$\boxed{13.2}$

(2) I'll do size 9. In $\mathbb{Z}_3[x]$ we have the following;
Let $h(x) = x^3 - x + \bar{1} = x^3 + \bar{2}x + \bar{1}$.
$h(\bar{0}) = \bar{1}$, $h(\bar{1}) = \bar{4} = \bar{1}$, $h(\bar{2}) = \bar{8} + \bar{4} + \bar{1} = \overline{13} = \bar{1}$.
Since $\deg(h) \leq 3$ and h has no zeroes in $\mathbb{Z}_3$, h is irreducible in $\mathbb{Z}_3[x]$.

$$\mathbb{F}_9 = \left\{ \bar{a} + \bar{b}\theta + \bar{c}\theta^2 \;\middle|\; \begin{array}{l} \theta^3 + \bar{2}\theta + \bar{1} = \bar{0}, \quad \bar{a}, \bar{b}, \bar{c} \in \mathbb{Z}_3 \\ \theta^3 = -\bar{2}\theta - \bar{1} = \theta + \bar{2} \end{array} \right\}$$

$\bar{1} + \theta$

$(\bar{1}+\theta)^2 = \bar{1} + \bar{2}\theta + \theta^2$

$$\begin{aligned}(\bar{1}+\theta)^3 = (\bar{1}+\theta)(\bar{1}+\bar{2}\theta+\theta^2) &= \bar{1} + \bar{2}\theta + \theta^2 + \theta + 2\theta^2 + \theta^3 \\ &= \bar{1} + \theta + \bar{2} = \theta\end{aligned}$$

$(\bar{1}+\theta)^4 = (\bar{1}+\theta)\theta = \theta + \theta^2$

$$\begin{aligned}(\bar{1}+\theta)^5 = (\bar{1}+\theta)(\theta+\theta^2) = \theta + \theta^2 + \theta^2 + \theta^3 &= \theta + 2\theta^2 + \theta + \bar{2} \\ &= \bar{2} + \bar{2}\theta + 2\theta^2\end{aligned}$$

$$\begin{aligned}(1+\theta)^6 = (\bar{1}+\theta)(\bar{2}+\bar{2}\theta+2\theta^2) &= \bar{2} + 2\theta + 2\theta^2 + \bar{2}\theta + \bar{2}\theta^2 + 2\theta^3 \\ &= \bar{2} + \theta + \theta^2 + \bar{2}(\theta + \bar{2}) \\ &= \theta^2\end{aligned}$$

$(\bar{1}+\theta)^7 = (\bar{1}+\theta)\theta^2 = \theta^2 + \theta^3 = \theta^2 + \theta + \bar{2}$

$$\begin{aligned}(\bar{1}+\theta)^8 = (\bar{1}+\theta)(\bar{2}+\theta+\theta^2) &= \bar{2} + \theta + \theta^2 + \bar{2}\theta + \theta^2 + \theta^3 \\ &= \bar{2} + 2\theta^2 + \theta + 2 = \bar{1}\end{aligned}$$

So, $\langle \bar{1}+\theta \rangle = \mathbb{F}_9^{\times} = \mathbb{F}_9 \setminus \{\bar{0}\}$.

(3) $\theta = 1+i$

$(\theta-1)^2 = i^2$

$\theta^2 - 2\theta + 1 = -1$

$\theta^2 - 2\theta + 2 = 0$

Let $f(x) = x^2 - 2x + 2$.

Then f is irreducible over $\mathbb{Q}$ since the roots of f are $1 \pm i$ which are not in $\mathbb{Q}$ and f has degree 2.

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(14) If $[F(\alpha):F]$ is odd, then $F(\alpha) = F(\alpha^2)$.

Note that $F(\alpha^2) \subseteq F(\alpha)$ since $\alpha^2 \in F(\alpha)$.

So, $[F(\alpha):F] = [F(\alpha):F(\alpha^2)][F(\alpha^2):F]$.

Suppose $F(\alpha) \neq F(\alpha^2)$. $\boxed{\text{Then } \alpha \notin F(\alpha^2).}$ Then $x^2 - \alpha^2$ is irreducible over $F(\alpha^2)$ since it is degree 2 and its roots are $\pm\alpha$.

Hence $[F(\alpha):F(\alpha^2)] = 2$. But this contradicts that $[F(\alpha):F]$ is odd.

Hence $F(\alpha) = F(\alpha^2)$.