Thursday
9/26

Last time

Find abs max/min of $\boxed{f(x,y) = x^2 - 2xy + 2y}$

on $S = \left\{ (x,y) \;\middle|\; \begin{matrix} 0 \le x \le 3 \\ 0 \le y \le 2 \end{matrix} \right\}$

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① Found critical point $(1,1)$

$$f(1,1) = 1$$

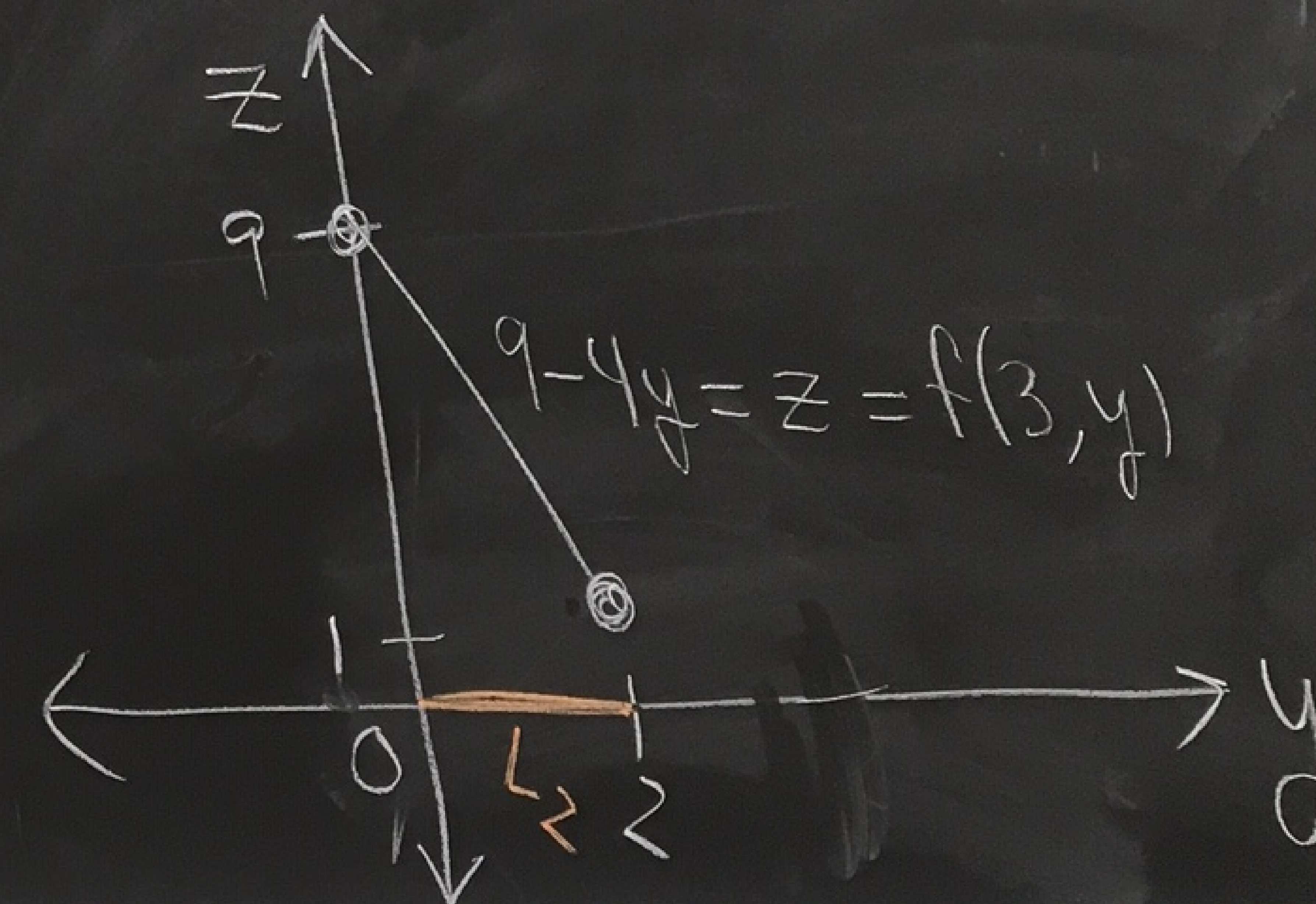
② **On L_1**

min at $(0,0)$, $f(0,0) = 0$

max at $(3,0)$, $f(3,0) = 9$

$\boxed{L_2}$ $(x = 3,\ 0 \le y \le 2)$

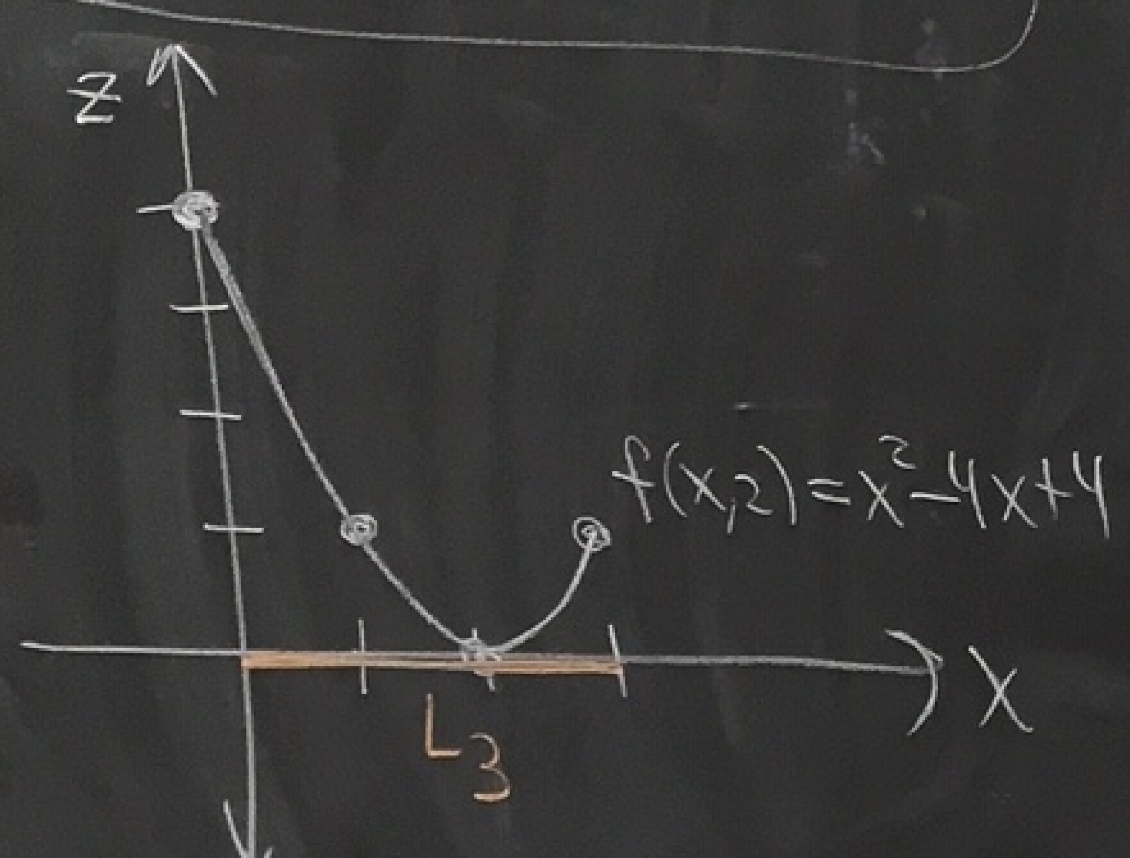
$$f(3,y) = 9 - 6y + 2y = 9 - 4y$$



$\boxed{L_3}$ $(0 \le x \le 3,\ y = 2)$

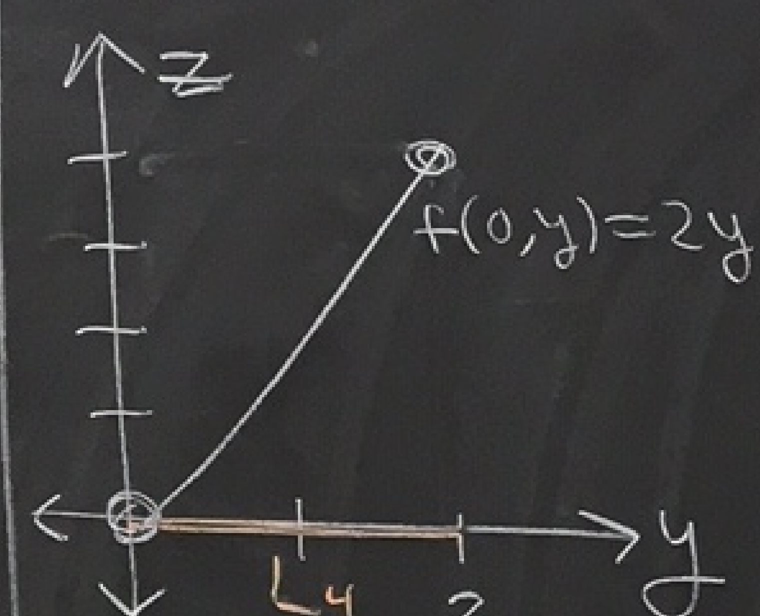
$f(x,2) = x^2 - 4x + 4$

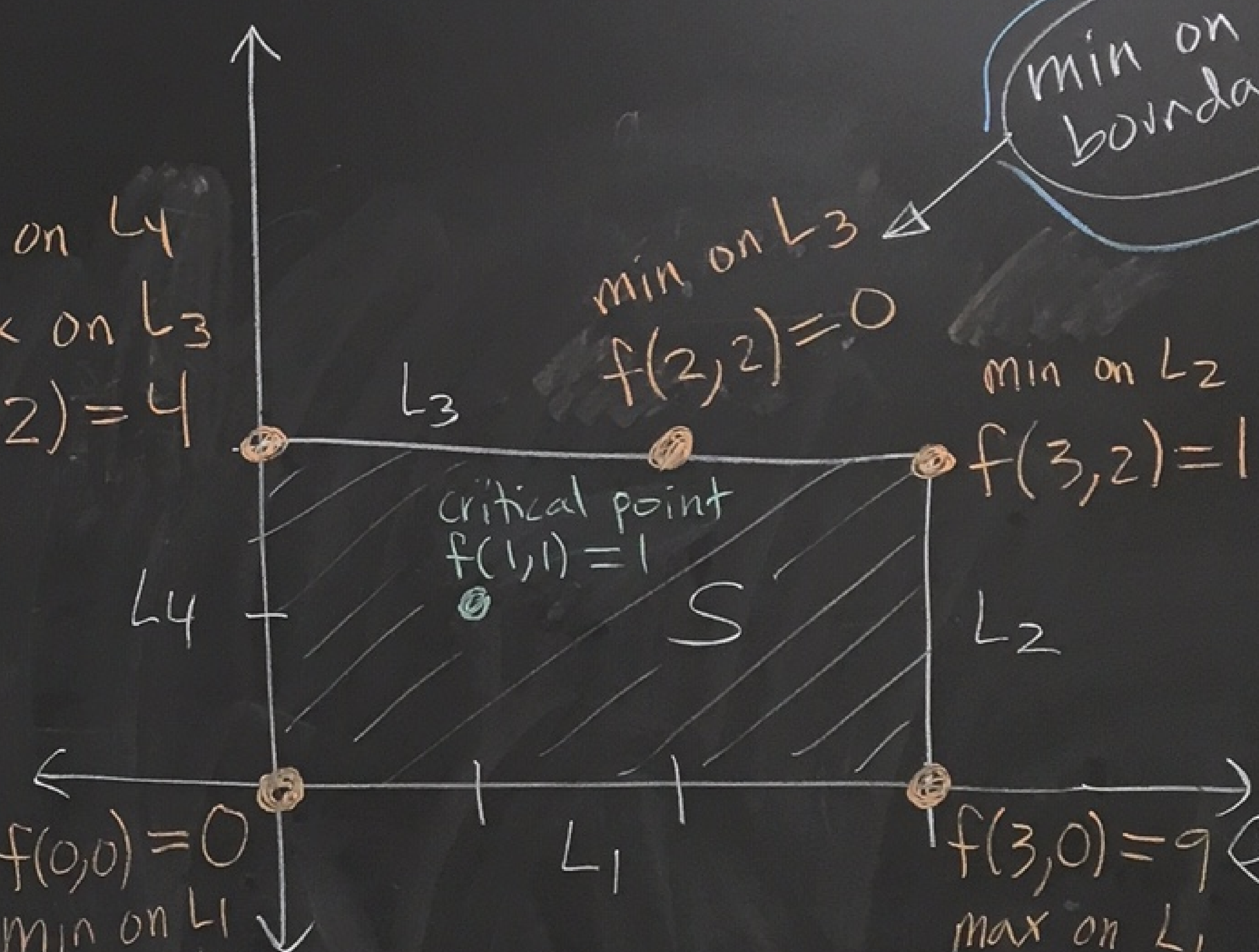
min of $x^2 - 4x + 4$ when
$(x^2-4x+4)' = 0$
$2x - 4 = 0$
$x = 2$



$\boxed{L_4}$ $(x = 0,\ 0 \le y \le 2)$

$f(0,y) = 2y$





ANSWER

absolute max	absolute min
9 at (3,0)	0 at (0,0) and (2,2)

12.9 - Lagrange Multipliers

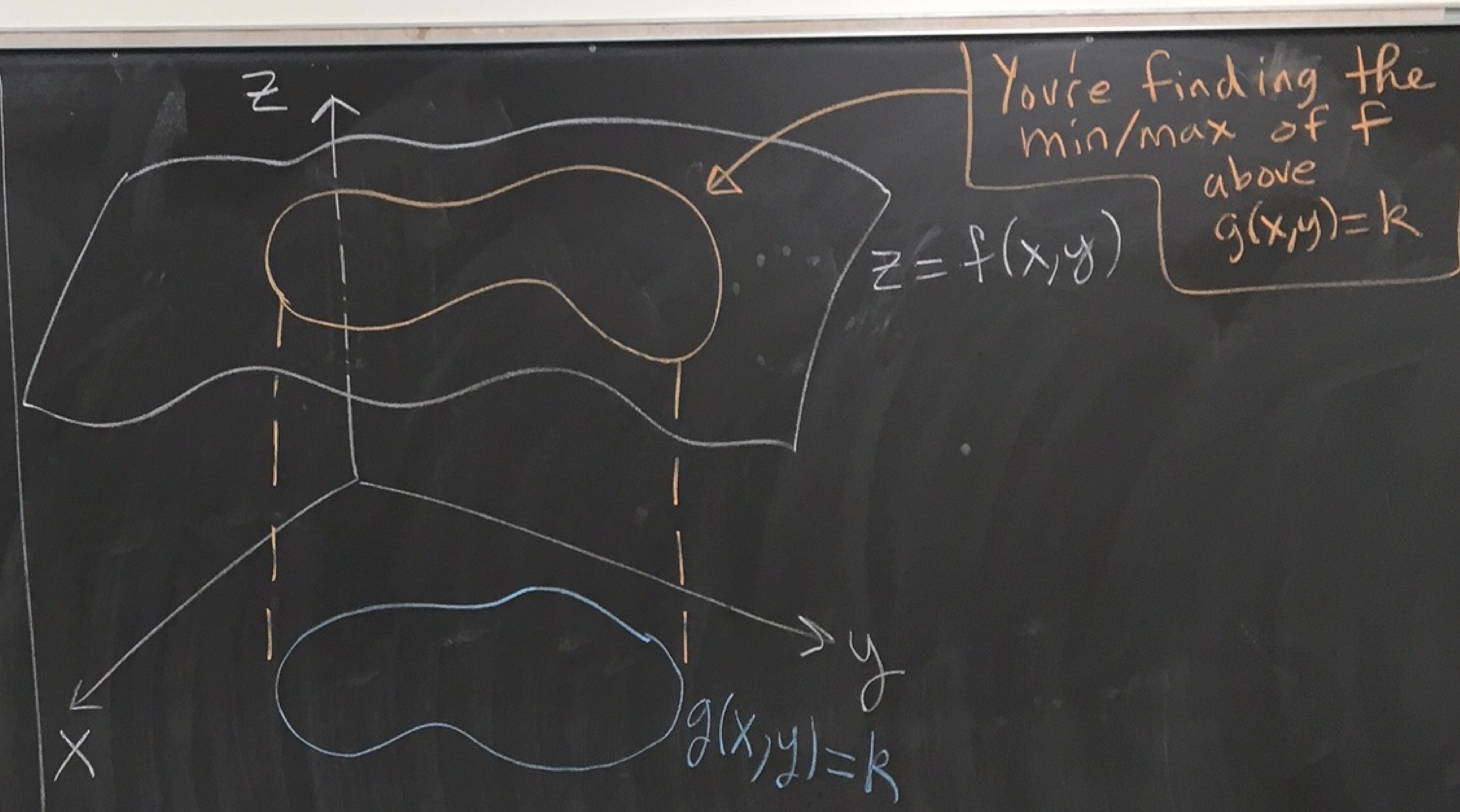
Method: To find the maximum and minimum values of $f(x,y)$ subject to the constraint $g(x,y)=k$ [assuming that these extreme values exist and $\nabla g \neq \vec{0}$ on $g(x,y)=k$]:

① Find all the values of x, y, λ such that

$$\nabla f(x,y) = \lambda \nabla g(x,y)$$

and $$g(x,y) = k$$

② Evaluate f at all the points (x,y) that result from step 1. The largest of these values is the max of f subject to $g(x,y)=k$. The smallest of these values is the min of f subject to $g(x,y)=k$.



Ex: Find the max and min of

$$f(x,y) = 2x^2 + y^2 + 2$$

subject to the constraint

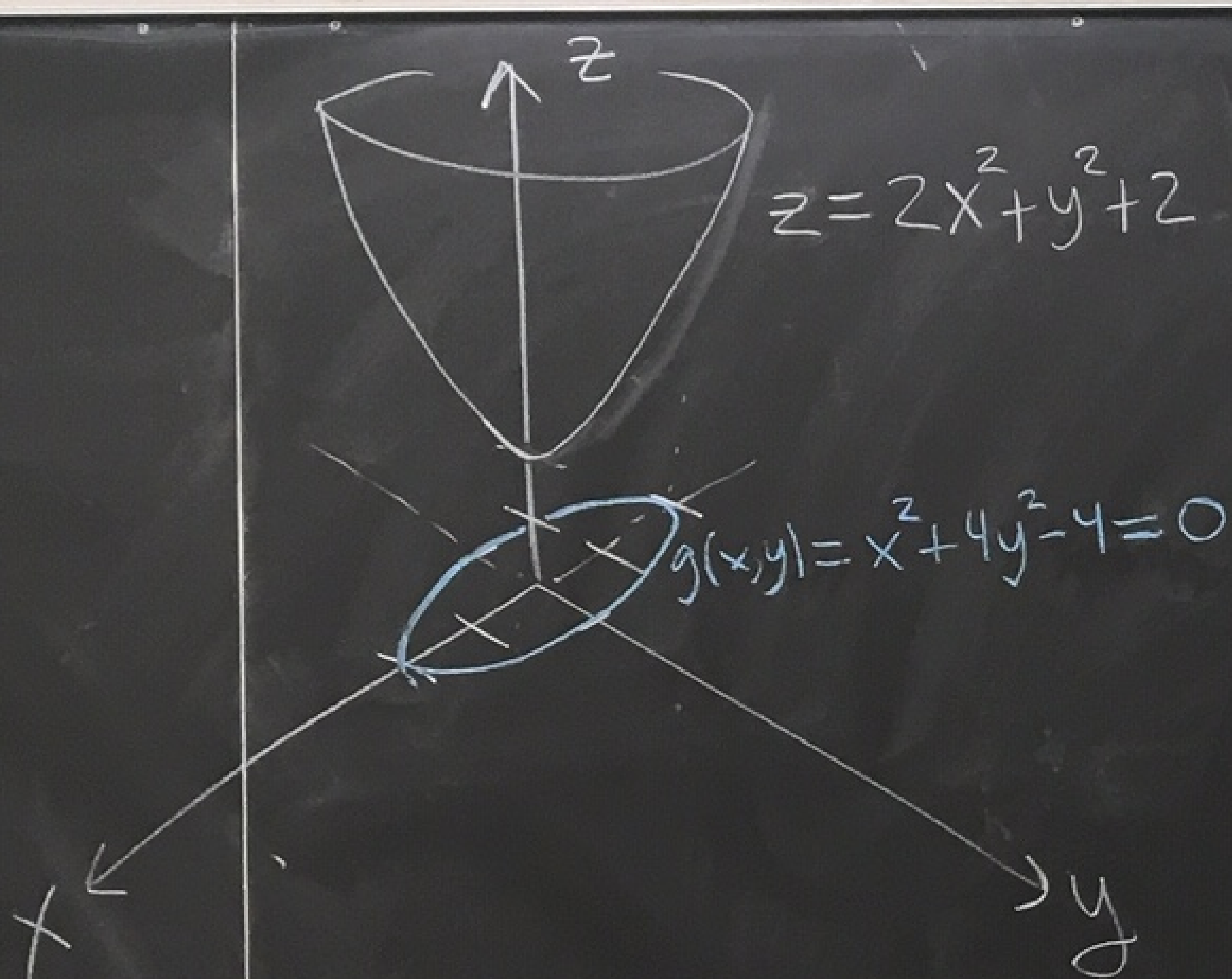
$$g(x,y) = x^2 + 4y^2 - 4 = 0$$

$$x^2 + 4y^2 - 4 = 0$$

$$\frac{x^2}{2^2} + \frac{y^2}{1^2} = 1$$

$$\boxed{g(x,y) = 0}$$

x



Lagrange time!

① $\boxed{\text{Solve} \quad \begin{aligned} \nabla f &= \lambda \nabla g \\ g &= k \end{aligned}}$

$$\boxed{\begin{aligned} \langle 4x, 2y \rangle &= \langle 2\lambda x, 8\lambda y \rangle \\ x^2 + 4y^2 - 4 &= 0 \end{aligned}}$$

$$\nabla f = \langle 4x, 2y \rangle$$

$$\nabla g = \langle 2x, 8y \rangle$$

$$\lambda \nabla g = \langle 2\lambda x, 8\lambda y \rangle$$

$$\boxed{\begin{aligned} 4x &= 2\lambda x \\ 2y &= 8\lambda y \\ x^2 + 4y^2 - 4 &= 0 \end{aligned}}$$

Need to solve for x, y, λ

① $2x(2-\lambda)=0$
② $2y(1-4\lambda)=0$
③ $x^2+4y^2-4=0$

$\boxed{\text{① Solutions: } x=0,\ \lambda=2}$

case 1: $\boxed{x=0}$

Plug $x=0$ into ③ to get

$$0^2+4y^2-4=0$$

So, $\boxed{y=\pm 1}$

So, $(x,y)=(0,1)$ solves ① & ③

Then eqn ② becomes $2(1-4\lambda)=0$
which is solved by $\lambda=\frac{1}{4}$.

Thus, $\boxed{x=0,\ y=1,\ \lambda=\frac{1}{4}}$ solves ①,②,③

If
becom
Again
So,

$x=$

solves

If $x=0, y=-1$, then ② becomes $-2(1-4\lambda)=0$. Again $\lambda=\frac{1}{4}$ solves ②.

So,

$$\boxed{x=0, y=-1, \lambda=\frac{1}{4}}$$

solves ①, ②, and ③.

Case 2: $\lambda=2$

Plug $\lambda=2$ into ② to get $2y(1-8)=0$ or $-14y=0$ or $\boxed{y=0}$.

Plug $y=0$ into ③ to get $x^2-4=0$ or $\boxed{x=\pm 2}$

So, $\boxed{\begin{array}{l} x=2, y=0, \lambda=2 \\ x=-2, y=0, \lambda=2 \end{array}}$ Solve ①, ②, ③.

②, ③