Math 455

Homework # 4 - Cyclic Groups

1. Suppose that $\phi : U_5 \to \mathbb{Z}_5$ is a homomorphism with $\phi(e^{2\pi i/5}) = \overline{2}$. Find all the values of ϕ and draw a picture of ϕ.

2. Is there an isomophism $\phi : U_8 \to \mathbb{Z}_8$ with $\phi(e^{\pi i/4}) = \overline{2}$? Explain why or why not.

3. Find all homomorphisms $\phi : \mathbb{Z} \to \mathbb{Z}$. Which ones are isomorphims?

4. Find all homomorphisms $\phi : \mathbb{U}_6 \to \mathbb{Z}_3$.

5. Find all homomorphisms $\phi : \mathbb{Z} \to \mathbb{Z}_4$.

6. Find all homomorphisms $\phi : \mathbb{Z}_8 \to \mathbb{Z}_6$.

7. Find all the subgroups of U_6.

8. Find all the subgroups of $\mathbb{Z}_8$.

9. Does there exist a cyclic group with exactly one generator? Does there exist a cyclic group with exactly two generators?

10. Find all the generators of U_6.

11. Find all the generators of $\mathbb{Z}_8$.

12. Let G be a group and $x \in G$. Prove that $\langle x \rangle = \langle x^{-1} \rangle$.

13. Prove that the set of rational numbers $\mathbb{Q}$ under addition is not a cyclic group. This is an example of an infinite abelian group that is not cyclic.

14. Let G be a group and let x be an element of G. Prove that the order of x equals the order of x^{-1}.

15. Find all homomorphisms $\phi : \mathbb{Z}_6 \to D_6$.

16. Are the following pairs of groups isomorphic? If so, find an isomorphism. If not, explain why no isomorphism exists.

(a) $\mathbb{R}$ and $\mathbb{Z}$

(b) U_5 and $\mathbb{Z}_5$

(c) D_8 and $\mathbb{Z}_8$

(d) $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ and $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$.