

Math 4680 - Homework # 3

Some Topology

1. Let $z_0 \in \mathbb{C}$. Let $r \in \mathbb{C}$ with $r > 0$. Prove that the r -neighborhood

$$D(z_0; r) = \{z \mid |z - z_0| < r\}$$

is open.

2. For each of the following sets, prove (a) whether or not the set is open, and (b) whether or not the set is closed.

(a) $S = \{z \in \mathbb{C} \mid |z| < 2\}$

(b) $S = \{z \in \mathbb{C} \mid |z| \leq 1\}$

(c) $S = \{z \in \mathbb{C} \mid \text{Im}(z) > 0\}$

(d) $S = \{z \in \mathbb{C} \mid \text{Im}(z) \geq 0\}$

(e) $S = \{z \in \mathbb{C} \mid 2 \leq \text{Re}(z) \leq 3\}$

3. Prove the following.

(a) \mathbb{C} is open.

(b) The empty set \emptyset is open.

(c) \mathbb{C} is closed.

(d) The empty set \emptyset is closed.

(e) If $z_0 \in \mathbb{C}$, then the set $\{z_0\}$ is closed.

(f) If A and B are open sets in \mathbb{C} then $A \cap B$ is open.

(g) If A and B are closed sets in \mathbb{C} then $A \cup B$ is closed.