Math 4740
Homework 7
Solutions

1 (a)

Let X be the number of 4's that occur in $n = 100$ rolls.
On one roll of the die the probability of rolling a 4 is $p = \frac{1}{6}$.

Thus,

$$P(0 \le X \le 15)$$

$$= P\left(\frac{0 - 100(\frac{1}{6})}{\sqrt{100(\frac{1}{6})(\frac{5}{6})}} \le \frac{X - 100(\frac{1}{6})}{\sqrt{100(\frac{1}{6})(\frac{5}{6})}} \le \frac{15 - 100(\frac{1}{6})}{\sqrt{100(\frac{1}{6})(\frac{5}{6})}} \right)$$

$$\approx P\left(-4.47 \le \frac{X - 100(\frac{1}{6})}{\sqrt{100(\frac{1}{6})(\frac{5}{6})}} \le -0.45 \right)$$

$$\approx \Phi(-0.45) - \Phi(-4.47)$$

$$\approx [1 - \Phi(0.45)] - [1 - \Phi(4.47)]$$

$$\approx \underbrace{[1 - 0.6736]}_{0.3264} - \underbrace{[1 - 1]}_{0} \approx \boxed{0.3264}$$

1 (b) Same setup as in part (a).

$$P(X=15) = P(14.5 \le X \le 15.5)$$

$$= P\left(\frac{14.5 - 100(\frac{1}{6})}{\sqrt{100(\frac{1}{6})(\frac{5}{6})}} \le \frac{X - 100(\frac{1}{6})}{\sqrt{100(\frac{1}{6})(\frac{5}{6})}} \le \frac{15.5 - 100(\frac{1}{6})}{\sqrt{100(\frac{1}{6})(\frac{5}{6})}}\right)$$

$$\approx P\left(-0.58 \le \frac{X - 100(\frac{1}{6})}{\sqrt{100(\frac{1}{6})(\frac{5}{6})}} \le -0.31\right)$$

$$\approx \Phi(-0.31) - \Phi(-0.58)$$

$$\approx [1 - \Phi(0.31)] - [1 - \Phi(0.58)]$$

$$\approx [1 - 0.6217] - [1 - 0.7190]$$

$$\approx 0.3783 - 0.281$$

$$\approx \boxed{0.0973}$$

(2) Let X be the number of 3's that occur in $n=50$ rolls of a 4-sided die. On one roll the probability of a 3 is $p = 1/4$.

Thus,

$$P(10 \le X \le 15)$$

$$= P\left(\frac{10 - 50\cdot\frac{1}{4}}{\sqrt{50\cdot\frac{1}{4}\cdot\frac{3}{4}}} \le \frac{X - 50\cdot\frac{1}{4}}{\sqrt{50\cdot\frac{1}{4}\cdot\frac{3}{4}}} \le \frac{15 - 50\cdot\frac{1}{4}}{\sqrt{50\cdot\frac{1}{4}\cdot\frac{3}{4}}} \right)$$

$$= P\left(-0.82 \le \frac{X - 12.5}{75/8} \le 0.82\right)$$

$$\approx \Phi(0.82) - \Phi(-0.82)$$

$$\approx \Phi(0.82) - [1 - \Phi(0.82)]$$

$$\approx -1 + 2\Phi(0.82) \approx -1 + 2(0.7939)$$

$$\approx \boxed{0.5878}$$

③ Let X be the number of wins in $n = 50$ spins of the Roulette wheel when you bet on black each time.

On a single spin of the wheel, the probability of winning on black is $p = \frac{18}{38} = \frac{9}{19}$

Thus,

$$P(26 \le X \le 50)$$

$$= P\left(\frac{26-(50)\left(\frac{9}{19}\right)}{\sqrt{(50)\left(\frac{9}{19}\right)\left(\frac{10}{19}\right)}} \le \frac{X-(50)\left(\frac{9}{19}\right)}{\sqrt{(50)\left(\frac{9}{19}\right)\left(\frac{10}{19}\right)}} \le \frac{50-(50)\left(\frac{9}{19}\right)}{\sqrt{(50)\left(\frac{9}{19}\right)\left(\frac{10}{19}\right)}}\right)$$

$$\approx P\left(0.66 \le \frac{X-(50)\left(\frac{9}{19}\right)}{\sqrt{(50)\left(\frac{9}{19}\right)\left(\frac{10}{19}\right)}} \le 7.45\right)$$

$$\approx \Phi(7.45) - \Phi(0.66)$$

$$\approx 1 - 0.7454 \approx \boxed{0.2546}$$