California State University – Los Angeles
Mathematics
Masters Degree Comprehensive Examination

Complex Analysis Spring 2017

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Do five of the following seven problems.
If you attempt more than 5, the best 5 will be used.
Please

(1) Please write in a fairly soft pencil (number 2) (or in ink if you wish) so that your work will duplicate well. There should be a supply available.
(2) Write on one side of the paper only
(3) Begin each problem on a new page
(4) Assemble the problems you hand in in numerical order

Exams are being graded anonymously, so put your name only where directed and follow any instructions concerning identification code numbers.

Notation: $\mathbb{C}$ denotes the set of complex numbers.
$\mathbb{R}$ denotes the set of real numbers.
$\mathrm{Re}(z)$ denotes the real part of the complex number z.
$\mathrm{Im}(z)$ denotes the imaginary part of the complex number z.
$\overline{z}$ denotes the complex conjugate of the complex number z.
$|z|$ denotes the absolute value of the complex number z.
$\mathrm{Log}\, z$ denotes the principal branch of $\log z$.
$\mathrm{Arg}\, z$ denotes the principal branch of $\arg z$.
$D(z;r)$ is the open disk with center z and radius r.
A *domain* is an open connected subset of $\mathbb{C}$.

MISCELLANEOUS FACTS

$$2\sin a \sin b = \cos(a-b) - \cos(a+b) \qquad 2\cos a \cos b = \cos(a-b) + \cos(a+b)$$

$$2\sin a \cos b = \sin(a+b) + \sin(a-b) \qquad 2\cos a \sin b = \sin(a+b) - \sin(a-b)$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b \qquad \cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\sin^2 a = \frac{1}{2} - \frac{1}{2}\cos(2a) \qquad \cos^2 a = \frac{1}{2} + \frac{1}{2}\cos(2a)$$

Spring 2017 # 1. Let S be the infinite strip $\{z \in \mathbb{C} : 0 \le \operatorname{Im} z \le \pi/3\}$ and $f(z) = e^z$. Find and sketch the image set $f(S)$.

Spring 2017 # 2. Let $B = \{z \in \mathbb{C} : |\, z - i\,| < 1\}$ and $f(z) = 2/z$. Find and sketch the image set $f(B)$.

Spring 2017 # 3. Evaluate the integral $\displaystyle\int_\gamma \frac{e^z}{(z-2)(z+4)}\,dz$ around each of the following curves. Give reasons for your answers.

a. The circle of radius 1 centered at 0 travelled once counterclockwise.
b. The circle of radius 3 centered at 0 travelled once counterclockwise.
c. The circle of radius 5 centered at 0 travelled once counterclockwise.
d. The path following straight line segments from $5-i$ to $5+i$ to $-5-i$ to $-5+i$ and returning to $5-i$.

Spring 2017 # 4. For $R > 0$, let γ_R be the square composed of straight line segments from $R+Ri$ to $-R+Ri$ to $-R-Ri$ to $R-Ri$ and returning to $R+Ri$. Suppose $f : \mathbb{C} \to \mathbb{C}$ is analytic on $\mathbb{C}$ and that for each R, $|\, f(z)\,| \le R$ for all z on γ_R.

a. (14 pts) Show that there are constants a and b such that $f(z) = az + b$ for all z in $\mathbb{C}$.

b. (6 pts) What are a and b in terms of f?

(Suggestion: What can you do with a Taylor series?)

Spring 2017 # 5. How many solutions, counting possible multiplicity, are there to the equation $e^z = z^3$ in the disk $B = \{z \in \mathbb{C} : |\, z\,| < 3\}$? (Recall that $e \approx 2.71828 < 3$.)

Spring 2017 # 6. Evaluate each of the following integrals. Show any contours and discuss any estimates needed to justify your method.

$$\textbf{a.}\quad \int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2}\,dx \qquad \textbf{b.}\quad \int_0^{2\pi} \frac{1}{8 - 2\sin\theta}\,d\theta$$

Spring 2016 # 7. Find the Laurent series expansions for $f(z) = \dfrac{1}{z^2(z^2-9)}$ around $z_o = 0$ valid in each of the following regions

a. $A = \{z \in \mathbb{C} : 0 < |\, z\,| < 3\}$ **b.** $B = \{z \in \mathbb{C} : |\, z\,| > 3\}$

End of Exam