# Math 456

## Homework # 2 - Integral Domains

1. Calculate the elements of $\mathbb{Z}_n^\times$ where $n = 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14$.

2. Determine which of the following rings are integral domains:

   (a) $n\mathbb{Z}$ where $n \geq 2$

   (b) $\mathbb{Z} \times \mathbb{Z}$

   (c) $\mathbb{Z}_2 \times \mathbb{Z}_3$

   (d) $\mathbb{Z}_5$

   (e) $\mathbb{Z}_{106}$

   (f) $M_2(\mathbb{R})$

3. Let $R_1$ and $R_2$ be integral domains. Prove that $R_1 \times R_2$ is NOT an integral domain.

4. (a) Let $R$ be an integral domain with identity 1 and $S$ be a subring of $R$ satisfying $1 \in S$. Prove that $S$ is an integral domain. (b) What if $1 \notin S$ but $S$ is still a subring of $R$?

5. Let $R$ and $S$ be subdomains of an integral domain $T$. Prove that $R \cap S$ is a subdomain of $T$.

6. Let $R$ be a ring. We say that $x \in R$ is an **idempotent** of $R$ if $x \cdot x = x$. Show that if $R$ is an integral domain then it only has two idempotents.