

## Math 456

### Homework # 3 - Polynomial Rings

1. Find the sum and product of the given polynomials in the given polynomial ring.

(a)  $f(x) = \bar{2}x^2 + x$  and  $g(x) = x^2 + \bar{2}x + \bar{1}$  in  $\mathbb{Z}_3[x]$

(b)  $f(x) = x^3 + x^2 + x + \bar{1}$  and  $g(x) = x^2 + \bar{1}$  in  $\mathbb{Z}_2[x]$

2. List all of the polynomials in  $\mathbb{Z}_3[x]$  of degree less than or equal to one.

3. List all of the polynomials in  $\mathbb{Z}_2[x]$  of degree less than or equal to two.

4. Find all of the zeros of  $f(x) = x^2 + \bar{1}$  in  $\mathbb{Z}_2[x]$ .

5. Find all of the zeros of  $f(x) = x^2 + \bar{2}$  in  $\mathbb{Z}_3[x]$ .

6. Let  $R$  be an integral domain.

(a) If  $p(x), q(x) \in R[x]$  are nonzero elements of  $R[x]$ , then  $\deg(p(x)q(x)) = \deg(p(x)) + \deg(q(x))$ .

(b) Prove that  $R[x]$  is an integral domain.

(c) The units of  $R[x]$  are the units of  $R$ . That is,  $(R[x])^\times = R^\times$ .