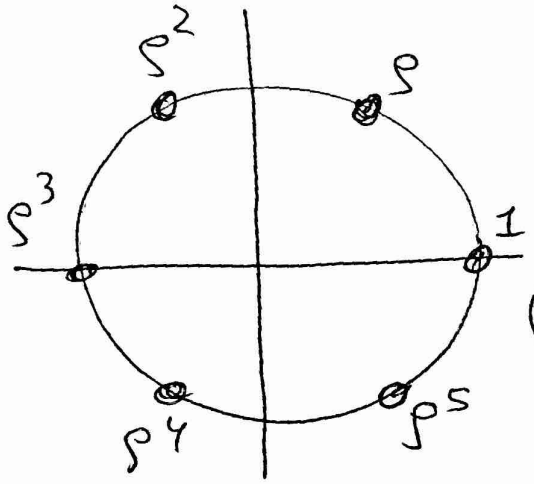
HW #1 (Better way to do #2, #3, #13)

(2)



U_6

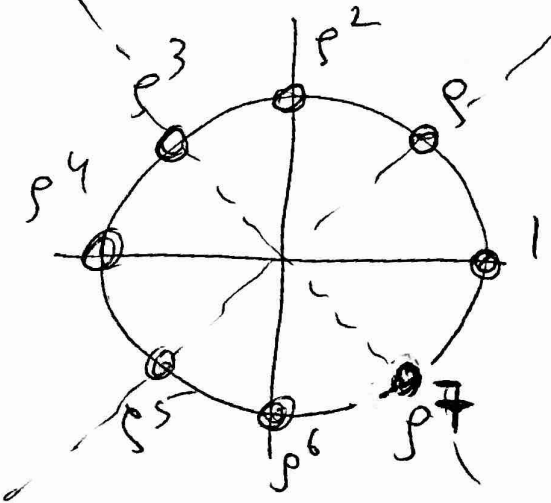
Let $\rho = e^{\frac{2\pi i}{6}} = e^{\frac{\pi i}{3}}$. Note that $\rho^6 = 1$

$U_6 = \{1, \rho, \rho^2, \rho^3, \rho^4, \rho^5\}$

Example calculation in U_6

$\rho^3 \cdot \rho^4 = \rho^7 = \rho^6 \cdot \rho = \rho$

($\rho^6 = 1$)



U_8

Let $\rho = e^{2\pi i/8} = e^{\pi i/4}$. Note $\rho^8 = 1$

$U_8 = \{1, \rho, \rho^2, \rho^3, \rho^4, \rho^5, \rho^6, \rho^7\}$

Example calculation in U_8

$\rho^5 \rho^6 = \rho^{11} = \rho^8 \rho^3 = \rho^3$

($\rho^8 = 1$)

(3) $U_4 = \{1, \rho, \rho^2, \rho^3\}$ where $\rho = e^{2\pi i/4} = e^{\pi i/2}$. Note $\rho^4 = 1$.

U_4	1	ρ	ρ^2	ρ^3
1	1	ρ	ρ^2	ρ^3
ρ	ρ	ρ^2	ρ^3	1
ρ^2	ρ^2	ρ^3	1	ρ
ρ^3	ρ^3	1	ρ	ρ^2

$\mathbb{Z}_4$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{0}$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$
$\bar{1}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{0}$
$\bar{2}$	$\bar{2}$	$\bar{3}$	$\bar{0}$	$\bar{1}$
$\bar{3}$	$\bar{3}$	$\bar{0}$	$\bar{1}$	$\bar{2}$

Note: The tables are the same if you make this correspondence

$1 \leftrightarrow \bar{0}$, $\rho \leftrightarrow \bar{1}$, $\rho^2 \leftrightarrow \bar{2}$, $\rho^3 \leftrightarrow \bar{3}$

(13) U_6 and U_8 are as in #2.

U_6 (Recall $\rho^6 = 1$ here)

1 is its own inverse (since $1 \cdot 1 = 1$)
ρ and ρ^5 are inverses (since $\rho \cdot \rho^5 = \rho^6 = 1$)
ρ^2 and ρ^4 are inverses (since $\rho^2 \rho^4 = \rho^6 = 1$)
ρ^3 is its own inverse (since $\rho^3 \rho^3 = \rho^6 = 1$)

U_8 (Recall $\rho^8 = 1$ here)

1 is its own inverse (since $1 \cdot 1 = 1$)
ρ and ρ^7 are inverses (since $\rho \cdot \rho^7 = \rho^8 = 1$)
ρ^2 and ρ^6 are inverses (since $\rho^2 \rho^6 = \rho^8 = 1$)
ρ^3 and ρ^5 are inverses (since $\rho^3 \cdot \rho^5 = \rho^8 = 1$)
ρ^4 is its own inverse (since $\rho^4 \cdot \rho^4 = \rho^8 = 1$)