4740

9/16/24

CA SuperLotto Plus

A ticket consists of

- 5 "lucky" numbers chosen from 1–47
- 1 "mega" number chosen from 1–27

- No repeat #s amongst the lucky #s. But the mega # can repeat a lucky #.
- Order doesn't matter for the lucky #s. They are always written in sequential order on a ticket.

Example tickets:

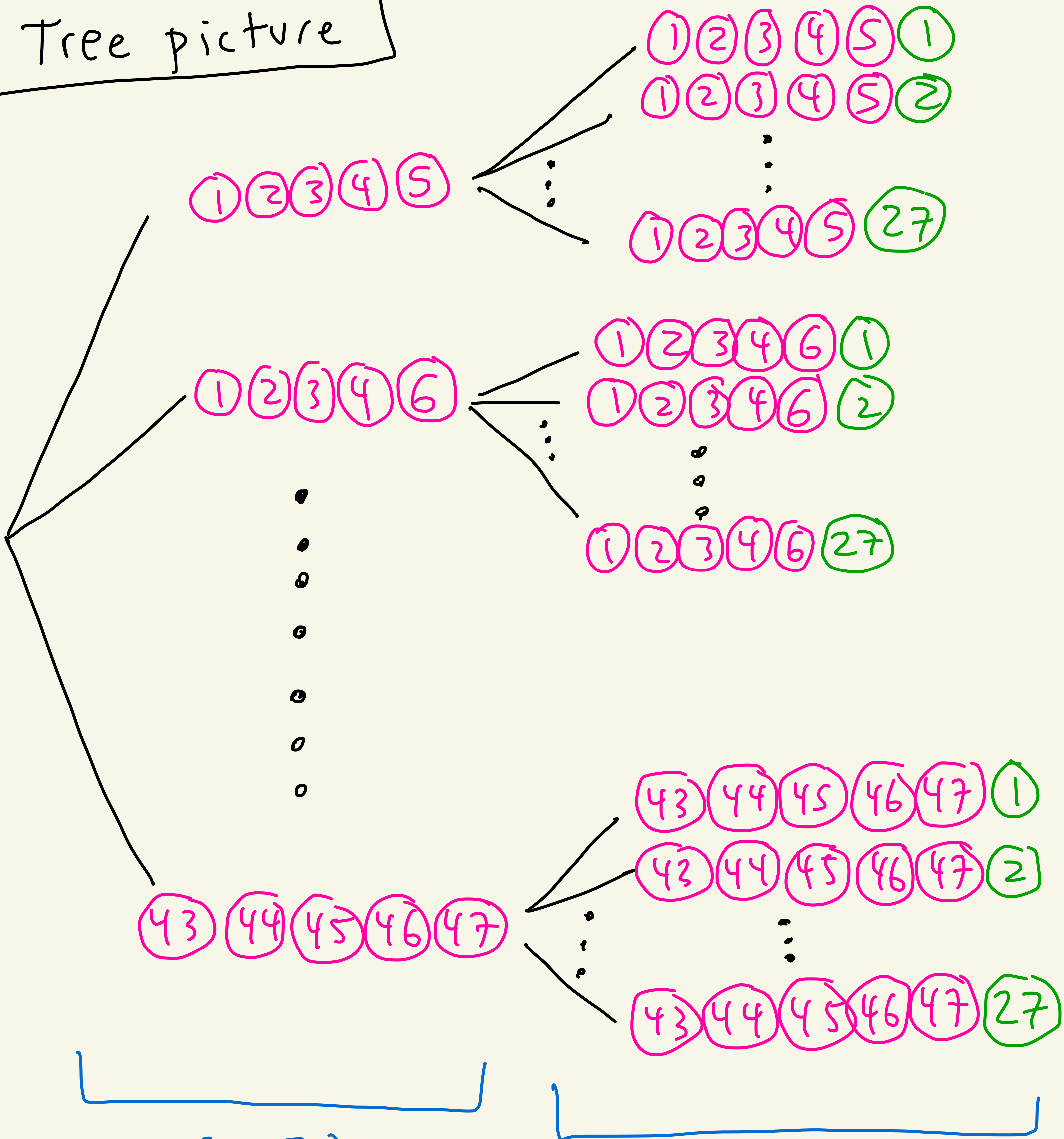
$$\underbrace{1\quad 7\quad 15\quad 21\quad 47}_{\text{lucky \#s}}\qquad \underbrace{11}_{\text{mega \#}}$$

$$\underbrace{2\quad 3\quad 19\quad 22\quad 25}_{\text{lucky \#s}}\qquad \underbrace{25}_{\text{mega \#}}$$

How many possible tickets are there?

$$\underbrace{\binom{47}{5}}_{\text{choose 5 of the 47 lucky \#s}} \cdot \overbrace{\binom{27}{1}}^{\text{choose 1 mega \# from the 27}}$$

Tree picture



$\binom{47}{5}$ branches

each has $\binom{27}{1}$ subbranches

The total of tickets is

$$\binom{47}{5}\cdot\binom{27}{1} = \frac{47!}{5!(47-5)!}\cdot 27$$

$$\binom{n}{1} = \frac{n!}{1!(n-1)!}$$

$$= \frac{n\left[\cancel{(n-1)!}\right]}{\cancel{(n-1)!}}$$

$$= n$$

Ex:

$$5! = 5\left[4!\right]$$

$$= \frac{47!}{5!\,42!}\cdot 27$$

$$= \frac{47\cdot 46\cdot 45\cdot 44\cdot 43\cdot \cancel{42!}}{5!\,\cancel{42!}}\cdot 27$$

$$= \frac{47\cdot 46\cdot 45\cdot 44\cdot 43\cdot 27}{120}$$

$$= \boxed{41{,}416{,}353 \text{ possible tickets}}$$

Q: What's the probability that if you buy 1 ticket you win the jackpot, ie you get all 5 lucky #s correct and the mega # correct?

Let S be the sample space of all possible outcomes that the magical lottery machines can create. So, $|S| = 41{,}416{,}353$.

Let E be the event that all the numbers match your ticket.

Then $|E| = 1$.

$$\text{So, } P(E) = \frac{|E|}{|S|} = \frac{1}{41{,}416{,}353}$$

$\approx 0.000002414\ \%$

Q: What is the probability that you get exactly 3 of the 5 lucky numbers correct and not the mega # correct? (You bought 1 ticket.)

Your ticket: 3 12 15 41 42 17

Let E be the event that the lottery machines output exactly 3 of your lucky #s and not your mega #.

$$|E| = \underbrace{\binom{5}{3}}_{\text{choose 3 of our lucky \#s}} \cdot \overbrace{\binom{47-5}{2}}^{\text{choose 2 that aren't our lucky \#s}} \cdot \underbrace{\binom{26}{1}}_{\text{choose mega not ours}}$$

Ex's:

	match ours	don't match	mega
Ex 1	3 12 15	1 7	5
Ex 2	12 15 42	18 32	21

You will get:

$$|E| = \frac{5!}{3!(5-3)!} \cdot \frac{42!}{2!(42-2)!} \cdot 26$$

$$= 223{,}860$$

So,

$$P(E) = \frac{|E|}{|S|} = \frac{223{,}860}{41{,}416{,}353}$$

$$\approx 0.540511\,\%$$

The lottery website says the probability is 1 in 185

or $\dfrac{1}{185} \approx 0.00540541\ldots$

$\approx 0.540541\%$

Ex: Suppose five 6-sided die are rolled. What's the probability that exactly two of the die have 6's showing?

Ex:

$\boxed{2}$	$\boxed{6}$	$\boxed{6}$	$\boxed{4}$	$\boxed{2}$
die 1	die 2	die 3	die 4	die 5

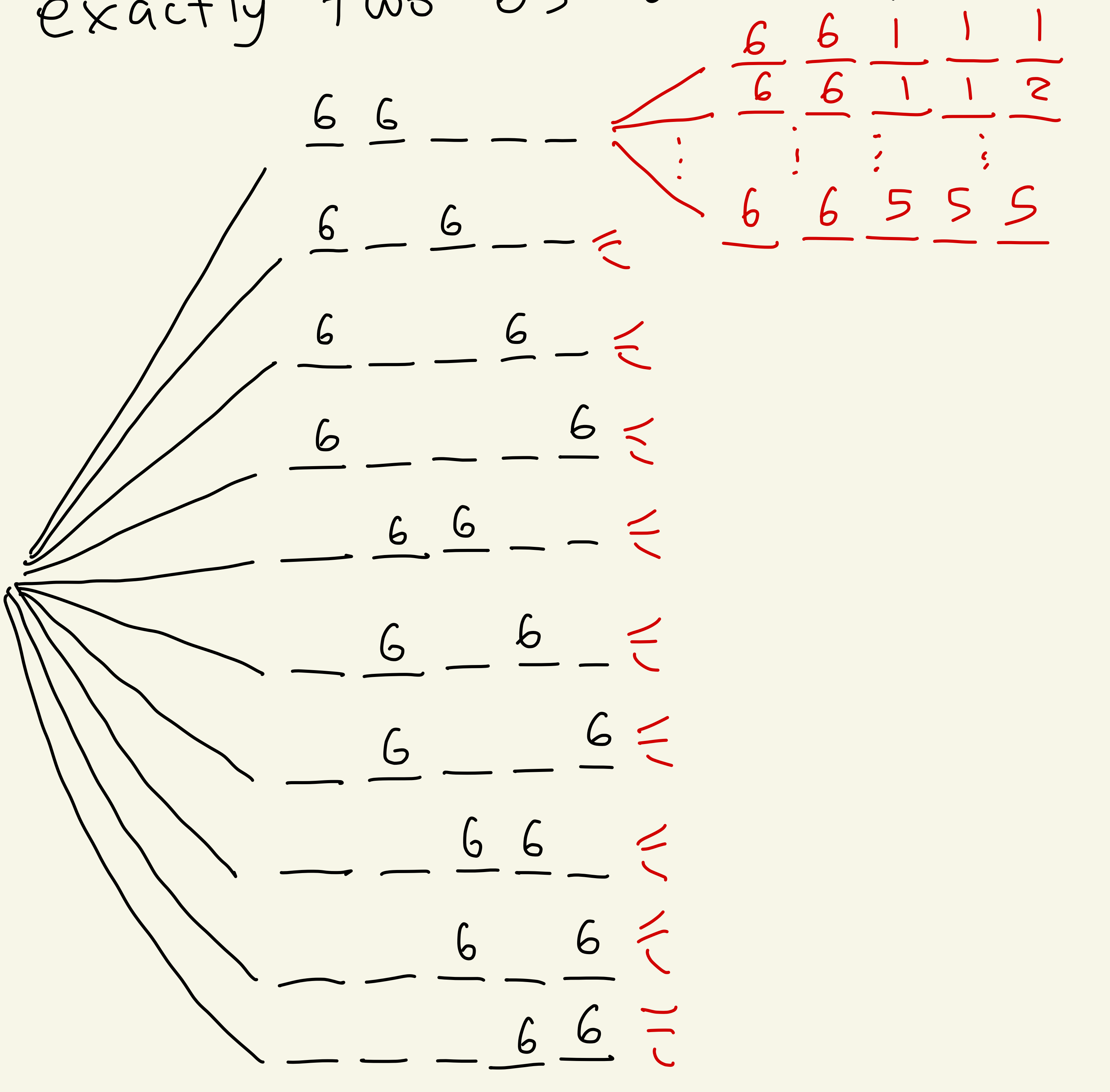
Sample space size:

$$\underset{\text{die } 1}{\underline{1-6}} \quad \underset{\text{die } 2}{\underline{1-6}} \quad \underset{\text{die } 3}{\underline{1-6}} \quad \underset{\text{die } 4}{\underline{1-6}} \quad \underset{\text{die } 5}{\underline{1-6}}$$

$$|S| = 6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5$$

= 7,776 possible outcomes

Let E be the event that exactly two 6's occured.



$\binom{5}{2}$ ways to get which die have the 6's

$5 \cdot 5 \cdot 5 = 125$ ways to fill in remaining die

$$|E| = \binom{5}{2} \cdot 5^3 = 10 \cdot 125 = 1250$$

$$p(E) = \frac{|E|}{|S|} = \frac{1250}{7776} \approx 16.075\%$$

Or way:

Step 1: Pick where the 2 6's go:

___ 6 6 ___ ___

$\binom{5}{2}$ ways

Step 2: Fill in the remaining #s

1 6 6 2 5

$5 \cdot 5 \cdot 5$ ways

<u>Combined:</u> $\binom{5}{2} \cdot 5 \cdot 5 \cdot 5$