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Fictions and Models

New Essays

Foreword by Nancy Cartwright

Philosophia

III Mark Balaguer
Fictionalism, Mathematical Facts and Logical/Modal Facts[137]

1 Introduction

Mathematical fictionalists--or as I'll call them, fictionalists--think that ordinary mathematical sentences like '3 is prime' are, strictly speaking, untrue. The reason, in a nutshell, is that they think that (a) these sentences are supposed to be about abstract objects, and (b) there are no such things as abstract objects. Nonetheless, despite their eschewal of ordinary mathematical truth, fictionalists seem to be committed to the idea that there are important facts "lurking behind" sentences like '3 is prime'--facts that could reasonably be thought of as mathematical facts, or at least proto-mathematical facts, or pseudo-mathematical facts, or something along these lines.

This is worrying because it's not obvious what fictionalists could

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take these proto-mathematical facts to *be*. The problem is that, because of their ontological scruples, fictionalists just don't have access to very many kinds of facts. I won't run through the argument for this, but it seems to me that when fictionalists go to give an account of the nature of the proto-mathematical facts they're committed to, they will have access, at most, to two different kinds of facts, namely, physical-empirical facts and logical/modal facts. That's the best-case scenario. The worst-case scenario is that they'll have access only to physical-empirical facts. For one might argue that fictionalists can't countenance the existence of logical or modal facts because such facts are best thought of as involving metaphysically spooky objects of some kind, e.g., abstract objects or Lewisian non-actual possible worlds. (I'll assume throughout that fictionalists reject the existence of both abstract objects and non-actual possible worlds; and I'll use 'spooky' to cover both sorts of objects.)

(Actually, one might wonder whether fictionalists can countenance the existence of any facts at all; for one might think that facts are best thought of as abstract objects. I won't worry about this here. I'll assume that fictionalists can either develop an anti-platonistic view of facts or simply jettison 'fact' talk from their view; e.g., instead of talking about the fact that snow is white, they could simply say that snow is white.)

In this paper, I provide a response to the above worry about the kinds of mathematical facts, or proto-mathematical facts, that fictionalists are committed to. In section 2, I provide some background; in particular, I formulate the fictionalist view of mathematics in a bit more detail and explain why fictionalists are committed to the existence of something like mathematical facts, or proto-mathematical facts, despite their rejection of ordinary mathematical truth. In section 3, I briefly sketch a theory of what these fictionalistic proto-mathematical facts *are*. In section 4, I argue that these facts decompose into logical/modal facts and physical-empirical facts. And in section 5, I respond to the objection that fictionalists can't appeal to logical/modal facts in their account of mathematics; I do this by developing a non-spooky theory of logical/modal facts,

i.e., a theory that's consistent with the rejection of abstract objects and non-actual possible worlds.

Before commencing, I want to make two points. First, the theory of logical/modal facts developed in section 5 can be used not just by fictionalists, but by any nominalist about mathematics. I think it can be argued that all of the mainstream nominalistic views of mathematics need to be supplemented by a nominalistic account of logic. The view I develop here can be combined with any of these views. Indeed, it can be endorsed by anyone, nominalist or not. As we'll see, I think it's just the right view to adopt.

Second, one might have thought that fictionalism entails that mathematical facts reduce entirely--not just partially--to logical facts. On Field's version of fictionalism, this is close to true. But as we'll see in section 2.2, Field's view can't be right. Fictionalists need a different theory, and on the view I'll be developing, mathematical facts do not reduce entirely to logical facts--although one might argue that on this view just about all of the mathematically *interesting* facts do reduce to logical facts.

2. Background

2.1 *Mathematical fictionalism*

Fictionalism is best defined in terms of--or as a reaction to--mathematical platonism. Platonism can be thought of as the view that (a) there exist abstract mathematical objects, i.e., objects that are non-spatiotemporal and wholly non-physical and non-mental, e.g., numbers, sets, functions, and so on; and (b) our mathematical theories provide true descriptions of such objects. For instance, on this view, '3 is prime' says something true about the number 3,which is an abstract object; i.e., it's a real and objective thing that exists independently of us, outside of space and time, and it is wholly non-physical, non-mental, and causally inert.

Fictionalism is essentially the view that the platonists' semantic hypothesis is true while their ontological hypothesis is false. More precisely, it's the view that (a) our mathematical theories do *purport* to be about abstract objects, as platonists claim (e.g., '3 is prime' should be interpreted as purporting to make a claim about the number 3); but (b) there are no such things as abstract objects; and so (c) our mathematical theories are not literally true. Thus, on this view, '3 is prime' is not literally true for the same reason that 'Santa Claus lives at the North Pole' is not literally true--because just as there is no such person as Santa Claus, so too, there is no such thing as the number 3.[138]

There are other ways to avoid platonism, but I think it can be argued that fictionalism is the best version of anti-platonism. I don't have the space to argue this point here, but in a nutshell, the problem with other versions of anti-platonism is that they all involve a rejection of the following (extremely plausible) hypothesis:

> *The platonistic semantic hypothesis*: Our mathematical theories are best interpreted as being about (or at least purporting to be about) abstract objects--e.g., numbers and sets and so on--and in order for these theories to be literally true, there must actually exist abstract objects like numbers and sets and so on.

If we rejected this hypothesis, we would have to endorse one of the following four views, all of which are problematic. First, we could endorse the *physicalistic* view that sentences like '3 is prime' and 'There are infinitely many transfinite cardinals' are about physical objects. Second, we could endorse the *psychologistic* view that such sentences are about mental objects. Third, we could endorse the *paraphrase nominalist* view that sentences like '3 is prime' should be read not at face value--i.e., not as being of the form '*Fa*'-- but, rather, as having a logical form with no ontological commitments, i.e., as not really making claims about objects at all. (For instance, one might hold that 3 is prime' should be read as really saying that *if there were numbers, then 3 would be prime*, so that it could be true even if there were no such thing as the number 3.) Fourth and finally, we could endorse the *neo-Meinongian* view that sentences of the form

[138] This view was introduced by Field (1980, 1989) and developed further by me (1996, 1998a, 2009), Rosen (2001), Yablo (2002), and Leng (2010).

'*Fa*' can be literally true even if the singular term '*a*' doesn't refer to anything, so that, again, '3 is prime' could be true even if there were no such thing as the number 3.[139]

I think there are good arguments against all four of these views, and more generally, I think we have good reason to endorse the platonistic semantic hypothesis and, hence, to conclude that fictionalism is the best version of anti-platonism. The arguments I have in mind here are empirical arguments based on facts about ordinary mathematical practice and what mathematicians (and ordinary folk) actually mean when they say things like '3 is prime' and 'There are infinitely many transfinite cardinals.' But, again, I cannot develop these arguments here. I include these remarks simply to explain why I think anti-platonists--i.e., those who reject the existence of abstract objects--should endorse fictionalism.[140]

2.2 Why fictionalists are committed to mathematical facts, or proto-mathematical facts:

There is an obvious worry about fictionalism that can be put like this:

Since fictionalism entails that sentences like '3 is prime' and '2 + 2 = 4' are untrue, it seems to give us no account of the difference between, e.g., '2 + 2 = 4' and '2 + 2 = 5'. It seems beyond doubt that '2 + 2 = 4' is--in some sense or other--*right* or *correct*, whereas '2 + 2 = 5' is incorrect. This seems to be an objective fact that we can't simply ignore, and so fictionalists need to say what the correctness of sentences like '2 + 2 = 4' consists in. And in doing this, it would seem they are going to have to commit themselves to the existence of some facts. The facts in question might not be *mathematical* facts, strictly speaking, but, presumably, they will at least be characteriza-

[139] The neo-Meinongian view has been developed by Routley (1980), Priest (2003, 2005), Azzouni (2004), and Bueno (2005). Paraphrase nominalism has been developed by Putnam (1967a,b), Hellman (1989), and Chihara (1990). Physicalistic views have been endorsed by Mill (1843) and Kitcher (1984). And versions of psychologism have been defended by Husserl (1891), Brouwer (1912), and Heyting (1956).

[140] For quick arguments against the various anti-platonistic alternatives to fictionalism, see my 2008.

ble as pseudo-mathematical, or proto-mathematical, or something along these lines.

Field (1989) responded to this worry by claiming that the sense in which '2 + 2 = 4' is correct and '2 + 2 = 5' is incorrect is roughly equivalent to the sense in which 'Alice entered Wonderland by falling down a rabbit hole' is correct and 'Alice entered Wonderland by falling down a manhole' is incorrect. More specifically, on Field's view, the so-called correctness of '2 + 2 = 4' consists in the fact that it's *true in the story of mathematics*, or *part of* the story of mathematics.

This, I think, is a good start, but fictionalists need to say more. In particular, they need to say what the story of mathematics consists in. According to Field (1998), the story of mathematics consists (roughly) in the various axiom systems that are accepted in the various branches of mathematics. Thus, on Field's view, the relevant sort of mathematical correctness--what we might call fictionalistic mathematical correctness--comes down (roughly) to following from accepted axioms. And so on this view, the facts that fictionalists are committed to--the proto-mathematical facts behind our mathematical theories--are facts about what follows from accepted axioms.

But Field's view can't be right. The problem is that it can't account for how there could be objectively correct answers to mathematical questions that are undecidable in current mathematical theories, e.g., the question of whether the continuum hypothesis (CH) is true.[141] Suppose, for instance, that (i) some mathematician, call her Zoey, found a new set-theoretic axiom candidate A that was accepted by mathematicians as an intuitively obvious claim about sets, and (ii) Zoey proved CH from ZF+A (where ZF is Zermelo-Fraenkel set theory). Then mathematicians would say that Zoey had discovered the answer to the CH question. And, of course, they would be right. But Field's view cannot account for this. Given that CH and ~CH are both consistent with currently accepted set theories, Field's view entails that neither is true in the story of mathematics and, hence, that there is no objectively correct answer to the CH question. Now, of course, he could say that if mathematicians accepted A as a new axiom of set theory, then CH would *become* correct, but this seems to get things wrong. Given that A is an intuitively obvious claim about sets that all mathematicians accept as soon as they hear it, it seems clear that, in this scenario, CH was right all along and that Zoey *discovered* this fact. But this is precisely what Field's view doesn't allow us to say.

[141] For present purposes, it doesn't matter what CH says. All that matters is that it can't be proven true or false in any standard (or accepted) set theory, e.g., ZF.

So fictionalists need a different theory of what the story of mathematics (and fictionalistic mathematical correctness) consist in. In the next section, I will briefly sketch a view that fictionalists can endorse. And we'll see at the end that this view easily accounts for the case that refutes Field's view, i.e., the Zoey case.

3. A theory of fictionalistic mathematical correctness

When I say I'll "briefly sketch" a theory of fictionalistic mathematical correctness, I mean it. What I give here is essentially a summary of a view that I develop in more detail elsewhere (see my 2009 and, to a lesser extent, my 1998a and 2001). In particular, I will say just enough about this to set up the discussion in sections 4 and 5, i.e., the discussion of the nature of fictionalistic mathematical facts, or proto-mathematical facts.

Fictionalists can arrive at a better theory of what the story of mathematics (and fictionalistic mathematical correctness) consist in by recalling the original idea behind fictionalism, the idea of stealing as much as possible from platonism without admitting the existence of abstract objects. Ideally, fictionalists would like a theory of fictionalistic correctness that enables them to account for the Zoey case in essentially the same way that platonists do. I think fictionalists can do this if they adopt the following view:

(SM) The story of mathematics consists in the claim that there actually exist abstract mathematical objects of the kinds that platonists have in mind--i.e., the kinds that our mathematical theories are about, or at least purport to be about.

If fictionalists accept this, then they can also endorse the following:

(FC) A pure mathematical sentence is *correct, or fictionalistically orrect,* iff it's true in the story of mathematics, as defined in the above way; or, what comes to essentially the same thing, iff it would have been true if there had actually existed abstract mathematical objects of the kinds that platonists have in mind, i.e., the kinds that our mathematical theories purport to be about. (But keep this in mind: the view here is *not* that '2 + 2 = 4' *really says* that '2 + 2 = 4' is true in the story of mathematics. That would make the view a version of paraphrase nominalism. Fictionalists maintain that (a) '2 + 2 = 4' says that 2 + 2 = 4, and (b) this is false. But then they go on to claim that while it's false, it's also fictionalistically correct, because it's true in the story of mathematics.)

I will call fictionalists who endorse (SM) and (FC) *theft-over-honest-toil fictionalists*, or for short *T-fictionalists*. If fictionalists endorse this view, then, as we'll see below, they can handle the Zoey case in essentially the same way that platonists do. Indeed, they'll be able to mimic any minimally reasonable story that platonists might decide to tell about why CH is true in the Zoey scenario.

Since T-fictionalists maintain that a mathematical sentence is fictionalistically correct just in case it would have been true if platonism had been true, it seems that if we want to get clear on what mathematical correctness ultimately consists in, according to T-fictionalism, we first need to get clear on what mathematical truth ultimately consists in, according to platonism. In what follows, I will sketch the view that I think platonists should accept here.

It might seem that platonists can simply say that a mathematical theory is true iff it accurately describes a collection of abstract

objects. But this can't be right. For since there are hierarchies in which ZF+CH is true and hierarchies in which ZF+~CH is true, platonism seems to entail that ZF+CH and ZF+~CH *both* accurately describe collections of abstract objects.

Another stance platonists might adopt here is that a mathematical theory is true iff it accurately describes the *intended* objects, or the intended structure. But we cannot assume that in every branch of mathematics, there will always be a unique intended structure (up to isomorphism). For if our intentions aren't perfectly precise, they might fail to zero in on a unique structure (up to isomorphism). In other words, there could be multiple structures in a given branch of mathematics (that aren't isomorphic to one another) that all count as intended because they all fit perfectly with our intentions. Thus, platonists cannot simply say that a mathematical sentence is true iff it's true in *the* intended structure.

Nonetheless, I think the idea of defining mathematical truth in terms of truth in intended structures is right; platonists just need to figure out how to do this in a way that doesn't assume that there is always a unique intended structure (up to isomorphism) in every branch of mathematics. In order to do this, the first thing platonists need to get clear on is what it is for a structure to be intended--i.e., what determines which structures count as intended in a given branch of mathematics. We can start out here, somewhat roughly, by saying that a part P of the mathematical realm (e.g., a structure) counts as intended in a branch B of mathematics iff P fits with our intentions in B. But we need to make this more precise. We can think of our intentions in a given branch of mathematics as being captured by the *full conception* that we have of the objects, or purported objects, being studied in that branch of mathematics. For instance, on this view, our arithmetical intentions are captured by our full conception of the natural numbers (FCNN); and our set-theoretic intentions are captured by our full conception of the universe of sets (FCUS); and so on. If we think of these full conceptions--or as I'll call them, FCs--as consisting of bunches of sentences, then we can say the following:

Intendedness: A part P of the mathematical realm counts as intended in a branch B of mathematics iff all the sentences that are built into the FC in B--i.e., the full conception that we have of the purported objects in B--are true in P. (Actually, this isn't quite right because it assumes that all of our FCs are consistent. If we dropped this assumption, we would need to complicate things somewhat, but for the sake of simplicity, we can ignore this complication and work with the assumption that our FCs are consistent.)

To make this more precise, I need to say a few words about the kinds of sentences that are included in our FCs. And to do this, I need to distinguish two different kinds of cases, namely, (a) cases in which mathematicians work with an axiom system, and there is nothing behind that system, i.e., we don't have any substantive pretheoretic conception of the objects (or purported objects) being studied, so that any structure that satisfies the relevant axioms is *thereby* an intended structure (or a structure of the kind that the given theory is supposed to be *about*, or some such thing); and (b) cases in which we *do* have an intuitive, pretheoretic conception of the (purported) objects being studied, so that a structure S could satisfy the relevant axiom system but still fail to be an intended structure (or a standard model, or a structure of the kind that the given theory was supposed to be about) because the axiom system failed to zero in on the kind of structure we had in mind intuitively and because S didn't fit with our intuitive or pretheoretic conception.[142] Given this distinction, we can characterize our FCs with the following two claims:

Non-pretheoretic FCs: In cases in which we don't have any substantive pretheoretic conception of the (purported) objects being studied, the so-called full conception of these objects, or the FC, is essentially exhausted by the given axiom system, so that any structure that satisfies the axioms is thereby an intended structure.

[142] One might question whether both of these kinds of theorizing actually go on in real mathematics. I think it's pretty obvious that they do, but I don't need this result here.

Pretheoretic FCs: In cases in which we (or the relevant people or theorists, whoever they are) do have an intuitive, pretheoretic conception of the (purported) objects being studied, a sentence S is included in the relevant FC iff (roughly) either (a) S is a claim about the relevant objects (i.e., the objects that the theory in question purports to be about) that is accepted (uncontroversially and nonspeculatively) by the relevant people; or (b) S follows from some claim about the relevant objects that's accepted (uncontroversially and nonspeculatively) by the relevant people. (Three quick points: First, notice that our FCs are not precisely defined and so there can be cases where it's not clear whether some sentence is part of some FC. Second, while I'm calling these FCs "pretheoretic", they can be, and usually are, theoretically informed; our theorizing in a branch of mathematics will typically affect our intuitive conceptions. Third, it's important to note that the relevant FC in a given branch of mathematics can go beyond the relevant axiom systems. For instance, our full conception of the natural numbers (or FCNN) seems to go beyond the axioms and theorems of standard arithmetical theories like Peano Arithmetic; FCNN does include the axioms and theorems of such theories, but it seems to include other sentences as well, e.g., 'The Gödel sentences of the standard axiomatic theories of arithmetic are true' and perhaps 'The number 4 is not identical to Julius Caesar'.[143])

All of this is a bit simplified, but for present purposes, it's good enough.

We saw above that platonists cannot say that a mathematical sentence is true just in case it accurately describes *the* intended structure, because there might not be a unique intended structure (up to isomorphism) in every branch of mathematics. Given the above discussion, we can now bring this point out with an

[143] I suppose one might question the assumption that arithmetic is a case of the relevant kind; i.e., one might question the idea that we have an intuitive, pretheoretic conception of the natural numbers. It seems pretty obvious to me that we do, but I don't need to argue for this claim here.

example. In the case of arithmetic, it seems pretty plausible to suppose that there is a unique intended structure up to isomorphism. In other words, it's plausible to suppose that our full conception of the natural numbers--i.e., FCNN--zeros in on a unique structure up to isomorphism. When it comes to set theory, however, things aren't so clear. It's not at all obvious that FCUS (i.e., our full conception of the universe of sets) zeros in on a unique part of the mathematical realm, up to isomorphism. Consider, e.g., hierarchies in which ZF+CH is true and hierarchies in which ZF+~CH is true. It *may* be that there are hierarchies of both of these kinds that are perfectly consistent with *everything* that's built into FCUS. And if this is indeed the case, then it would seem that at least some hierarchies of both kinds ought to count as intended. Thus, in this scenario, CH would be true in some intended parts of the mathematical realm and false in others. And this is why platonists should not say that a mathematical sentence is true iff it's true in *the* intended part of the mathematical realm.

What, then, should platonists say? How should they define mathematical truth? I think it can be argued that the best platonist view here is the following:

(IBP) A (pure) mathematical sentence S is *true* iff it's true in *all* the parts of the mathematical realm that count as intended in the given branch of mathematics (and there is at least one such part of the mathematical realm); and S is *false* iff it's false in all such parts of the mathematical realm (or there is no such part of the mathematical realm[144]); and if S is true in some intended parts of the mathematical realm and false in others, then it's neither true nor false (or perhaps, there is no fact of the matter whether it's true or false).

[144] We actually don't need this parenthetical remark because if there are no such parts of the mathematical realm, then it will be true (vacuously) that S is false in all such parts of the mathematical realm. Also, one might want to say (a la Strawson) that if there's no such part of the mathematical realm, then S is neither true nor false, because it has a false presupposition. I prefer the view that in such cases S is false; but nothing important turns on this.

Now, since this view allows for the possibility of bivalence failures in mathematics, one might worry that it entails that mathematicians can't use classical logic in their proofs. However, in my 2009, I argue that this is false, that IBP-platonism is perfectly consistent with the use in mathematics of classical logic. Indeed, I argue that the way in which IBP-platonism allows for the possibility of bivalence failures actually gives rise to a powerful argument in *favor* of this view.[145] And more generally, I argue that IBP-platonism is the best version of platonism there is.

I won't rehearse the defense of IBP-platonism here because in the present context it doesn't really matter. As long as fictionalists embrace T-fictionalism, they can sit back and wait for it to be determined what the best platonistic theory of mathematical truth is, and then they can endorse an exactly parallel theory of mathematical correctness. I think that IBP-platonism is the best version of platonism, and so I am going to assume this result here and develop an analogous version of fictionalism. But if it turned out that platonists should abandon (IBP) and endorse some other view, then T-fictionalists could make an analogous move.

(Actually, there's one (wildly implausible) version of platonism that T-fictionalists might have a hard time mimicking. Consider the following view:

Crazy Platonism: It's not the case that ZF+CH hierarchies and ZF+~CH hierarchies both exist in platonic heaven. Hierarchies of only one of these kinds actually exist. So whether CH is true is settled by brute existence facts. Moreover, it's also true that hierarchies of either of the two kinds *might have* existed; but the brute fact is that hierarchies of one of the two kinds just *don't* exist. Thus, while fictionalists might be able to mimic the views of other platonists, they can't mimic our view. They can't get the same results we do by say-

[145] More specifically, I argue that (a) whenever IBP-platonism entails that there's no fact of the matter about a mathematical sentence S, there really isn't any fact of the matter about S; and (b) allowing bivalence failures in mathematics in the way that IBP-platonism does is the only way to avoid endorsing a revisionistic philosophy of mathematics.

ing that a set-theoretic sentence is correct iff it would have been true if sets had existed. For hierarchies of either of the above kinds might have existed. In point of actual fact, however, hierarchies of only one of the two kinds actually do exist, and this is what determines whether CH is true or false.

Fictionalists might not be able to mimic this view, but it doesn't matter because, again, this view is wildly implausible; e.g., it's pretty easy to show that if ZF+CH hierarchies exist, then ZF+~CH hierarchies do too, and vice versa; moreover, even if this weren't the case, Crazy Platonism would still be untenable, because it makes mathematical truth arbitrary and mathematical discovery impossible.)

Let's assume, then, that IBP-platonism is the best version of platonism. Given this, fictionalists should endorse the following view:

(IBF) A pure mathematical sentence S is *correct*, or *fictionalistically correct*, iff, in the story of mathematics, S is true in all the parts of the mathematical realm that count as intended in the given branch of mathematics; and S is *incorrect* iff, in the story of mathematics, S is false in all intended parts of the mathematical realm; and if, in the story of mathematics, S is true in some intended parts of the mathematical realm and false in others, then it's neither correct nor incorrect (or perhaps, there is no fact of the matter whether it's correct or incorrect).

In my 2009, I argue that just as IBP-platonism is the best version of platonism, so too IBF-fictionalism is the best version of fictionalism. But, again, I don't want to rehearse the argument for this result here. Instead, I want to assume that fictionalists should endorse IBF-fictionalism, and I want to explore the nature of the mathematical *facts*--or proto-mathematical facts--that IBF-fictionalists are committed to. I turn to this issue in section 4, but before I do that I want to explain how IBP-platonists and IBF-fictionalists can account for the case that refuted Field's view--i.e., the Zoey case.

IBP-platonists can say that Zoey discovered the answer to the CH

question because they can say that the intuitive obviousness of her new axiom--i.e., A--is good reason to think that sentence is *true*. Now, you might wonder why intuitive obviousness is evidence of mathematical truth. Why should the fact that A is intuitively obvious to *us* suggest that it accurately describes an independently existing hierarchy of abstract objects? The answer, according to IBP-platonists, is that truth has to do with accurately describing the *intended* objects. If A is completely obvious to us, or to set theorists, that's evidence that it's built into our conception of set, or FCUS. But if A is built into FCUS, then it's true in all intended structures and, hence, on the IBP-platonist view, it's true. But if A is true, and if ZF+A entails CH (and if--what seems obvious--ZF is built into FCUS and, hence, true), then it follows that CH is true. Thus, IBP-platonists can easily explain why mathematicians would take Zoey to have *discovered* that CH is true. And, of course, IBF-fictionalists can tell an exactly analogous story here.

In this section, I have responded to just one of the worries people might have about fictionalism, namely, the worry about objectivity and correctness. There are of course other worries about fictionalism, e.g., the Quine-Putnam worry that our mathematical theories must be true because they form an indispensable part of our empirical theories; but I cannot discuss these other worries here.[146]

4. The decomposition of fictionalistic proto-mathematical facts into physical-empirical facts and logical/modal facts

Given the above remarks, it should be clear that while IBF-fictionalists deny that sentences like '3 is prime' are literally true, they are nonetheless committed to the existence of something like mathematical facts. For instance, on their view, it is a fact that if there were objects of the kinds that platonists have in mind--i.e., the kinds that our mathematical theories purport to be about--then '3 is prime' would be true, i.e., it would be true in all intended parts of the mathematical realm. But what kind of fact is this? Are facts of this kind consistent with fictionalism? In this section, I

[146] For a discussion of the various worries about fictionalism, and the responses that fictionalists have given to these worries, see my 2008.

will argue that the facts that IBF-fictionalists are committed to here decompose into two different kinds of facts, namely, logical or modal facts and physical-empirical facts about our heads, in particular, our intentions. I will argue for this result by discussing the following three kinds of facts that IBF-fictionalists are committed to:

Axiomatic Facts: The facts behind the axioms of our mathematical theories; e.g., the fact that if there existed abstract objects of the kinds that platonists have in mind, then the axioms of arithmetic (e.g., '0 is a number') would be true, i.e., true in all structures that count as intended in arithmetic.

Theorem Facts: Facts about the theorems of our mathematical theories following from the axioms of those theories; or put slightly differently, the fact that if the axioms of these theories were true, then the theorems would also be true; e.g., the fact that if the axioms of arithmetic were true, then '3 is prime' would be true.

Undecidable Facts: Facts corresponding to certain undecidable sentences. For instance, it seems to be a fact that if there were abstract objects of the sort that platonists have in mind, then the Gödel sentence for Peano Arithmetic would be true. And it *may* be a fact that if there were such things as abstract objects (in particular, sets), then CH would be true; or it may be a fact that if there were such objects, then ~CH would be true; or it may be that there is no objectively correct answer to this question.

I will discuss these three kinds of facts in turn. In particular, I will show that they all boil down to logical/modal facts and/or empirical facts about our intentions, or our heads.

The easiest of these three kinds of facts to deal with are theorem facts. For since theorem facts are entailment facts, they are pretty obviously logical facts. And this, one might argue, already gives us the result that on the IBF-fictionalist view, most mathematically *interesting* facts boil down to logical facts. (I'm using 'logical fact' a bit loosely here. The entailments in question--from the axioms of our mathematical theories to the theorems--aren't strictly logical entailments. But we could turn them into strictly logical entailments by simply throwing in a few stipulative definitions.)

Axiomatic facts require a bit more discussion. There are two different kinds of cases that need to be discussed, namely, (a) cases in which we have an intuitive, pretheoretic conception of the (purported) objects being studied, or more precisely, cases in which a structure could satisfy the relevant axiom system but still fail to be an intended structure because it didn't fit with our intuitive or pretheoretic conception; and (b) cases in which we don't have any substantive pretheoretic conception of the (purported) objects being studied, so that any structure that satisfies the relevant axioms is *thereby* an intended structure. I will begin with the former sort of case, and to make the discussion concrete, I will focus on Peano Arithmetic, or PA. We can think of the facts behind the axioms in PA as being lumped together into a single fact, namely,

Fact-Behind-PA: The fact that if there actually existed abstract objects of the kinds that platonists have in mind, then the axioms of PA would be true--i.e., they would be true in all the parts of the mathematical realm that count as intended in arithmetic.

Now, of course, it *may* be that this isn't a fact at all; e.g., it may be that PA is inconsistent. But we can ignore this possibility here. I'm trying to provide an account of ordinary mathematical facts, or ordinary cases of mathematical correctness; PA is just an example. If it turned out that PA wasn't correct, that wouldn't be a problem for fictionalists--it would be a problem for the mathematical community. In that case, I could just switch to a different example. Thus, for our purposes, we can simply assume that PA is in fact correct.

If we assume that PA is correct, then Fact-Behind-PA can be thought of as decomposing into the following three facts:

Fact (i): The fact that PA is consistent.

Fact (ii): The fact that the axioms of PA are built into FCNN, i.e., our full conception of the natural numbers. (I am assuming here--and I make the same assumption in connection with Fact (iii)--that FCNN is consistent. If we dropped this assumption, fictionalists would have to change things a bit, but I won't go into this here.)

Fact (iii): The fact that if (a) there were objects of the kinds that platonists have in mind, and (b) PA were consistent, and (c) the axioms of PA were built into FCNN, then (d) the axioms of PA would be true, i.e., they would be true in all the parts of the mathematical realm that count as intended in arithmetic. (Alternatively, fictionalists could formulate this fact as the fact that, necessarily, if (a), (b), and (c), then (d). Or they could formulate it as the fact that clauses (a)-(c) entail clause (d).)

There are two different points I want to argue here. First, I want to argue that Facts (i)-(iii) are either logical/modal facts or empirical facts about our heads. And second, I want to argue that this is the right way for IBF-fictionalists to proceed--i.e., that they should indeed maintain that Fact-Behind-PA decomposes into Facts (i)-(iii). I will argue the second point first.

The main point that needs to be made here is that in cases like this--i.e., cases in which we have an intuitive or pretheoretic conception of a mathematical structure, or a collection of mathematical objects, and we construct an axiom system that's supposed to characterize those objects--what's needed in order for the axioms to be true (or fictionalistically correct) is precisely that the axioms be built into the full conception (or the FC) that we have of the given objects. More precisely, we can say this:

BICONDITIONAL: In cases like the one's we're discussing here--where we have an intuitive, pretheoretic conception of the relevant objects--as long as the given axiom system and the corresponding FC are both internally consistent, *the axioms will be true (or fictionalistically correct) if and only if they are built into the corresponding FC.*

This is something that IBP-platonists and IBF-fictionalists should both accept. Let me argue the point for platonists first. Both directions of BICONDITIONAL are obvious. Consider first the claim that if the axioms are built into the corresponding FC, then they're true, i.e., true in all intended structures. To appreciate this, it's enough to remember that a structure S counts as intended in a given branch of mathematics just in case all the sentences built into the relevant FC are true in S. Thus, it's entirely obvious that if a given axiom is built into the relevant FC, then it will be true in all intended structures and, hence, true.

The other direction (if the axioms are true, then they're built into the corresponding FC) is just as trivial. Let me motivate this by arguing for the contrapositive--i.e., for the thesis that if the axioms aren't built into the relevant FC, then they aren't true, i.e., they aren't true in all intended structures, or all intended parts of the mathematical realm. To bring this point out, suppose that AX is the conjunction of all the axioms in a given axiom system. If AX isn't built into the relevant FC, then there is at least one structure S, or at least one part of the mathematical realm, such that (i) all the sentences built into the given FC are true in S, but (ii) AX isn't true in S. It follows from (i) that S is an intended structure in the given branch of mathematics. Thus, combining this with (ii), we get the result that there is at least one intended structure in which AX isn't true. Thus, AX isn't true in all intended structures, or all intended parts of the mathematical realm, and so, on the IBP-platonist view it it isn't true.[147] - So it seems to me that IBP-platonists should endorse BICONDITIONAL. And given this, it should be obvious that IBF-fictionalists should endorse it as well; for again, on their view, we should take what IBP-platonists say about mathematical truth and say exactly analogous things about fictionalistic mathematical correctness. In short, according to IBF-fictionalism, a mathema-

[147] This argument assumes that on the IBP-platonist view, the mathematical realm is plenitudinous, i.e., that on this view, there are as many mathematical objects as there could be. I think it can be argued that this is the only tenable way to develop platonism (see my 1998a), but I can't argue this point here.

tical sentence is correct just in case it would have been true if IBP-platonism had been true.

Given all this, it seems that IBF-fictionalists should indeed say that Fact-Behind-PA decomposes into Facts (i)-(iii). The other point I want to argue is that Facts (i)-(iii) are either logical/modal facts or empirical facts about our heads. All three points here are pretty obvious. Fact (i) is a consistency fact, so it is obviously a logical fact. Fact (ii)--i.e., the fact that the axioms of PA are built into our full conception of the natural numbers--is obviously a fact about our intentions, or our concept of the natural numbers; thus, it's an empirical fact about our heads. (Actually, as I've formulated Fact (ii) here, it's about *sentences*; but this is just a convenient way of talking about our heads; to fully address this worry, I would need to respond to the Quine-Putnam indispensability argument; i.e., I would need to provide an IBF-fictionalist account of how we can successfully use abstract-object talk to characterize the physical world, even if there are no such things as abstract objects; I don't have the space to do this here, but see my 1998a.) Finally, Fact (iii) is pretty obviously a logical fact (in particular, an entailment fact) or, if you'd rather, a modal fact (in particular, a necessity fact, or an impossibility fact). I will discuss facts of this kind in detail in section 5. (Note that, once again, I'm using 'logical fact' a bit loosely here; to make Fact (iii) a strictly logical fact, we'd have to introduce a few definitions, but I won't bother with this here.) Finally, I should note that Fact (iii) is entirely *trivial*. Indeed, I've already argued for Fact (iii), because it's essentially equivalent to the first direction of BICONDITIONAL.

Let's move on now to cases in which we don't have any substantive pretheoretic conception of the objects (or purported objects) being studied. To make the discussion concrete, let's suppose that (i) *AxSys-with-no-Pre-Con* is an axiom system of pure mathematics that we are currently working with, and (ii) we don't have any substantive pretheoretic conception of the objects that AxSys-with-no-Pre-Con characterizes, or purports to characterize. Moreover, we can also suppose that (iii) AxSys-with-no-Pre-Con is fictionalistically correct. (Of course, there can be axiom systems like this that *aren't* correct because they can be inconsistent. But, again, we can ignore this here. Since I'm trying to provide an account of ordinary mathematical facts, we can just focus on cases in which our axioms *are* correct.) In any event, if we assume that AxSys-with-no-Pre-Con is indeed correct, then we can think of the facts behind the axioms in this system as being lumped together into the following fact:

Fact-Behind-AxSys-with-no-Pre-Con: The fact that if there actually existed abstract objects of the kinds that platonists have in mind, then the axioms in AxSys-with-no-Pre-Con would be true--i.e., they would be true in all the parts of the mathematical realm that count as intended in the given branch of mathematics.

This fact can be thought of as decomposing into the following three facts:

Fact (A): The fact that AxSys-with-no-Pre-Con is consistent

Fact(B): The fact that AxSys-with-no-Pre-Con is an axiom system of pure mathematics that we work with (or that certain mathematicians work with) and that we do not have any substantive pretheoretic conception of the objects (or purported objects) that this system characterizes.

Fact (C): The fact that if (a) AxSys-with-no-Pre-Con were consistent, and (b) AxSys-with-no-Pre-Con were an axiom system of pure mathematics that we work with, and (c) we didn't have any substantive pretheoretic conception of the objects that AxSys-with-no-Pre-Con characterized, and (d) there actually existed abstract objects of the kinds that platonists have in mind, i.e., the kinds that our mathematical theories characterize, or purport to characterize, then (e) the axioms in AxSys-with-no-Pre-Con would be true, i.e., they would be true in all the parts of the mathematical realm that count as intended in the given branch of mathematics. (As was the case with Fact (iii) above, if fictionalists wanted to, they could formulate Fact (C) as the fact that, necessarily, if (a), (b), (c), and (d), then (e). Or as the fact that clauses (a)-(d) entail clause (e).)

All three of these facts are either logical/modal facts or empirical facts. Fact (A) is a consistency fact, so it's obviously a logical fact. Fact (B) is obviously an empirical fact about us and our heads. And as was the case with Fact (iii) above, Fact (C) is a logical fact or a modal fact. And, again, like Fact (iii), Fact (C) is trivial. The reason it's trivial is that, as we've seen, if we don't have any substantive pretheoretic conception of the objects we're studying, then the intended objects just *are* the objects picked out by, or characterized by, the given axiom system. And it follows trivially from this that the axioms in AxSys-with-no-Pre-Con are true in all intended structures and, hence, true.

Finally, moving on to undecidable facts, we can say the same thing here that we say about axiomatic facts in cases where we have a pretheoretic conception of the objects being studied. If, unbeknownst to us, it is a mathematical fact that, say, CH is true (or correct), then this is because CH is built into FCUS (i.e., our full conception of the universe of sets). If CH is not built into FCUS, then it's not correct; for in this scenario, ZF+~CH is perfectly consistent with FCUS, and so ~CH is true in at least some intended structures, and so CH isn't true, or correct, because it isn't true in all intended structures. (This view of CH is defended in much more detail in my 2009 and 2001.)

So if my arguments here are cogent, then IBF-fictionalists can claim that mathematical facts decompose into logical/modal facts and empirical facts about our heads. Just about all mathematically interesting facts collapse into logical facts, in particular, entailment facts about what follows from currently accepted axioms. But there are also some important empirical facts about our heads--about what's built into our various full conceptions, or FCs--that are relevant to determining which mathematical sentences are correct.

5. Logical/modal facts

The IBF-fictionalist view of mathematical facts relies on the thesis that there are logical facts, or modal facts. But it's not obvious that

fictionalists can accept this thesis. For starters, one might think that logical facts are best understood platonistically, i.e., as involving abstract objects. Now, I think it's pretty obvious that there are at least some cases in which this is right. Consider, for instance, the following sentence:

(PropEntail) The proposition that there are flying, talking donkeys entails the proposition that there are flying donkeys.

I think it can be argued pretty easily that sentences like this are best interpreted platonistically, as referring (or at least purporting to refer) to abstract objects, in particular, propositions. But, of course, it doesn't follow that we should endorse a platonistic ontology here. Instead, we can endorse a fictionalistic view and say that while (PropEntail) is strictly speaking untrue because there are no such things as propositions, it is still correct in an important sense of the term because it's true in the story of propositions. In other words, fictionalists can say that (PropEntail) is such that if there existed propositions of the kinds that platonists have in mind (or the kinds that our 'that'-clauses purport to refer to), then it would be true.

But, of course, fictionalists can't leave it at this. For as we've already seen, they're committed to the view that there are real logical/modal facts "lurking behind" fictionalistically correct platonistic sentences like '3 is prime', and the axioms of arithmetic, and (PropEntail). For instance, in connection with the axioms of arithmetic, we found in section 4 that fictionalists are committed to Facts (i) and (iii). And it's pretty easy to see that they will be committed to similar sorts of facts in connection with (PropEntail). Thus, fictionalists have to provide an account of facts like these that takes them to be nominalistically kosher and, more generally, metaphysically non-spooky. To see how we can do this, let's focus on some simpler instances of these kinds of facts. To this end, consider the following two sentences:

(Necessary) If there had been flying, talking donkeys, then there would have been flying donkeys.

(Possible) There could have been flying donkeys.

Prima facie, these two sentences don't seem to be about abstract objects. Indeed, at first glance, they seem not to have any ontological commitments at all, and so, *prima facie*, it seems that we might be able to make out a nominalistic, non-spooky view of these sentences. But, of course, there's a worry one might have here; one might think that modal sentences like (Necessary) and (Possible) are best interpreted as claims about possible worlds; e.g., one might think (Possible) is equivalent to

(Possible World) There is at least one possible world in which there are flying donkeys.

But possible worlds are either abstract objects of some kind (e.g., properties or sets of propositions) or they're Lewisian concrete objects; either way, they won't be acceptable to fictionalists, who, we're assuming here, reject all spooky objects.

I think there are good reasons to reject the (PossibleWorld) analysis of (Possible). For (a) read at face value, (Possible) doesn't seem to involve any reference to possible worlds; and (b) our intuitions seem to line up with this face-value reading. To appreciate point (b), imagine a scenario in which there are no such things as non-actual possible worlds and ask yourself whether, intuitively speaking, (Possible) is true in this scenario. My intuition is that it clearly is. So (Possible) is different from mathematical sentences like '3 is prime.' Intuitively, it seems that if there is no such thing as the number 3, then '3 is prime' isn't literally true. But (Possible) is different; intuitively, it seems that even if there are no such things as non-actual possible worlds, (Possible) is still true. And if this is right, then (Possible) is simply not equivalent to (PossibleWorld). In particular, the real existence of non-actual possible worlds is not needed for the literal truth of (Possible).

Let me make four quick points about this argument before going on. First, the argument works equally well whether we think of

possible worlds as abstract objects or Lewisian concrete objects (and a similar argument works against the Rosen-style view that sentences like (Possible) are about a (really existing) possible-worlds fiction). Second, I am aware that one might respond to this argument by claiming that possible worlds exist necessarily; I won't discuss this right now, but I will respond to it below. Third, this probably goes without saying, but I'll say it anyway: even though I think the possible-worlds analysis of ordinary modal claims is false, that doesn't mean I think the possible-worlds apparatus is useless, and in fact I don't. Fourth and finally, I don't think fictionalists *need* the result that the possible-worlds analysis of ordinary modal claims is false, and at the end of the paper, I'll explain how they can accept that analysis without committing to the existence of non-actual possible worlds. For now, though, I want to proceed with the assumption that the possible-worlds analysis is false, and I want to develop an alternative view that fictionalists can endorse instead.

The view I've got in mind begins (but certainly doesn't end) with the well-known idea that we can take possibility to be a primitiv notion.[148] Thus, e.g., instead of taking (Possible) to be equivalent to (Possible World), we can take it to be equivalent to

(Possible Primitive) Possibly*, there are flying donkeys (where 'possibly*' is--I hereby stipulate--a primitive sentential possi-bility operator).

On this view, expressions like 'possibly' and 'could have' function as sentential operators, and (Possible) is true not because there is *another* world in which there *are* flying donkeys, but rather, because in *this* world, there *could have been* flying donkeys.

But fictionalists can't leave it at this. They need to say what kind of *facts* are behind primitive possibility claims like (Possible).

[148] This idea has been defended by numerous people, including Kreisel (1967), Prior (1968), Fine (1977), Field (1989), Hellman (1989), Chihara (1990), and myself (1998a).

They think it's a fact that there could have been flying donkeys. But what sort of fact is this? Or to put the question differently, what are the *truth-makers* of modal claims like (Possible) and (Necessary)? Possible-worlds realists have a story to tell here: the facts in question are facts about possible worlds. But what can fictionalists say about this? More precisely, how can they endorse a realist view of modal sentences like (Necessary) and (Possible) without committing themselves to anything metaphysically spooky, like abstract objects or Lewisian non-actual possible worlds?

One thing fictionalists might try to say here is that (Necessary) and (Possible) are analytic, or conceptually true, or something along these lines. I think this is probably right--that is, I think there's probably an important sense in which (Necessary) and (Possible) are analytic--but I don't think fictionalists can use this to solve the problem of finding truth-makers for (Necessary) and (Possible). You might have thought we could use an appeal to analyticity to argue that (Necessary) and (Possible) don't have any substantive truth-makers--because they're true in virtue of meaning, or some such thing. But I don't think this will work. I'm actually partial to the idea that (Necessary) and (Possible) might not have truth-makers, and I'll discuss this below, but I don't see how an appeal to analyticity can help justify this claim. To see why I say this, notice first that the relevant kind of analyticity is probably going to have to be defined in terms of something like logical consequence, or *following from*; e.g., one might say that (Necessary) is analytic because it follows from the meanings of 'and' and 'not' (or from a statement of what these words mean) that contradictions can't be true, and hence that it can't be that there both are and aren't flying donkeys, and hence that (Necessary) is true. But if fictionalists say this, they'll be left with the job of finding truth-makers for these claims about what follows from the meanings of 'and' and 'not', and so they won't have made very much progress.

I suppose you might also have thought that we could use an appeal to analyticity to help us find truth-makers for sentences like (Possible) and (Necessary). But the only remotely plausible way

of doing this that I can think of is to endorse some version of the idea that the truth-makers of analytic sentences are facts about meanings (or concepts, or properties, or something along these lines), and this view is not available to fictionalists. Consider, for instance, the sentence

(Analytic) All bachelors are unmarried.

Platonists might claim that the truth-maker of this sentence is the fact that the first component of the proposition expressed by (Analytic)--i.e., the concept *bachelor*, or the meaning of 'bachelor', or the property of being a bachelor, or whatever platonists of this kind want to plug in here--contains (or some such thing) the second component of the proposition, i.e., the concept *unmarried* (or the meaning of 'unmarried', or whatever). But views like this are not available to fictionalists because the objects in question (concepts, meanings, etc.) are abstract objects. Now, I suppose you might try to get around this point by claiming that (a) concepts are mental objects existing in our heads, and facts about these mental objects are the truth-makers of analytic sentences, or (b) the truth-makers of analytic sentences are facts about our conventions, or what we mean by our words, or some such thing, or (c) the truth-maker of (Analytic) is the fact that if there *were* concepts, then the concept *bachelor* would contain the concept *unmarried*. But claim (c) is no help, for if fictionalists say this, they'll be right back where they started--with a modal fact that looks just like the one expressed by (Necessary). And claims (a) and (b) are simply untenable.[149]

[149] I don't need to argue here that (a) and (b) are untenable because they would only help me. But for whatever it's worth, they both seem implausible to me. In connection with (a), there are well-known reasons for thinking that what's in my head, vis-à-vis the concept *bachelor*, is best thought of as a *representation* of that concept, not the concept itself. And in connection with (b), there are good reasons for thinking that facts about our conventions, or about how we use our words, can only make it the case that the sentence 'All bachelors are unmarried' means in English that all bachelors are unmarried. They can't also make it the case that it's *true* that all bachelors are unmarried. Indeed, the fact that all bachelors are

(Aside from the fact that the view in question here--i.e., the view that sentences like (Analytic) and (Possible) and (Necessary) are made true by facts about meanings, or concepts--is unavailable to fictionalists, I also think there are some problems with this view. Some of the problems are analogous to problems with the view that modal claims are made true by facts about possible worlds. I'll say a few words about this below, but I won't discuss it in much detail because it goes beyond the scope of this paper. My aim here is simply to find a plausible view of modal facts that fictionalists can endorse; I'm not trying to refute all the alternatives.)

Anyhow, so much for the appeal to analyticity. A second view that fictionalists might endorse--vis-à-vis the problem of finding truth-makers for (Necessary) and (Possible)--is that these sentences are about the actual world (see, e.g., Fine 1977, 2003). (Or to put the idea a bit more precisely, the view I've got in mind is that modal claims like (Possible) are about the *concrete part* of the actual world; for ersatz-possible-worlds realists can say that (Possible) is about the actual world, but this view is obviously not available to fictionalists.) In any event, the claim that (Necessary) and (Possible) are "about the (actual) world" can be understood in numerous ways, and on some ways of understanding this claim, I have no problem with it. What seems wrong to me, however, or probably wrong, is the following:

AW-as-TM: The (concrete part of the) actual world is the truth-maker of modal sentences like (Necessary) and (Possible).

The reason I think this view is probably false is very simple: I just can't think of any features of the (concrete part of the) actual world that could plausibly be taken to be the truth-makers of these sentences. That's not a very powerful argument, I know, so I might be wrong. And, in fact, I would *welcome* this result--for if there were a feature of the (concrete part of the) actual world that

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unmarried is completely independent of facts about how we use our words.

was the truth-maker of (Possible), then we could presumably be non-spooky realists about ordinary modal claims like (Possible) and (Necessary), and fictionalists would be done. But, again, unless someone can tell me *which* features of the world could be doing the truth-making here, I am doubtful. But what I want to argue now is that fictionalists don't need to find truth-makers for sentences like (Possible). For I think it can be argued that *even if sentences like (Possible) didn't have any truth-makers, they would still be true.*

To appreciate this point, consider the following argument (I intend this argument to be understood in the tradition of intuitions-about-imagined-scenario arguments, although I admit, it's a somewhat odd instance of that argument type): Imagine the scenario in which there is no actual world. Indeed, imagine that there is *nothing*--no actual world, no non-actual possible worlds, no propositions or sentences or abstract objects of any kind. Now ask yourself whether (Possible) is true with respect to this completely empty scenario. I have a clear intuition that it is; in other words, even if there were literally *nothing*, it would still be the case that there could have been flying donkeys. And this suggests that even if (Possible) didn't have a truth-maker, it would still be true.

Let me make two points about this argument. First, one might try to respond to it by claiming that the actual world exists necessarily. But this seems really implausible. The world clearly doesn't exist of conceptual necessity, so the claim would have to be that it exists of metaphysical necessity. But what does this even *mean*? I think we can make sense of the idea that sentences like the following are metaphysically necessary: 'Muhammad Ali is Cassius Clay', 'Water is H_2O', 'This very desk is made of wood'. One way to proceed here is to claim that what it means to say that these sentences are metaphysically necessary is that any world in which Ali exists is a world in which he's Clay; and any world in which there's water is a world in which it's H_2O; and any world in which this very desk exists is a world in which it's made of wood. But, of course, this can't be what's behind the metaphysical necessity of existence claims like 'The actual world exists', and 'Non-actual possible worlds exist', and 'Abstract objects exist'. And as far as I know, no one has ever said what it could really amount to to say that existence claims like this are metaphysically necessary. (For more on this, see Field 1989 and my 1998a.) Also, even if we ignore the issue of what it could really mean to assert that there are metaphysically necessary existence claims, the idea that this assertion is *true* is unmotivated and implausible. There are clearly conceptually possible scenarios in which there are no Gods, no abstract objects, no non-actual possible worlds, no actual world, and so on. Why should we think these scenarios are "metaphysically impossible"? Intuitively, they just seem plain-old possible.

Second, you might think the above empty-scenario argument shows that (Possible) *doesn't have* a truth-maker. But, in fact, it doesn't show that. For even if (Possible) would remain true if there were nothing, it doesn't follow that it doesn't have a truth-maker in the actual world. Consider the sentence, 'Snow is white or it's not the case that snow is white.' This would remain true if there were nothing, but despite this, it seems to have a truth-maker in the actual world, namely, the fact that snow is white.[150]

Given this, I want to allow that we *might* someday find truth-makers (in the concrete part of the actual world) for modal claims like (Necessary) and (Possible). If we did, this would of course be a welcome result for fictionalists. But I seriously doubt that this is going to happen. It seems much more plausible to me that these sentences just don't have truth-makers. But I still think they're true. And so I want to propose the following view:

No-TM: Sentences like (Possible) and (Necessary) are substantively true, but they don't have any truth-makers.

Let me try to clarify this view. To begin with, when I say that according to No-TM, (Possible) is *substantively* true, I mean at least three things. First, it isn't vacuously true, or true in an empty way,

[150] I thank Steve Yablo for making this point to me.

along the lines of, say, 'All unicorns are purple.' Second, according to No-TM, (Possible) *says something*; in other words, to put the point how platonists would put it, (Possible) expresses a proposition; but fictionalists will want to capture this idea by saying that (Possible) is such that if there were propositions of the kinds that platonists have in mind, then it would express one of them--namely, the proposition that there could have been flying donkeys. So on the view I've got in mind, (Possible) says something in the same way that, e.g., 'There are donkeys' says something. (And notice that on this way of conceptualizing things, we get the happy result that strings like 'He the for' *don't* say anything; for even if there were propositions, such strings wouldn't express any of them.) So No-TM isn't an expressivist view, or a quasi-realist view, or anything else along those lines; it's a *cognitivist* view. Third and most important, on the view I've got in mind, (Possible) is *objectively* true. There are a few different ways to say what this means. One rather sloppy way is to say that it's true independently of us (and our thinking, and our conventions, and so on) that there could have been flying donkeys. A more careful way is to say that tokens of (Possible) are such that if there had been propositions, then these tokens would have expressed a proposition (in particular, the proposition that there could have been flying donkeys) that was true independently of us and our conventions and so on.[151]

Next, when I say that according to No-TM, (Possible) doesn't have a truth-maker, I'm not saying that on this view, (Possible) isn't *about* anything. For if they like, No-TM-ists can say that there are

[151] One might think it an uncomfortable result that fictionalists have to say that since in fact there are no such things as propositions, it follows that in fact (Possible) doesn't say anything. But this is really just a special case of the Quine-Putnam indispensability worry. We use proposition talk in doing empirical semantics (e.g., to say how one string says something whereas another does not) in the same way that we use number talk in doing physics. I think fictionalists can explain why this is good scientific practice--i.e., why we can successfully characterize the physical world using abstract-object talk--even if there are no such things as abstract objects. I can't discuss this here, but see my 1998a and 1998b.

various senses in which (Possible) is "about the world", or in which it's a fact about the world that it could have contained flying donkeys. When I say that according to No-TM, (Possible) doesn't have a truth-maker--and, in particular, that the actual world isn't its truth-maker--what I mean is that (according to this view) there's nothing about the way the world actually is that makes it the case that there could have been flying donkeys. Or perhaps better: the actual world doesn't have any substantive intrinsic features that make it the case that there could have been flying donkeys. There's no *stuff* that makes (Possible) true, and there are no features of any existent things that make it true. Finally, I should note that on the view I've got in mind, the no truth-maker claim isn't just a claim about the reification of facts, or anything else along those lines. It's much deeper than that. The idea is that sentences like (Necessary) and (Possible) are true, but reality doesn't make them true. Or better: *Being* doesn't make them true.

This might seem intuitively weird, and, indeed, I think this is the most important problem with the view--it just seems bizarre to claim that a sentence could be substantively true without anything making it true. Let's call this the incredulous-stare problem. I have two responses to it, one serious and the other not so serious. The serious response is this: I admit that, at first blush, No-TM feels intuitively weird; but I think this feeling is generated by an overgeneralization, by moving too quickly from (a) our ordinary thinking about truths about what's actually the case to (b) a conclusion about truth in general. It seems right that ordinary claims about what's actually the case can't be substantively true without something in the world making them true. But (Possible) is not a claim about what's actually the case; it's not a claim about Being. It's a claim about what *could have been*, or about what Being could have been like, or some such thing. And it does not seem bizarre to me to suppose that a claim about what *could have been* could be true without any existent stuff making it true. Because such claims aren't about existent stuff. (You might respond that (Possible) *is* a claim about what's actually the case, because it tells us that it's actually the case that there could have been flying donkeys. Fine--I won't quibble about this. If this is

right, then my point is that claims like this can be true without truth-makers because what they're *ultimately* making claims about (or something like that) is what could have been.)

The less serious response to the incredulous-stare problem is this: Even if my remarks in the preceding paragraph don't completely eliminate the intuitive weirdness of No-TM, that view is still attractive, it seems to me, for the simple reason that it's the least weird view out there. The only alternatives to No-TM are the following: (I) anti-realism (it's not genuinely or substantively true that there could have been flying donkeys); (II) Lewisianism (the truth-makers of modal claims are facts about concrete non-actual possible worlds); (III) platonism (the truth-makers of modal claims are facts about abstract objects, e.g., ersatz possible worlds, or concepts, or a possible-worlds fiction); and (IV) AW-as-TM (the truth-makers of modal claims are facts about the concrete part of the actual world). If these are the only alternatives to No-TM--and there don't seem to be any other options--then it seems to me that no one is getting out of here without inspiring an incredulous stare.

(Views (II) and (III) seem *doubly* weird to me. For aside from the fact that they commit to the existence of spooky objects, it's hard to see why we should go along with their claims about the truth-makers of modal sentences like (Possible). To see why I say this, compare (Possible) to the following:

(Possible World) There is at least one possible world in which there are flying donkeys.

(Possible Fiction) According to the fiction of possible worlds, there is at least one possible world in which there are flying donkeys.

(Concept) The concept *flying* and the concept *donkey* are compatible.

These sentences seem to be straightforwardly *about* possible worlds, fictions, and concepts; and if there were no such things as possible worlds, fictions, or concepts, these sentences would not be true. But (Possible) doesn't seem to be making a claim about objects of any of these kinds in any straightforward way,[152] and moreover, it would still be true even if these objects didn't exist. And this, I think, raises a *prima facie* worry about views (II) and (III). This isn't a refutation of these views, I know--one might maintain that facts about worlds, fictions, or concepts are the truth-makers of sentences like (Possible) even though these sentences would remain true if these objects didn't exist--but this does seem to me to bring out a certain weirdness in these views.)

There's another worry about No-TM: It seems not to explain why some modal claims are true and others are false. E.g., (Possible) is true and 'There could have been round squares' is not. Why? If there's nothing out there in the world that makes sentences like (Necessary) and (Possible) true, does that mean they're all *brutely* true, according to No-TM? No. For the truth of these sentences can be explained by appealing to more basic, lawlike modal/logical truths. To bring this point out, let me focus on the case of (Necessary). We might try to explain why this sentence is true by saying something like this:

(1) if (Necessary) were false, then it could be that there both are and aren't flying donkeys;

but

(2) in point of fact, it couldn't be that there both are and aren't flying donkeys,

Because

(3) contradictions, i.e., sentence tokens of the form 'A and not A', can't be true.

[152] This point is pretty obvious in connection with fictions and concepts; it might seem less obvious in connection with possible worlds, but I think that's just because we've been trained to think of modal claims in this way. To non-philosophers, I think the idea that (Possible) is making a substantive claim about the real existence of another possible world, different from this world, is pretty counterintuitive.

One might think that claim (3) is a bottom-level modal truth, or a logical law, or something along these lines, and so one might claim that it's brutely true. Or perhaps No-TM-ists will want to slightly alter their formulation of the bottom-level truth here. Perhaps they will prefer something along the lines of one of the following:

(4) If we assume an ordinary, classical interpretation of 'and' and 'not', then sentence tokens of the form 'A and not A' can't be true; (or: it follows from the meanings of 'and' and 'not' that contradictions aren't true).

(5) The world couldn't be set up so that a sentence of the form 'A and not A' was true.

(6) The world couldn't both be and not be F--i.e., $\sim\Diamond(Fw \;\&\; \sim Fw)$, where 'w' denotes the (actual) world and 'F' is any one-place predicate.

However No-TM-ists decide to formulate things here, it seems to me that they can plausibly maintain that some sentence along the lines of (3)-(6) is brutely true, or expresses a brute fact, or some such thing. (I am assuming here for the sake of simplicity that the correct logic for English is one that upholds the law of non-contradiction; if some other kind of logic were correct, I wouldn't need to change anything important; I would simply need to change what the bottom-level logical/modal facts are.)

If this appeal to brute facts seems unsatisfying, ask yourself whether possible-worlds realists gain anything here, in terms of genuine explanatory power, by positing the real existence of possible worlds. They think the reason (Possible) is true is that there exists a possible world in which there are flying donkeys; and they think the reason (Necessary) is true is that there's no possible world in which there both are and aren't flying donkeys. But suppose we ask them this: In virtue of what are there no possible worlds of the kind relevant to (Necessary)? Or more generally: In virtue of what are there no possible worlds in which contradictions are true? It seems to me that possible-worlds realists are going to have to say that this is a brute fact.

So here's the situation: (i) No-TM-ists claim that it's a brute fact that contradictions can't be true, or that the world can't both be and not be F; and (ii) possible-worlds realists claim it's a brute fact that there are no possible worlds that both are and aren't F. Have possible-worlds realists gained anything substantive here by positing the extra ontology of possible worlds? It seems to me that they haven't. Why not just say that there are some brute impossibility facts and be done with it?

Indeed, while I don't need this point here, one might argue that the brute facts I'm positing are more basic than the brute facts that possible-worlds realists need to posit. If one were to ask, "Why are there no possible worlds in which there both are and aren't flying donkeys?," it would not seem odd to me to reply: "Because that's impossible." Does this obviously get the explanatory order backwards? It doesn't seem to. Indeed, to me, intuitively, it seems to get the order just right. If someone were to ask, "Why is it impossible for there to both be and not be flying donkeys?," it would seem odd to me to respond: "Because there is no possible world in which there both are and aren't flying donkeys." This doesn't strike me as explanatory at all, and, if anything, it seems to get the explanatory order backwards. It seems to me that the possibility and impossibility facts we're discussing here would hold even if there were no such things as non-actual possible worlds. Moreover, if someone were to ask, "Why can't contradictions be true? Why can't the world both be and not be F?," the right answer wouldn't be "Because there are no possible worlds that both are and aren't F." The right answer, it seems to me, would be: "The world just *can't* both be and not be F. Period."

So it seems to me that No-TM is defensible. But my official stance here is not that No-TM is true. My official stance is as follows: (a) fictionalists *might* be able to find truth-makers (in the concrete part of the actual world) for modal claims like (Possible), and if they do, then they can endorse AW-as-TM; but (b) it seems unlikely to me that this is going to happen; it seems much more plausible that modal sentences like (Possible) just don't have truth-makers; but even if they don't, it's OK because these sentences would still be substantively true and so, in this scenario, No-TM would be true, and so fictionalists could endorse that view.

The last point I want to make here is this: While I think there are good reasons for endorsing the primitive-term analysis of ordinary modal claims, fictionalists don't actually need this result. If they wanted to, they could endorse the possible-worlds analysis of such claims, because they could say something like this:

Ordinary modal claims like (Possible) are strictly speaking untrue because they make hidden references to non-actual possible worlds, and there are no such things. But these sentences are still correct in an important sense of the term (in particular, they're fictionalistically correct), because they're true in the story of possible worlds--i.e., they're such that if there were possible worlds of the kinds that possible-worlds realists have in mind, then they would be true. Now, the real facts behind these sentences--the facts that make them fictionalistically correct instead of incorrect--decompose (at least partially) into non-spooky modal/logical facts. These underlying facts are not expressible in ordinary English, but they *are* expressible in English*, where English* is just like English except that it includes a primitive possibility operator, 'possible*' (or if you'd rather, 'necessary*'). For instance, in connection with (Possible), the underlying fact can be expressed by the following sentence of English*: "Necessarily*, if there are possible worlds of the kinds that possible-worlds realists have in mind, i.e., the kinds that our ordinary modal sentences purport to be about, then there's a world in which there are flying donkeys." Now, of course, this doesn't solve the problem of truth-makers, but we can presumably solve that problem by endorsing an analogue of AW-as-TM or No-TM with respect to the underlying modal truths (or if you'd rather, the underlying modal* truths).

How might one respond to this stance? Well, the only response I can think of is based on the idea that the primitive term 'possible*' (or 'necessary*') doesn't make sense. One might argue the point like this:

There are two kinds of primitive terms--those that have intuitive, pretheoretic meanings and those that don't. If a primitive doesn't have an intuitive, pretheoretic meaning, then it's uninterpreted or, at best, contextually defined by something like a set of axioms. This doesn't seem to be what fictionalists have in mind in the present context, so they must think we have an intuitive, pretheoretic understanding of 'possible*'. But if they say that the ordinary term 'possible' is best understood in terms of possible worlds, and not as a primitive, then how can they claim that we have an intuitive understanding of the primitive term 'possible*'?

In response to this, I simply want to deny the idea that if the English term 'possible' isn't best understood as a primitive, then we don't have an intuitive understanding of 'possible*'. I think that, right now, I understand claims about possible worlds *and* primitive possibility claims. Which of these best capture the meanings of folk possibility claims is an empirical question. But if the possible-worlds analysis turned out to be right, that wouldn't change the fact that, right now, I understand primitive possibility claims.

Now, I suppose you might try to deny my claim that, right now, I understand primitive possibility claims. But given that 'possible*' is a primitive, I don't see how you could motivate this. You couldn't demand a definition of 'possible*' from me. About the only thing you could say, it seems to me, is that *you* don't understand 'possible*'--or that you don't see how it's supposed to be used, or some such thing. But from this it follows only that *you* can't be a fictionalist of the present kind. The rest of us still can.

In any event, I frankly doubt that there's anyone out there who understands the issues here but doesn't understand how to use 'possible*'. Indeed, I think that just about everyone in the philosophy-of-mathematics debate is committed to something like a primitive notion of possibility. In the first place, just about all of the mainstream versions of nominalism in the literature are explicitly committed to such a notion; this includes not just fictionalism (see, e.g., Field 1989 and my 1998a), but other kinds of nominalism as well (see, e.g., Hellman 1989 and Chihara 1990). But more importantly, and perhaps a bit surprisingly, I think it can be argued that mathematical platonists are committed to a primitive notion of possibility as well. I can't argue this point here, but in brief, the idea is this: (i) in order to develop an acceptable epistemology of abstract objects, platonists need to claim that human beings can acquire knowledge of certain kinds of possibility facts without the aid of any epistemic contact with abstract objects or non-actual possible worlds, and so (ii) platonists cannot claim that all possibility facts are ultimately about abstract objects or non-actual possible worlds (see my 1998a, chapter 3 for more on this).

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