Math 465 - Homework # 5
Open and closed sets

1. Prove that $\mathbb{R}$ is an open set.

2. Prove that the empty set $\emptyset$ is an open set.

3. Let A and B be open subsets of $\mathbb{R}$.

    (a) Prove that $A \cap B$ is open.

    (b) Prove that $A \cup B$ is open.

4. Let A and B be closed subsets of $\mathbb{R}$.

    (a) Prove that $A \cap B$ is closed.

    (b) Prove that $A \cup B$ is closed.

5. Prove the following. Let a and b be real numbers.

    (a) Prove that $(-\infty, a)$ is an open set.

    (b) Prove that (b, ∞) is an open set.

    (c) If $a < b$, then $[a, b]$ is a closed set.

    (d) Prove that $[a, \infty)$ is closed.

    (e) Prove that $(-\infty, b]$ is closed.

6. Let $x_1, x_2, \ldots, x_n$ be real numbers. Let S be the finite set $S = \{x_1, x_2, \ldots, x_n\}$.

    (a) Prove that S is closed.

    (b) Prove that S has no limit points.

7. Give an example of an open set A and a limit point x of A such that $x \notin A$.

8. Give an example of an infinite family of open sets A_n such that $\cap_n A_n$ is not open.

9. Give an example of an infinite family of closed sets B_n such that $\cup_n B_n$ is not closed.

10. Let $D \subseteq \mathbb{R}$. Suppose that D is closed and bounded. Prove that $a = \inf(D)$ and $b = \sup(D)$ exist and $a \in D$ and $b \in D$.