

8.2

(3) Let P be a prime ideal of R .

Since P is a prime ideal we know that R/P is an integral domain. So, we just need to show that ~~each~~ each ideal of R/P is principal.

Let $\bar{I} \subseteq R/P$ be an ideal. Consider

the map $\pi: R \rightarrow R/P$ where $\pi(x) = x + P$.

Let $I = \pi^{-1}(\bar{I})$. Then I is an ideal of R . Since R is a PID, $I = (z)$ where $z \in R$. ~~where~~

Claim: $\bar{I} = (z + P)$.

pf: ~~It is~~ clearly $z + P \in \bar{I}$ since $z \in I$ and ~~where~~ $z + P = \pi(z)$. Since \bar{I} is an ideal $(z + P) \subseteq \bar{I}$.

Now let $y + P \in \bar{I}$. Then $y = \pi^{-1}(y + P) \in I$.

So, $y = rz$ for some $r \in R$. Hence

$y + P = rz + P = (r + P)(z + P) \in (z + P)$.

So, $\bar{I} \subseteq (z + P)$.

