California State University – Los Angeles
Mathematics
Masters Degree Comprehensive Examination

Complex Analysis Fall 2019
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Do five of the following seven problems.
If you attempt more than 5, the best 5 will be used.

Please
(1) Write in a fairly soft pencil (number 2) (or in ink if you wish) so that your work will duplicate well. There should be a supply available.
(2) Write on one side of the paper only.
(3) Begin each problem on a new page.
(4) Assemble the problems you hand in in numerical order.

Exams are being graded anonymously, so put your name only where directed and follow any instructions concerning identification code numbers.

Notation: **C** denotes the set of complex numbers.
R denotes the set of real numbers.
Re(z) denotes the real part of the complex number z.
Im(z) denotes the imaginary part of the complex number z.
$|z|$ denotes the absolute value of the complex number z.
Log z denotes the principal branch of log z.
Arg z denotes the principal branch of arg z.
$D(z; r)$ denotes the open disk with center z and radius r.
A *domain* is an open connected subset of **C**.

Miscellaneous facts

$2\sin a \sin b = \cos(a - b) - \cos(a + b)$
$2\sin a \cos b = \sin(a + b) + \sin(a - b)$
$\sin(a + b) = \sin a \cos b + \cos a \sin b$
$2\cos a \cos b = \cos(a - b) + \cos(a + b)$
$2\cos a \sin b = \sin(a + b) - \sin(a - b)$
$\cos(a + b) = \cos a \cos b - \sin a \sin b$

Fall 2019 # 1. Let $R = \{z : 0 < \text{Re}(z) < 1,\ -\pi/2 \le \text{Im}(z) \le \pi/2\}$, and let $f(z) = e^{2z}$.
a. Draw R and $f(R) = \{w: w = f(z), z \in R\}$.
b. Determine if each R and $f(R)$ is open, connected, and/or simply connected. Explain.

Fall 2019 # 2. Evaluate $\displaystyle\int_C \frac{dz}{z^2 + 2z - 8}$, where

a. C is the circle $\{z: |z| = 1\}$ traversed in the counterclockwise direction.

b. C is the circle $\{z: |z| = 3\}$ traversed in the counterclockwise direction.

c. C is the circle $\{z: |z| = 5\}$ traversed in the counterclockwise direction.

Fall 2019 # 3. **a**. Find the residue of the function $f(z) = \dfrac{1}{z^4 \sin z}$ at $z_0 = 0$.

b. Find the residue of the function $f(z) = \dfrac{e^{1/z}}{z+1}$ at $z_0 = 0$.

Fall 2019 # 4. **a**. Evaluate the integral $\displaystyle\int_{-\pi}^{\pi} \frac{d\theta}{2-\cos\theta}$.

b. Evaluate the integral $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{x^4+1}dx$. Show any contour and estimates needed to justify your method.

Fall 2019 # 5. **a**. Let $f(z) = \dfrac{z-i}{z+i}$. What is the image under f of the circle with center 0 and radius 1?

b. Find a conformal map $f(z)$ which takes the disk $\{z : |z| < 3\}$ onto the left half plane $\{w : \text{Re}(w) < 0\}$ and satisfies $f(3) = 0$.

Fall 2019 # 6. Suppose u, v, U, and V are harmonic functions, such that, v is a harmonic conjugate of u, and V is a harmonic conjugate of U. Show that $uV + vU$ is harmonic, and find its harmonic conjugate.

Fall 2019 # 7. Suppose f is analytic on some domain (open and connected) which contains a segment of the real axis and whose lower half is the reflection of the upper half with respect to that axis. Prove that $\text{Re}(f(x)) = 0$ for each point x on the segment if and only if $\overline{f(z)} = -f(\bar{z})$.

End of Exam