# Math 455

## Homework # 2 - Subgroups

1. Is $\{1, s, sr, sr^2\}$ a subgroup of $D_6$?

2. Compute the order of every element in the following groups: $\mathbb{Z}_5$, $\mathbb{Z}_8$, $U_3$, and $D_6$.

3. Is $4\mathbb{Z} = \{4n \mid n \in \mathbb{Z}\}$ a subgroup of $\mathbb{Z}$ under addition? Prove or disprove.

4. Is $\{2^n \mid n \in \mathbb{Z}\}$ a subgroup of $\mathbb{Q}$ under addition? Prove or disprove.

5. Is $\mathbb{Z}^* = \mathbb{Z} \setminus \{0\}$ a subgroup of $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$ under multiplication? Prove or disprove.

6. Is $\{2^n \mid n \in \mathbb{Z}\}$ a subgroup of $\mathbb{Q}^* = \mathbb{Q} \setminus \{0\}$ under multiplication? Prove or disprove.

7. Describe the elements of the subgroup generated by 3 in $\mathbb{Z}$.

8. Describe the elements of the subgroup generated by 5 in $\mathbb{R}^*$.

9. Calculate the subgroup of $U_6$ generated by $e^{2\pi i/3}$.

10. Calculate the subgroup of $U_8$ generated by $e^{3\pi i/4}$.

11. Calculate the subgroup of $\mathbb{Z}_8$ generated by the element $\overline{2}$. Do the same thing for $\overline{4}$ and $\overline{5}$.

12. Compute the subgroup of $D_{2n}$ that is generated by $r$.

13. Let $H$ and $K$ be subgroups of an abelian group $G$. Prove that

$$HK = \{hk \mid h \in H \text{ and } k \in K\}$$

is a subgroup of $G$.

14. Let $G$ be an abelian group. Let $H = \{x \in G \mid x^2 = e\}$. Prove that $H$ is a subgroup of $G$.

15. Let $G$ be an abelian group. Let $H = \{x^2 \mid x \in G\}$. Prove that $H$ is a subgroup of $G$.

16. Let $G$ be a group. The center of $G$ is the set

$$Z(G) = \{x \in G \mid xy = yx \text{ for all } y \text{ in } G\}.$$

Prove that $Z(G)$ is a subgroup of $G$.

17. Let $G$ be a group. Let $H$ and $K$ be subgroups of $G$. Prove that $H \cap K$ is a subgroup of $G$.