Math 4740

9/23/24

Last day of topic 2

How do we make a probability function when you do two experiments in a row where the outcome of the first experiment doesn't influence the outcome of the second experiment?

Ex: Suppose you flip a coin and then roll a 4-sided die. Let's make a probability space for this.

Sample space

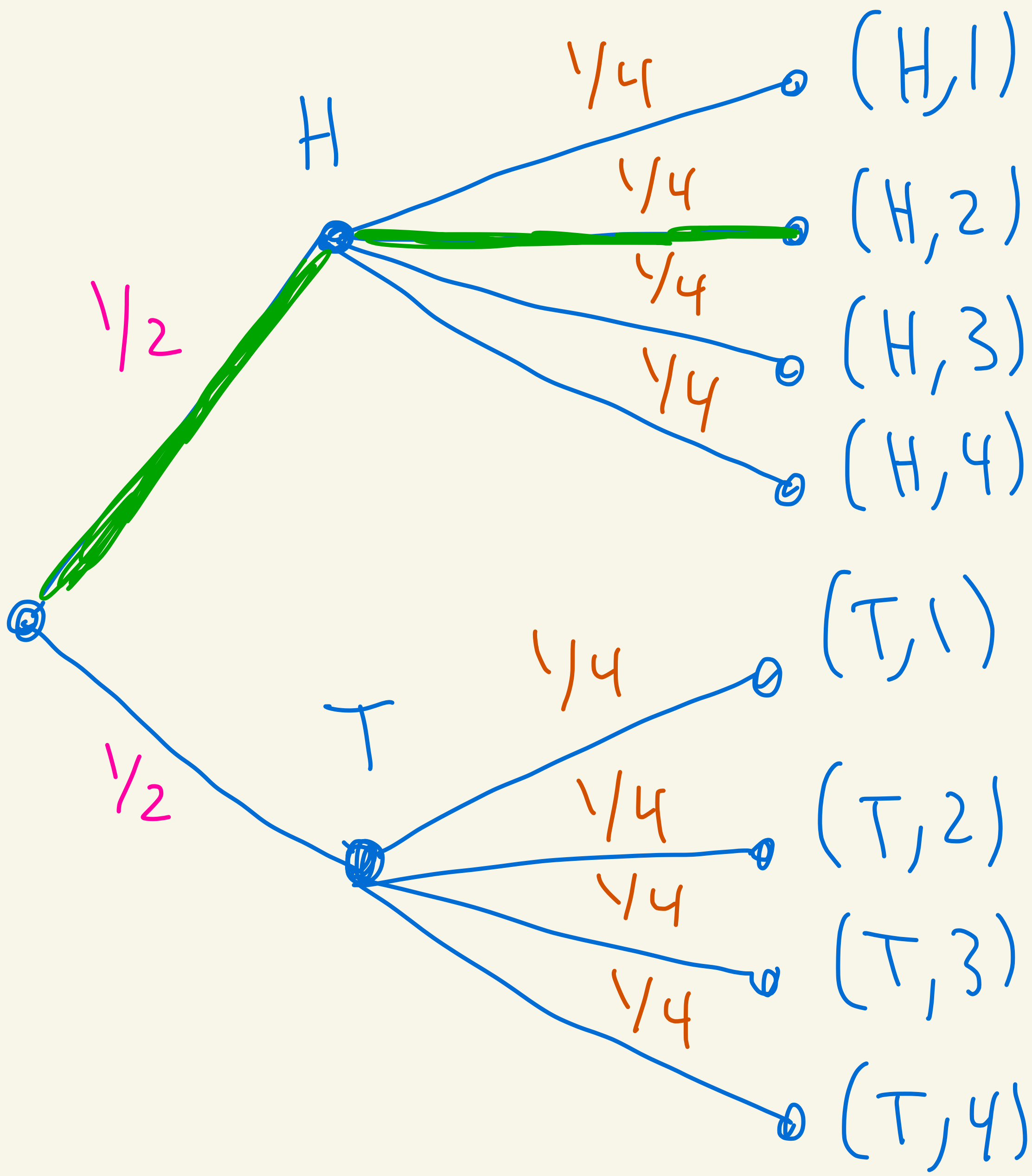
$$S = \{H, T\} \times \{1, 2, 3, 4\}$$

$$= \{(H,1), (H,2), (H,3), (H,4), (T,1), (T,2), (T,3), (T,4)\}$$

$(T,2) \leftarrow$ means tails on coin flip
2 on die roll

events: Ω is set of all subsets of S

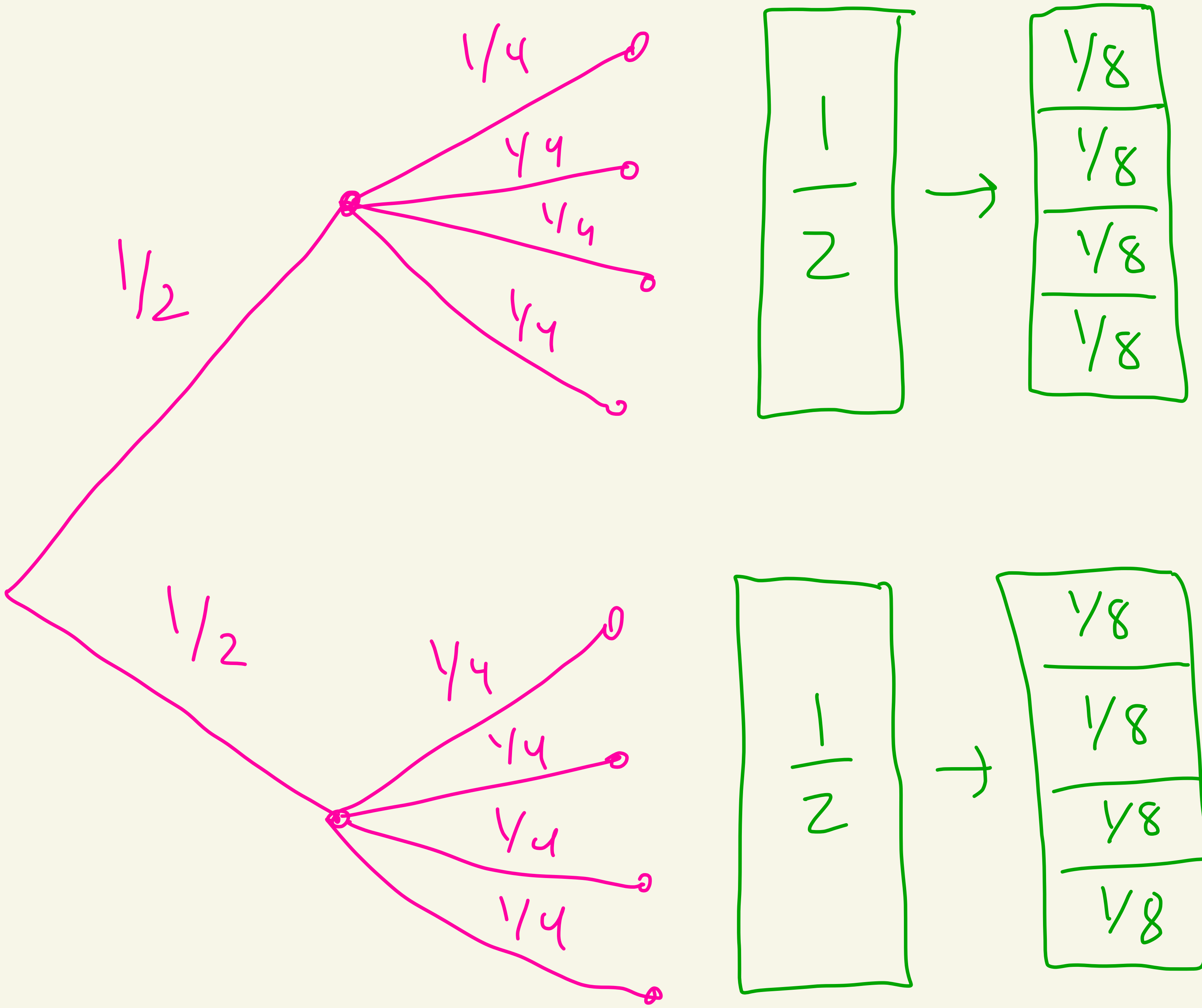
Let's make the probability function using a tree diagram.



Then

$$\left.p((H,2)) = \underbrace{\frac{1}{2}}_{\text{prob } H} \cdot \underbrace{\frac{1}{4}}_{\text{prob } 2} = \frac{1}{8}\ \right\}\ \text{follow the path}$$

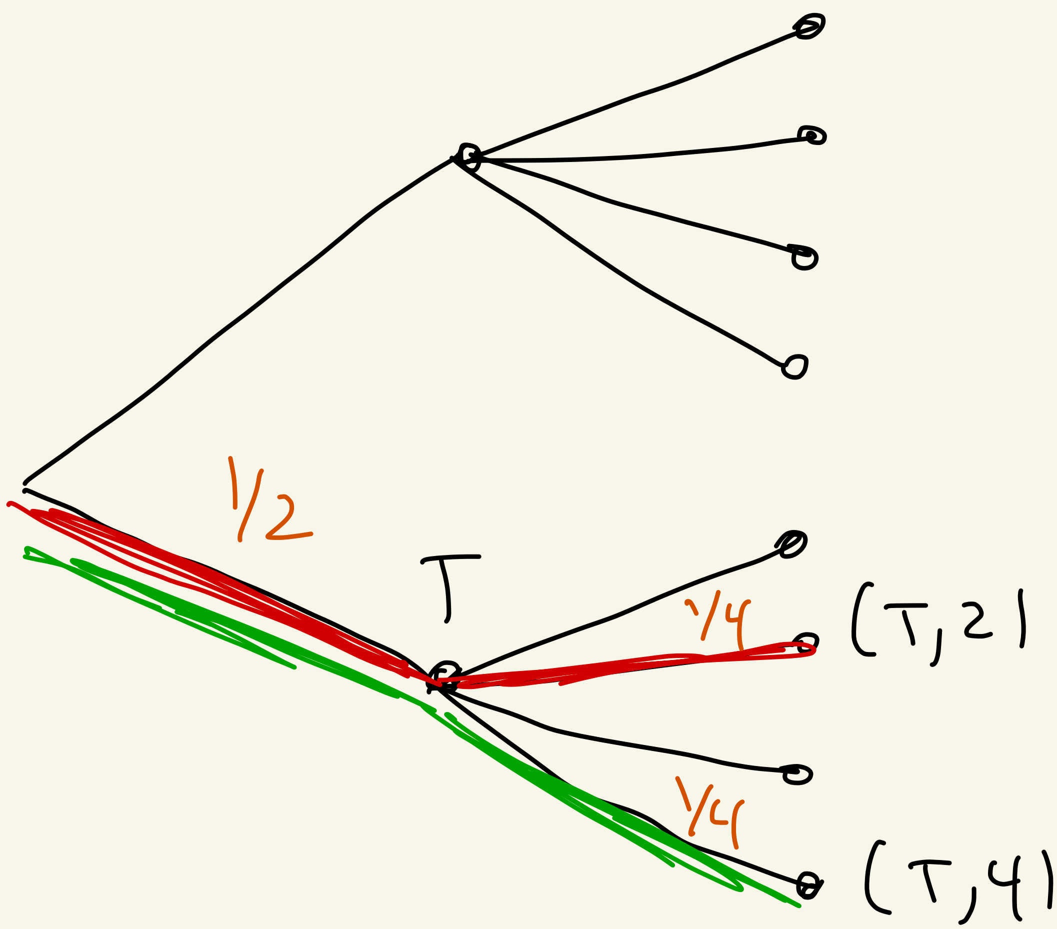
Why does this work?
Why multiply?



There is a rigorous way to abstract this. See notes online if interested.

Q: In the above, what is the probability of $E = \{(T,2),(T,4)\}$

$$P(E) = P((T,2)) + P((T,4)) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$



Ex: Suppose you have 4-sided weighted die labeled 1, 2, 3, 4. From rolling the die lots of times you estimate the following probabilities:

# on die	1	2	3	4
probability	1/8	1/4	1/2	1/8

Let's model rolling this die and then flipping a normal coin.

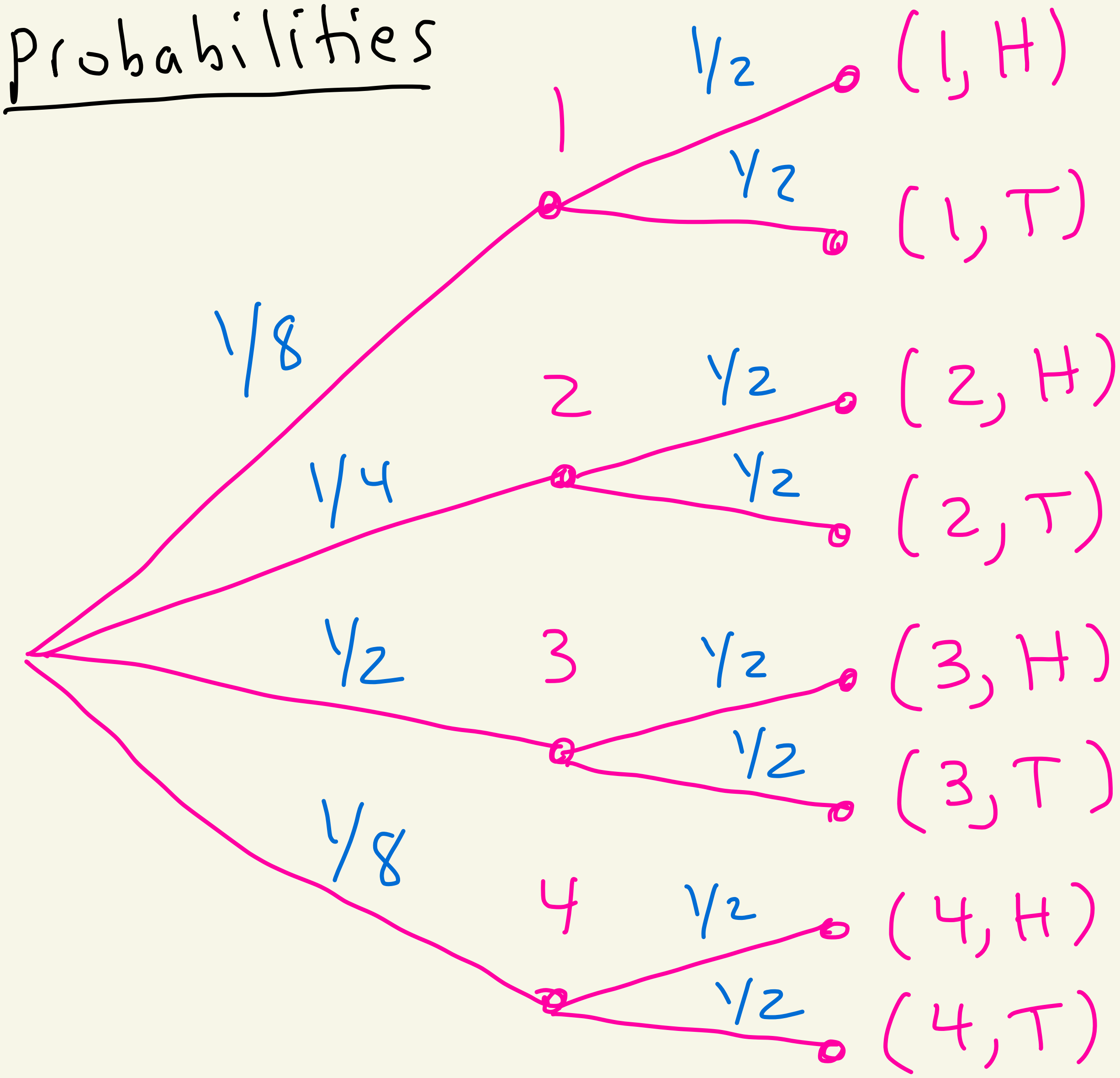
Sample space:

$$S = \{(1,H), (2,H), (3,H), (4,H), (1,T), (2,T), (3,T), (4,T)\}$$

events:

Ω is set of all subsets

Probabilities



$$P(1,H) = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$

$$P(1,T) = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$

$$P(2,H) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(2,T) = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(3,H) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(3,T) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$P(4,H) = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$

$$P(4,T) = \frac{1}{8} \cdot \frac{1}{2} = \frac{1}{16}$$

Let $E = \{(3,H), (2,T), (2,H)\}$

$$P(E) = P(3,H) + P(2,T) + P(2,H)$$

$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

HW 2

(14) Suppose five numbers are selected from 1–20. Each # is equally likely to be selected. No repeated #s picked, order doesn't matter.

What's the probability that the smallest # picked is larger than 6? Ie all the #s you pick are larger than 6.

Ex:

#'s picked	Are all of them larger than 6?
{11, 5, 10, 20, 1}	NO
{7, 11, 12, 20, 15}	Yes

Size of sample space is

$$\underbrace{\binom{20}{5}}_{\text{pick 5 \#'s out of 20}} = \frac{20!}{5!\,15!} = \frac{20\cdot 19\cdot 18\cdot 17\cdot 16\cdot \cancel{15!}}{5\cdot 4\cdot 3\cdot 2\cdot 1\cdot \cancel{15!}}$$

$$= \frac{20\cdot 19\cdot 18\cdot 17\cdot 16}{5\cdot 4\cdot 3\cdot 2\cdot 1}$$

$$= \boxed{15{,}504}$$

\# ways to pick five numbers from 1-20

Let E be the event where all the numbers selected are greater than 6.

$$|E| = \binom{14}{5} = \frac{14!}{5!\,9!} = 2{,}002.$$

picking five from 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Then,

$$P(E) = \frac{|E|}{|S|} = \frac{2{,}002}{15{,}504}$$

$$\approx 0.129 \approx \boxed{12.9\%}$$

$\boxed{\text{HW 2}}$

$\boxed{12(a)}$ Suppose a coin is tossed 20 times. What is the probability that at least 2 heads occur?

$$P\Big(\underbrace{\text{at least 2 heads}}_{E}\Big) = 1 - P\Big(\underbrace{\text{exactly 0 or 1 head}}_{\overline{E}}\Big)$$

$$|S| = 2^{20}$$

We want $|\overline{E}|$.

$$\overline{E} = \Big\{ \underbrace{(T,T,T,\ldots,T)}_{\text{0 heads}},\ \left.\begin{matrix}(H,T,T,\ldots,T),\\ (T,H,T,\ldots,T),\\ (T,T,H,\ldots,T),\end{matrix}\right\} \text{1 hea}$$

$(T, T, T, \dots, H)\}$

So, $|\bar{E}| = 1 + 20 = 21$

Thus,

$$
\begin{aligned}
P(E) &= 1 - P(\bar{E}) \\
&= 1 - \frac{21}{2^{20}} \\
&= 1 - \frac{21}{1{,}048{,}576} \\
&\approx 0.99997997 \\
&\approx 99.998\%
\end{aligned}
$$