$\boxed{13.4}$

4) $f(x) = x^6 - 4$

roots of $f(x)$: $x = 4^{1/6} e^{\left(0 + \frac{2\pi k}{6}\right) i}$, $k = 0,1,2,3,4,5$

$= 2^{1/3}, 2^{1/3}\omega, 2^{1/3}\omega^2, 2^{1/3}\omega^3, 2^{1/3}\omega^4, 2^{1/3}\omega^5$

where $\omega = e^{\pi i/3} = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

So, the splitting field of f over $\mathbb{Q}$ is
$E = \mathbb{Q}(2^{1/3}, \omega)$.

Note that $\mathbb{Q}(2^{1/3}, \omega) = \mathbb{Q}(2^{1/3}, \sqrt{3} i)$

since $\sqrt{3} i = 2\left(\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) - \frac{1}{2}\right)$.

Note that $\mathbb{Q}(2^{1/3}) = \left\{ a + b(2^{1/3}) + c(2^{1/3})^2 \mid a,b,c \in \mathbb{Q} \right\}$

since $\min_{2^{1/3}, \mathbb{Q}}(x) = x^3 - 2$.

Since $\mathbb{Q}(2^{1/3}) \subseteq \mathbb{R}$, we have that $\pm\sqrt{3} i \notin \mathbb{Q}(2^{1/3})$.

Hence $\min_{\sqrt{3} i, \mathbb{Q}(2^{1/3})}(x) = x^2 + 3$.

So, $[\mathbb{Q}(2^{1/3}, \sqrt{3} i) : \mathbb{Q}] = [\mathbb{Q}(\sqrt{3} i, 2^{1/3}) : \mathbb{Q}(2^{1/3})][\mathbb{Q}(2^{1/3}) : \mathbb{Q}]$

$= 2 \cdot 3 = 6$