Math 455

Homework # 6 - Cosets and Lagrange's Theorem

1. For the following groups G and subgroups H compute the left cosets and the right cosets. Are they equal?

(a) $G = \mathbb{Z}_{12}$ and $H = \langle \overline{4} \rangle$.

(b) $G = \mathbb{Z}$ and $H = 4\mathbb{Z}$

(c) $G = S_3$ and $H = \langle (1,2) \rangle$.

(d) $G = S_3$ and $H = \langle (1,2,3) \rangle$.

(e) $G = D_8$ and $H = \langle r \rangle$.

(f) $G = D_8$ and $H = \langle s \rangle$.

2. Let G be a group and H be a subgroup of G. Let $a, b \in G$. Prove that if $aH = bH$, then $Ha^{-1} = Hb^{-1}$.

3. Let G be a group where $|G| = pq$ where p and q are primes. Let H be a proper subgroup of G. Prove that H is cyclic.

4. Let G be a group with identity element e. Suppose that $|G| = n$. Prove that $x^n = e$ for all $x \in G$.