Math 4680 - Homework # 10

Cauchy Integral Formula

1. Evaluate the following integrals.

   (a) $\displaystyle\int_\gamma \frac{z^2}{z-1}\,dz$ where γ is the circle of radius 2, centered at 0, oriented counterclockwise.

   (b) $\displaystyle\int_\gamma \frac{\sin(z)}{z^2}\,dz$ where γ is the unit circle oriented clockwise.

   (c) $\displaystyle\int_\gamma \frac{z^2-1}{z^2+1}\,dz$ where γ is the circle of radius 2, centered at 0, oriented counterclockwise.

   (d) $\displaystyle\int_\gamma \frac{z^{10}+5z^3+1}{z^4}\,dz$ where γ is the square with vertices $-1-i$, $1-i$, $1+i$, $-1+i$, oriented counterclockwise.

   (e) $\displaystyle\int_\gamma \frac{1}{(z^2+z+1)^2}\,dz$ where γ is the circle $|z|=2$ oriented counterclockwise.

   (f) $\displaystyle\int_\gamma \frac{z}{(9+z^2)(z+i)^2}\,dz$ where γ is the circle $|z|=4$ oriented counterclockwise.

2. Let γ be the circle $|z|=3$ oriented counterclockwise. Define

$$g(w)=\int_\gamma \frac{2z^2-z-2}{z-w}\,dz$$

   for all w with $|w|\neq 3$.

   (a) Show that $g(2)=8\pi i$.

   (b) What is the value of $g(w)$ when $|w|>3$?

3. Suppose that f is analytic within and on a simple, closed smooth curve γ. Further suppose that $f(w)=0$ for all w on γ. Prove that $f(z)=0$ for all z inside of γ.