Math 455
Homework # 1 - Groups

1. Compute the addition and multiplication tables for $\mathbb{Z}_3$ and $\mathbb{Z}_6$.

2. Calculate the elements of U_6 and U_8 and draw a picture of them.

3. Compute the multiplication table for U_4. Do you see any similarity between this table and the addition table of Z_4?

4. Determine whether or not $3\mathbb{Z} = \{3n \mid n \in \mathbb{Z}\}$ is a group under the operation $a * b = a + b$.

5. Let $\mathbb{R}^+$ denote the set of positive real numbers. Determine whether or not $\mathbb{R}^+$ is a group under the operation $a * b = \sqrt{ab}$.

6. Let $\mathbb{R}^*$ denote the set of nonzero real numbers. Determine whether or not $\mathbb{R}^*$ is a group under the operation $a * b = a/b$.

7. Let $G = \mathbb{R} \setminus \{-1\}$. Prove that G is a group under the operation $a * b = a + b + ab$ on G.

8. Let G be a group with operation $*$. We say that x is an idempotent element of G if $x * x = x$. Show that G has only one idempotent element.

9. Let G be a group where every element of the group is its own inverse. Prove that G is abelian.

10. Let G be an abelian group. Let $a, b \in G$. Prove by induction that $(a * b)^n = (a^n) * (b^n)$ for all positive integers n.

11. Compute the group table for D_6. Compute as much of the group table for D_8 as you have patience for.

12. Find the inverses of each of the elements of D_6. Find the inverses of the following elements in D_8: r, r^2, sr, and sr^2. What is the inverse of r in D_{2n}? What is the inverse of sr^i in D_{2n}?

13. Find the inverse of each element in U_6. Do the same thing for U_8.

14. Find the inverse of each element in $\mathbb{Z}_6$.