Erratum to: Fractional chromatic number and circular chromatic number for distance graphs with large clique size

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There is a subtle error within a proof in our paper [1]. In the 16th line on page 136 (middle of Case 1 in the proof of Theorem 3.1 [1]), we "let $i_2 = i + 2b - a$" and claimed that "$i_2 \in U$." This is not always true, as it might be that $i + 2b - a < a + c$, resulting in $i_2 \notin U$.

To mend this, we give below an argument to replace the part on page 136, starting from the 9th line till the end of Case 1. The rest of the original proof remains the same.

Let $m \geq 0$ be the smallest integer such that $S \cap T_{i-a+(m-1)(b-a)} = \emptyset$ or $i+b+m(b-a) \geq a+c$. If $S \cap T_{i-a+(m-1)(b-a)} = \emptyset$, let $i_2 = i-a+(m-1)(b-a)$. Otherwise, let $i_2 = i+b+m(b-a)$. If it is the former case, then $m \geq 1$ and $i_2 \in I$ (since $i+b+(m-1)(b-a) < a+c$, so $i_2 = i-a+(m-1)(b-a) < c-b = a$).

We now show that if it is the latter case, then $i_2 \in U$. Assume $i_2 = i+b+m(b-a)$. Then $i+b+(m-1)(b-a) < a+c \leq i+b+m(b-a)$, which implies

$$a + c \leq i + b + m(b-a) = i_2 < b + c. \qquad (*)$$

We claim that for $0 \leq m' \leq m$,

$$i + m'(b-a) \in S. \tag{**}$$

If $m' = 0$, then (**) follows from our assumption that $i \in T$. Assume $0 \leq m' \leq m-1$ and $i + m'(b-a) \in S$. Then $i - a + m'(b-a) \notin S$ and $i + m'(b-a) + b = i - a + m'(b-a) + c \notin S$. Consider $T_{i-a+m'(b-a)} = \{i - a + m'(b-a), i - a + m'(b-a) + b, i - a + m'(b-a) + c\}$. Note, $T_{i-a+m'(b-a)}$ is well-defined since, by definition, $i + b + m'(b-a) < a + c$, so $i - a + m'(b-a) < a$. By the definition of i_2, we have $T_{i-a+m'(b-a)} \cap S \neq \emptyset$. Hence, it must be that $i - a + m'(b-a) + b = i + (m'+1)(b-a) \in S$, so (**) holds. In particular, we have $i + m(b-a) \in S$, which implies that $i_2 = i + b + m(b-a) \notin S$. Combining this with (*), we have $i_2 \in U$.

It remains to show that for any $i \in T$, $i_1 \neq i_2$, and for any $i \neq j \in T$, $\{i_1, i_2\} \cap \{j_1, j_2\} = \emptyset$. Assume to the contrary that $i_p = j_q$, where $p, q \in \{1, 2\}$ and $(i,p) \neq (j,q)$. If $i_p = j_q \in I$, then by definition, $p = q = 2$, and $i_p = i - a + m(b-a) = j_q = j - a + m'(b-a)$ for some $m, m' \geq 0$. As $(i,p) \neq (j,q)$ and $p = q = 2$, we have $i \neq j$ and hence $m \neq m'$. Assume $m' > m$. Then $i = j + (m'-m)(b-a) \geq a + 1 + b - a = b + 1$ (as $j \geq a+1$), contradicting the assumption that $i \in T \subseteq \{a+1, a+2, \cdots, b-1\}$.

Assume $i_p = j_q \in U$. Then $i_p \in \{i + c, i + b + m(b-a)\}$ and $j_q \in \{j + c, j + b + m'(b-a)\}$ for some $m, m' \geq 0$. By the same argument in the above, we cannot have both $i_p = i + b + m(b-a)$ and $j_q = j + b + m'(b-a)$. Moreover, as $(i,p) \neq (j,q)$, it is impossible that $i_p = i + c = j_q = j + c$. This leaves the only possibility (by symmetry) that $i_p = i + c = i + b + a = j_q = j + b + m'(b-a)$, which implies that $i + a = j + m'(b-a)$. This is again impossible, as by (**), $j + m'(b-a) \in S$, and by the assumption that $i \in S$. ∎

References

[1] D. Liu and X. Zhu, *Fractional chromatic number and circular chromatic number for distance graphs with large clique size*, J. Graph Theory, 47 (2004), 129-146.