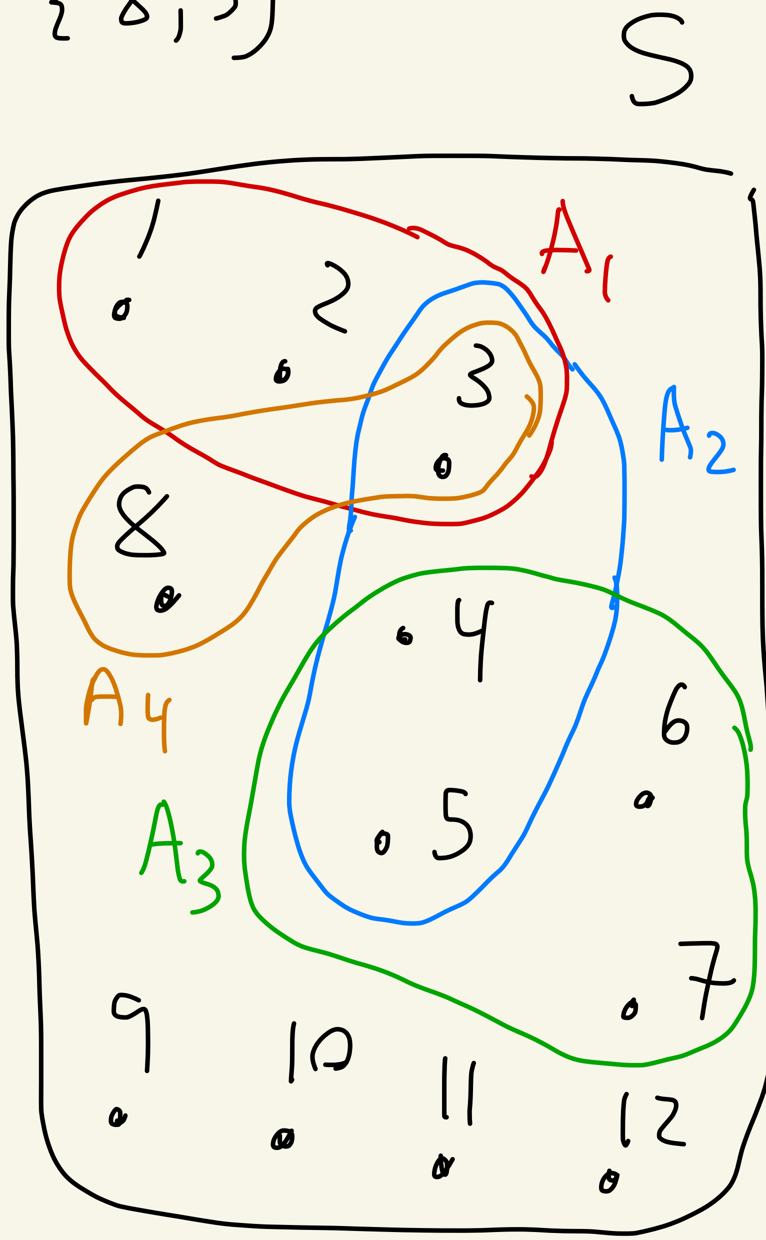
$S = \{1,2,3,4,5,6,7,8,9,10,11,12\}$

(this could represent rolling a 12-sided die)
dodecahedron

Let

$A_1 = \{1,2,3\}$ $\quad A_3 = \{5,6,7,4\}$

$A_2 = \{3,4,5\}$ $\quad A_4 = \{8,3\}$



Then

$$\bigcup_{i=1}^{4} A_i = A_1 \cup A_2 \cup A_3 \cup A_4$$

$$= \{1,2,3,4,5,6,7,8\}$$

$$A_1 \cup A_2 \cup A_4 = \{1,2,3,4,5,8\}$$

$$A_1 \cap A_2 \cap A_4 = \{3\}$$

$$A_2 \cap A_3 \cap A_4 = \phi$$

$A_1 \cap A_2 \cap A_3 \cap A_4 = \phi$

Def: Let A and B be two sets. The Cartesian product of A and B is

$$\underbrace{A \times B}_{} = \{(a,b) \mid a \in A \text{ and } b \in B\}$$

read: "A cross B"

Ex: Let $A = \{H, T\}$
$B = \{1, 2, 3, 4\}$

Then

$$A \times B = \{(H,1),(H,2),(H,3),(H,4),(T,1),(T,2),(T,3),(T,4)\}$$

$$A \times A = \{(H,H),(H,T),(T,H),(T,T)\}$$

$$B \times A = \{(1,H),(2,H),(3,H),(4,H),(1,T),(2,T),(3,T),(4,T)\}$$

$$B \times B = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$$